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Cyclic Codes Over $F_2+uF_2+vF_2+uvF_2+u^2F_2+v^2F_2+u^2v^2F_2$

Mohammad A.M. Hamoda[1], Abd Elkarim Nagi[2*]

[1] Department of Mathematics, Al-Aqsa University, 4051, Gaza, Palestine.

[2*] Department of Mathematics, Al-Aqsa University, 4051, Gaza, Palestine.

Abstract.

In this paper, we study the structure of cyclic codes of length n over the ring $F_2+uF_2+vF_2+uvF_2+u^2F_2+v^2F_2+u^2v^2F_2$. We characterize a set of generators for each cyclic code. We study the rank for these codes, and we find their minimal spanning sets. Lee weights and Gray maps for these codes over $F_2+uF_2+vF_2+uvF_2+u^2F_2+v^2F_2+u^2v^2F_2$ are also studied.

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1 Introduction

Codes over finite rings have been studied in the early 1970's. A great deal of attention has been given to codes over finite rings from 1990, because of their new role in algebraic coding theory and their successful applications. The structure of cyclic codes over rings of odd length n has been discussed in Bonnecaze and Udaya [5], Conway and Sloan [8], and Grasst [9]. Wolfmann [12], and other papers [10], [6] presented a complete structure of cyclic codes over Z_4 of odd length. Calderbank [7] has shown that certain nonlinear binary codes with excellent error-correcting capabilities can be identified as images of linear codes over Z_4 under the Gray map. Cyclic codes of arbitrary length over finite chain rings in different contexts have been studied by numerous authors [4],[1], and [2]. The structure of linear codes and cyclic codes over the ring $F_2+uF_2+vF_2+uvF_2$, where $u^2=v^2=0$ and $uv=vu$ was studied by Yildis and Karadeniz [13], [14]. Furthermore, in [13], cyclic codes over that ring were studied by them. They showed that some good binary codes are obtained as the images of cyclic codes over $F_2+uF_2+vF_2+uvF_2$ under two Gray maps that are defined. In this paper we focus on cyclic codes over the ring $F_2+uF_2+vF_2+uvF_2+u^2F_2+v^2F_2+u^2v^2F_2$ which is not a chain ring. We use ideas from group rings to characterize the cyclic codes over the ring $F_2+uF_2+vF_2+uvF_2+u^2F_2+v^2F_2+u^2v^2F_2$. The remaining part of this paper is organized as follows:

In section 2, we analyze the structure of the ring and then construct the generators for cyclic codes over the ring $F_2+uF_2+vF_2+uvF_2+u^2F_2+v^2F_2+u^2v^2F_2$. In section 3, we find the minimal spanning sets for these codes. In section 4, we study the Lee weight and the Gray map of linear codes over the ring $F_2+uF_2+vF_2+uvF_2+u^2F_2+v^2F_2+u^2v^2F_2$.

2 Generators for cyclic codes over $F_2+uF_2+vF_2+uvF_2+u^2F_2+v^2F_2+u^2v^2F_2$

The ring $R = F_2 + uF_2 + vF_2 + uvF_2 + u^2F_2 + v^2F_2 + u^2v^2F_2$ is defined as a ring of characteristic two subject to the restrictions $u^3 = v^3 = 0$ and $uv = vu$. The ring R is $local$, $Frobenius ring$ that is not chain or principal.

Definition 2.1 *A linear code C of length n over the ring $R = F_2 + uF_2 + vF_2 + uvF_2 + u^2F_2 + v^2F_2 + u^2v^2F_2$ is an $R-$submodule of $R^n = (F_2 + uF_2 + vF_2 + uvF_2 + u^2F_2 + v^2F_2 + u^2v^2F_2)^n$.*

Definition 2.2 *A free module C over the ring R is a module with a basis (a linearly independent spanning set for C).*

Definition 2.3 *A linear code of length n over R is cyclic if it is invariant under the automorphism σ which is defined as*

$$\sigma(c_0, c_1, \ldots, c_{n-1}) = (c_{n-1}, c_0, \ldots, c_{n-2}).$$

The following proposition is the analogy of well-known result for cyclic codes over finite fields. The proof is also similar, so we omit it here. [See ch.7 in [3]].

