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# A Note on a Comparison of Methods for the Estimation of Weibull Distribution Parameters of Nwobi and Ugomma

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Felix Noyanim Nwobi and Chukwudi Anderson Ugomma published an article in 2014 titled *"A Comparison of Methods for the Estimation of Weibull Distribution Parameters"* (Nwobi and Ugomma [1]). The paper compared the performance of the following methods of model parameter estimation: graphical method (mean rank, median rank and symmetric cumulative distribution function *cdf*) and the analytical method (maximum likelihood estimate (*mle*), method of moments, and least squares estimate (*lse*) methods) in estimating the parameters of the two parameter Weibull distribution. Subsequently, we shall refer either the authors or the paper as NU. NU assessed the performance of the various estimation methods by comparing their mean square errors (*mse*) and using the Kolmogorov-Smirnov (*ks*) test. We have noted and outlined several analytical inconsistencies and other mistakes in various sections of the paper by NU and corrections have been given.

## Notes

- In Sub-section 2.1 of $NU$, the $cdf$ of the three parameter Weibull distribution in Equation (2.2) given by $F(x) = 1 - e^{-\frac{x-\nu}{\alpha}}$ is wrong.

- Under Sub-subsection 2.2.2 of $NU$ the likelihood function of the two parameter Weibull distribution in Equation (2.10) given by $L(x;\alpha,\beta) = \left(\frac{\beta}{\alpha}\right)\left(\frac{x_t}{\alpha}\right)^{n\beta-n}\sum_{t=1}^{n} x_t^{\beta-1} e^{-\sum_{t=1}^{n}\left(\frac{x_t}{\alpha}\right)^{\beta}}$ is wrong because $\left(\frac{\beta}{\alpha}\right)$ is a constant and is multiplied $n$ times, hence, should be raised to the power $n$; again the quantity $\left(\frac{x_t}{\alpha}\right)^{n\beta-n}$ is wrong because $x_t$ is a random variable not a constant and the quantity $\sum_{t=1}^{n} x_t^{\beta-1}$ is wrong under the likelihood function and should be $\prod_{t=1}^{n} x_t^{\beta-1}$ , consequently, Equations (2.11), (2.12), (2.13), (2.14), and (2.15) of $NU$ are all wrong.

- Under the method of moments in Sub-subsection 2.2.2 of $NU$ the analytical expression of the $kth$ raw moment of the two parameter Weibull distribution in Equation (2.17) given by $\mu_k = \left(\frac{1}{\alpha^k}\right)^{-\frac{k}{\beta}}\Gamma(1+\frac{k}{\beta})$ is wrong, consequently, Equations (2.18), (2.19), (2.20) and (2.21) are all wrong.

- Under the Kolmogorov-Smirnov test in Subsection 3.2 of $NU$, the mathematical expression of the *cdf* defined by $F(x) = \int_{\infty}^{x} f(y, \boldsymbol{\theta})dy$ is wrong because the lower support of the integral should be $-\infty$.

- The tabulated results in Table 2 of $NU$ was obtained using the simulated random variables that results from applying the **R** command

  ```
  "replicate(N,mean(rweibull(n,shape=0.54)))"
  ```

  in **R** and not with the return data set as was supposed. That is why the value of the shape parameter ($\beta$) stagger about the 0.54 value that was supplied in the shape argument of the **R** code above. Also, from the same **R** code $NU$ simulated random variables from the one-parameter Weibull distribution without any motivation; whereas, they were studying the two-parameter Weibull distribution.

- And lastly, $NU$ assessed the performance of the various methods of parameter estimation through *pdf* and *cdf* plots using the estimated shape and scale parameters from the various methods. As far as we can see, the plots did not reveal any significant difference in the methods as $NU$ claims because the curves are not distinguishable. $NU$ should have considered plotting the $mse$ of the various methods against the sample sizes and repeat same for $ks$.

## Some Corrections

The *cdf* of the three parameter Weibull distribution is given by $1 - e^{-\left(\frac{x-\nu}{\alpha}\right)^{\beta}}$.