Proposition 2.1 *A subset C of R^n is a linear cyclic code of length n if and only if its polynomial representation is an ideal of $R[x]/(x^n - 1)$.*

Definition 2.4 [3] *An ideal I of a ring R is called a principal ideal if there exists an element $g \in I$ such that $I =< g >$, where*

$$< g >= \{rg : r \in R\}.$$

The element g is called *a generator* of I and I is said to be generated by g.

Lemma 2.2 *The ring $R_n = R[x]/(x^n - 1)$ is not a principal ideal ring.*

Proof. Let RG be any group ring, where $G =< g \mid g^n = 1 >$ is cyclic group of order n. Define $\Upsilon : RG \to R$ by $\Upsilon(a_0 + a_1 g + \ldots + a_{n-1} g^{n-1}) = a_0 + a_1 + \ldots + a_{n-1}$. Υ is an epimorphism. Now, in our case we can define $\Gamma : R_n \to R$ as

$$\Gamma(b_0 + b_1 x + \ldots + b_{n-1} x^{n-1}) = b_0 + b_1 + \ldots + b_{n-1}.$$

Consider the ideal $I =< u, v >$ of R. I is not a principal ideal. Let $J = \Gamma^{-1}(I)$. Obviously J is an ideal in R_n. Since Γ is an epimorphism, so

$$\Gamma(J) = \Gamma(\Gamma^{-1}(I)) = I.$$

So, if J were principal ideal, then its homomorphic image under Γ would have to be principal as well. But $\Gamma(J) = I$ is not a principal ideal. Hence $R_n = R[x]/(x^n - 1)$ is not a principal ideal ring. $\square$

Definition 2.5 [3] *A local ring is a ring that has a unique maximal ideal.*

The following two theorems are analogous to Theorem 2.5 and Theorem 2.7 in [13].

Theorem 2.3 *The ring R_n is not local ring when $n = 2^k.m$, where $m > 1$ is an odd integer.*

Lemma 2.4 *The ring R_n is a local ring when $n = 2^k$ for some $k \in \mathbb{N}$.*

Following the results in [11], R_n is isomorphic to the group ring $(F_2 + uF_2 + vF_2 + uvF_2 + u^2F_2 + v^2F_2 + u^2v^2F_2)G$ where $G =< g : g^n = 1 >$ is the cyclic group of order n.

The isomorphism is quite obvious in that we map $c_0 + c_1x + \ldots + c_{n-1}x^{n-1}$ to $c_0 + c_1g + \ldots + c_{n-1}g^{n-1}$. Now, for $(F_2 + uF_2 + vF_2 + uvF_2 + u^2F_2 + v^2F_2 + u^2v^2F_2)G$ we have a nice representation by matrices. For results in this section we refer to [11]. To every element in $(F_2 + uF_2 + vF_2 + uvF_2 + u^2F_2 + v^2F_2 + u^2v^2F_2)G$, and hence in R_n, corresponds a circulant matrix in the form :

$$\sigma(c_0 + c_1x + \ldots + c_{n-1}x^{n-1}) = \begin{bmatrix} c_0 & c_1 & c_2 & \ldots & & \ldots c_{n-1} \\ c_{n-1} & c_0 & c_1 & \ldots & & \ldots c_{n-2} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ c_1 & c_2 & c_3 & \ldots & & \ldots c_0 \end{bmatrix}$$

We focus at the units and zero divisors in R_n. The determinant function det is a multiplicative map from matrices over a commutative ring R to the ring R. The results in [11] for general group rings imply that $\alpha \in R_n$ is a unit if and only if $det(\sigma(\alpha))$ is a unit in $F_2 + uF_2 + vF_2 + uvF_2 + u^2F_2 + v^2F_2 + u^2v^2F_2$. So, we have the following corollary:

Corollary 2.5 *An element* $\alpha = c_0 + c_1x + \ldots + c_{n-1}x^{n-1}$ *is a unit in* R_n *if and only if* $det(\sigma(\alpha))$ *is a unit in* $F_2 + uF_2 + vF_2 + uvF_2 + u^2F_2 + v^2F_2 + u^2v^2F_2$.