*Proof.*

$$\begin{aligned}
F(x) &= \int_0^x \left(\frac{x-\nu}{\alpha}\right)^{\beta-1} e^{-\left(\frac{x-\nu}{\alpha}\right)^{\beta}} dx; \text{ setting } y = \left(\frac{x-\nu}{\alpha}\right), \text{ we have} \\
&= \frac{\beta}{\alpha} \int_0^{\frac{x-\nu}{\alpha}} y^{\beta-1} e^{-y^{\beta}} \alpha dy, \text{ and setting } z = y^{\beta}, \text{ we obtain} \\
&= \beta \int_0^{\left(\frac{x-\nu}{\alpha}\right)^{\beta}} \left[z^{\frac{1}{\beta}}\right]^{\beta-1} e^{-z} \frac{1}{\beta} z^{\frac{1}{\beta}-1} dz, \\
&= \int_0^{\left(\frac{x-\nu}{\alpha}\right)^{\beta}} e^{-z} dz, \text{ thus} \\
&= 1 - e^{-\left(\frac{x-\nu}{\alpha}\right)^{\beta}}.
\end{aligned}$$

□

The $mle$ of $\alpha$ and $\beta$ could be obtained as follows:

The likelihood equation is

$$\begin{aligned}
\mathcal{L}(\alpha,\beta|x) &= \prod_{t=1}^{n}\left(\frac{\beta}{\alpha}\right)\left(\frac{x_t}{\alpha}\right)^{\beta-1} e^{-\left(\frac{x_t}{\alpha}\right)^{\beta}} \\
&= \left(\frac{\beta}{\alpha}\right)^{n}\left(\frac{1}{\alpha}\right)^{n(\beta-1)}\prod_{t=1}^{n} x_t^{\beta-1}e^{-\left(\frac{x_t}{\alpha}\right)^{\beta}}, \text{ and the log-likelihood is given by} \\
\ln(\mathcal{L}(\alpha,\beta|x)) &= n\ln(\beta) - n\ln(\alpha) - n(\beta-1)\ln(\alpha) + (\beta-1)\sum_{t=1}^{n}\ln(x_t) - \sum_{t=1}^{n}\left(\frac{x_t}{\alpha}\right)^{\beta} \\
\frac{\partial \ln(\mathcal{L}(\alpha,\beta|x))}{\partial\alpha} &= -\frac{n}{\alpha} - \frac{n(\beta-1)}{\alpha} + \frac{\beta}{\alpha^{\beta+1}}\sum_{t=1}^{n} x_t^{\beta} \qquad (1) \\
\frac{\partial \ln(\mathcal{L}(\alpha,\beta|x))}{\partial\beta} &= \frac{n}{\beta} - n\ln(\alpha) + \sum_{t=1}^{n}\ln(x_t) - \frac{1}{\alpha^{\beta}}\sum_{t=1}^{n} x_t^{\beta}\ln\left(\frac{x_t}{\alpha}\right) \qquad (2)
\end{aligned}$$

setting Equations (1) and (2) to zero and solving for the parameters through certain non-linear numerical optimization method yields the $mle$ of $\alpha$ and $\beta$.

The $kth$ raw moment of the two parameter Weibull distribution is given by $\mu_k = \alpha^k\Gamma(\frac{k}{\beta}+1)$.

*Proof.*

$$\begin{aligned}
\mu_k &= \int_0^{\infty} x^k\frac{\beta}{\alpha}\left(\frac{x}{\alpha}\right)^{\beta-1} e^{-\left(\frac{x}{\alpha}\right)^{\beta}}dx, \\
&= \frac{\beta}{\alpha^{\beta}}\int_0^{\infty} x^k x^{\beta-1}e^{-\left(\frac{x}{\alpha}\right)^{\beta}}dx; \text{ setting } y = \left(\frac{x}{\alpha}\right)^{\beta}, \text{ we have} \\
&= \frac{\beta}{\alpha^{\beta}}\int_0^{\infty}\left[\alpha y^{\frac{1}{\beta}}\right]^{k+\beta-1} e^{-y}\frac{\alpha}{\beta}y^{\frac{1}{\beta}-1}dy, \\
&= \alpha^k\int_0^{\infty} y^{\frac{k}{\beta}}e^{-y}dy, \text{ and} \\
&= \alpha^k\Gamma(\frac{k}{\beta}+1).
\end{aligned}$$

□

## References

[1] Nwobi FN, Ugomma CA. A Comparison of Methods for the Estimation of Weibull Distribution Parameters. Metodološki Zvezki. 2014 Jan 1;11(1):65-78.