Now, we will characterize the cyclic codes over $F_2 + uF_2 + vF_2 + uvF_2 + u^2F_2 + v^2F_2 + u^2v^2F_2$:

Define $\psi_1 : F_2 + uF_2 + vF_2 + uvF_2 + u^2F_2 + v^2F_2 + u^2v^2F_2 \to F_2 + uF_2 + vF_2 + uvF_2$ by

$\psi_1(a + ub + vc + uvd + u^2e + v^2f + u^2v^2t) = a + ub + vc + uvd$. ψ_1 is a ring homomorphism that can be extended to the homomorphism

$$\phi_1 : (F_2 + uF_2 + vF_2 + uvF_2 + u^2F_2 + v^2F_2 + u^2v^2F_2)[x]/(x^n - 1) \to (F_2 + uF_2 + vF_2 + uvF_2)[x]/(x^n - 1)$$

by

$$\phi_1(c_0 + c_1x + \ldots + c_{n-1}x^{n-1}) = \psi_1(c_0) + \psi_1(c_1)x + \ldots + \psi_1(c_{n-1})x^{n-1}.$$

Note that $ker(\psi_1) = u^2F_2 + v^2F_2 + u^2v^2F_2$.

Define $\psi_2 : F_2 + uF_2 + vF_2 + uvF_2 \to F_2 + uF_2$ by $\psi_2(a + ub + vc + uvd) = a + ub$. ψ_2 is a ring homomorphism that can be extended to the homomorphism

$\phi_2 : (F_2 + uF_2 + vF_2 + uvF_2)[x]/(x^n - 1) \to (F_2 + uF_2)[x]/(x^n - 1)$ by

$$\phi_2(c_0 + c_1x + \ldots + c_{n-1}x^{n-1}) = \psi_2(c_0) + \psi_2(c_1)x + \ldots + \psi_2(c_{n-1})x^{n-1}.$$

Not that $ker(\psi_2) = vF_2 + uvF_2 = v(F_2 + uF_2)$. Now let us assume that C is a cyclic code over $F_2 + uF_2 + vF_2 + uvF_2 + u^2F_2 + v^2F_2 + u^2v^2F_2$. Restrict ϕ_1 onto C. Since C is invariant under the cyclic shift, so is $\phi_1(C)$. This means that $Im(C)$ is cyclic over $F_2 + uF_2 + vF_2 + uvF_2$. So by results in [13], [1], we have $Im(C) =< g_2(x) + up_2(x) + vg_3(x) + uvp_3(x), ua_2(x) + vg_4(x) + uvp_4(x), vg_1(x) + uvp_1(x), uva_1(x) >$ where $g_i(x), p_i(x), a_i(x)$ are polynomials in $F_2[x]/(x^n - 1)$ with $a_2(x) | g_2(x) | x^n - 1$, $a_2(x) | p_2(x)\dfrac{x^n - 1}{g_2(x)}$, $a_1(x) | g_1(x) | x^n - 1$, $a_1(x) | p_1(x)\dfrac{x^n - 1}{g_1(x)}$. Also, $ker(\phi_1) = v^2 < g_1(x) + u^2p_1(x) >$. Thus we have proved the following main theorem for characterization of cyclic codes over $F_2 + uF_2 + vF_2 + uvF_2 + u^2F_2 + v^2F_2 + u^2v^2F_2$:

Theorem 2.6 *Let* C *be a cyclic code over* $F_2+uF_2+vF_2+uvF_2+u^2F_2+v^2F_2+u^2v^2F_2$ *of length* n*. Then* C *is an ideal in* R_n *that can be represented as*

$$C=< g_2(x)+up_2(x)+vg_3(x)+uvp_3(x)+u^2l_1(x)+v^2s_1(x)+u^2v^2l_2(x), ua_2(x)+vg_4(x)+ uvp_4(x)+u^2l_3(x)+v^2s_2(x)+u^2v^2l_4(x), vg_5(x)+uvp_5(x)+u^2l_5(x)+v^2s_3(x)+u^2v^2l_6(x), uvp_6(x)+ u^2l)_7(x)+v^2s_4(x)+u^2v^2l_8(x), u^2l_9(x)+v^2s_5(x)+u^2v^2l_{10}(x), v^2g_1(x)+u^2v^2p_1(x), u^2v^2a_1(x)>$$

where $g_i(x), p_i(x), a_i(x), l_i(x), s_i(x)$ are polynomials in $F_2[x]/(x_n-1)$ with

$$a_i(x)\,|\,g_i(x)\,|\,(x^n-1),\ a_i(x)\,|\,p_i(x)\frac{x^n-1}{g_i(x)},\ a_i(x)\,|\,l_i(x)\frac{x^n-1}{s_i(x)}$$

3 Ranks and minimal spanning sets of cyclic codes over $F_2+uF_2+vF_2+uvF_2+u^2F_2+v^2F_2+u^2v^2F_2$

Following Abualrub and Siap [1, p.p. 274], the parameters of an $(F_2+uF_2+vF_2+uvF_2+u^2F_2+v^2F_2+u^2v^2F_2)-$code C are k_1,k_2,k_3,k_4,k_5,k_6 and d, where k_1 refers to the free part and k_2,k_3,k_4,k_5,k_6 refer to non free part (u, v, uv, u^2, v^2 and u^2v^2 multiple generators of C), and minimum distance d. Such codes are often referred to as a code of type $\{k_1,k_2,k_3,k_4,k_5,k_6\}$.

Definition 3.1 *The rank of a code* C *over* $F_2+uF_2+vF_2+uvF_2+u^2F_2+v^2F_2+u^2v^2F_2$ *of type* $\{k_1,k_2,k_3,k_4,k_5,k_6\}$*, denoted by* $rank(C)$*, is the minimum number of generators of* C*.*

Definition 3.2 *The free rank of* C *over* $F_2+uF_2+vF_2+uvF_2+u^2F_2+v^2F_2+u^2v^2F_2$ *of type* $\{k_1,k_2,k_3,k_4,k_5,k_6\}$ *, denoted by f-* $rank(C)$ *, is the maximum of the ranks of* $(F_2+uF_2+vF_2+uvF_2+u^2F_2+v^2F_2+u^2v^2F_2)-$*free submodules of* C*.*

Theorem 3.1 *Let* C *be a cyclic code of length* n *over*

$$F_2+uF_2+vF_2+uvF_2+u^2F_2+v^2F_2+u^2v^2F_2$$

If

$$C=< g_2(x)+up_2(x)+vg_3(x)+uvp_3(x)+u^2l_1(x)+v^2s_1(x)+u^2v^2l_2(x), ua_2(x)+ vg_4(x)+uvp_4(x)+u^2l_3(x)+v^2s_2(x)+u^2v^2l_4(x), vg_5(x)+uvp_5(x)+u^2l_5(x)+v^2s_3(x)+ u^2v^2l_6(x), uvp_6(x)+u^2l)_7(x)+v^2s_4(x)+u^2v^2l_8(x), u^2l_9(x)+v^2s_5(x)+u^2v^2l_{10}(x), v^2g_1(x)+ u^2v^2p_1(x), u^2v^2a_1(x)>,$$

$\deg(g_2(x)+up_2(x)+vg_3(x)+uvp_3(x)+u^2l_1(x)+v^2s_1(x)+u^2v^2l_2(x))=r_1$, $\deg(ua_2(x)+vg_4(x)+uvp_4(x)+u^2l_3(x)+v^2s_2(x)+u^2v^2l_4(x))=r_2$, $\deg(vg_5(x)+uvp_5(x)+u^2l_5(x)+v^2s_3(x)+u^2v^2l_6(x))=r_3$, $\deg(uvp_6(x)+u^2l)_7(x)+v^2s_4(x)+u^2v^2l_8(x))=r_4$, $\deg(u^2l_9(x)+v^2s_5(x)+u^2v^2l_{10}(x))=r_5$, $\deg(v^2g_1(x)+u^2v^2p_1(x))=r_6$ and $\deg(u^2v^2a_1(x))=r_7$.

Then C *has rank*$(C)=n-r_7$ *and a minimal spanning set given by:*

$$\beta = \Big\{(g_2(x)+up_2(x)+vg_3(x)+uvp_3(x)+u^2l_1(x)+v^2s_1(x)+u^2v^2l_2(x)), x(g_2(x)+up_2(x)+vg_3(x)+uvp_3(x)+u^2l_1(x)+v^2s_1(x)+u^2v^2l_2(x)), \ldots, x^{n-r_1-1}(g_2(x)+up_2(x)+vg_3(x)+uvp_3(x)+u^2l_1(x)+v^2s_1(x)+u^2v^2l_2(x)), (ua_2(x)+vg_4(x)+uvp_4(x)+u^2l_3(x)+v^2s_2(x)+u^2v^2l_4(x)), x(ua_2(x)+vg_4(x)+uvp_4(x)+u^2l_3(x)+v^2s_2(x)+u^2v^2l_4(x)), \ldots, x^{r_1-r_2-1}(ua_2(x)+vg_4(x)+uvp_4(x)+u^2l_3(x)+v^2s_2(x)+u^2v^2l_4(x)), (vg_5(x)+uvp_5(x)+u^2l_5(x)+v^2s_3(x)+u^2v^2l_6(x)), x(vg_5(x)+uvp_5(x)+u^2l_5(x)+v^2s_3(x)+u^2v^2l_6(x)), \ldots, x^{r_2-r_3-1}(vg_5(x)+uvp_5(x)+u^2l_5(x)+v^2s_3(x)+u^2v^2l_6(x)), \ldots\ldots\ldots\ldots, (u^2v^2a_1(x)), x(u^2v^2a_1(x)), x^{r_6-r_7-1}(u^2v^2a_1(x))\Big\}$$

Proof. Suppose

$$C = < g_2(x)+up_2(x)+vg_3(x)+uvp_3(x)+u^2l_1(x)+v^2s_1(x)+u^2v^2l_2(x), ua_2(x)+vg_4(x)+uvp_4(x)+u^2l_3(x)+v^2s_2(x)+u^2v^2l_4(x), vg_5(x)+uvp_5(x)+u^2l_5(x)+v^2s_3(x)+u^2v^2l_6(x), uvp_6(x)+u^2l)_7(x)+v^2s_4(x)+u^2v^2l_8(x), u^2l_9(x)+v^2s_5(x)+u^2v^2l_{10}(x), v^2g_1(x)+u^2v^2p_1(x), u^2v^2a_1(x) >.$$

Since the lowest degree polynomial in C is $u^2v^2a_1(x)$, then it suffices to show that β spans

$$\gamma = \Big\{(g_2(x)+up_2(x)+vg_3(x)+uvp_3(x)+u^2l_1(x)+v^2s_1(x)+u^2v^2l_2(x)), x(g_2(x)+up_2(x)+vg_3(x)+uvp_3(x)+u^2l_1(x)+v^2s_1(x)+u^2v^2l_2(x)), \ldots, x^{n-r_1-1}(g_2(x)+up_2(x)+vg_3(x)+uvp_3(x)+u^2l_1(x)+v^2s_1(x)+u^2v^2l_2(x)), (ua_2(x)+vg_4(x)+uvp_4(x)+u^2l_3(x)+v^2s_2(x)+u^2v^2l_4(x)), x(ua_2(x)+vg_4(x)+uvp_4(x)+u^2l_3(x)+v^2s_2(x)+u^2v^2l_4(x)), \ldots, x^{r_1-r_2-1}(ua_2(x)+vg_4(x)+uvp_4(x)+u^2l_3(x)+v^2s_2(x)+u^2v^2l_4(x)), (vg_5(x)+uvp_5(x)+u^2l_5(x)+v^2s_3(x)+u^2v^2l_6(x)), x(vg_5(x)+uvp_5(x)+u^2l_5(x)+v^2s_3(x)+u^2v^2l_6(x)), \ldots, x^{r_2-r_3-1}(vg_5(x)+uvp_5(x)+u^2l_5(x)+v^2s_3(x)+u^2v^2l_6(x)), \ldots\ldots\ldots\ldots, (u^2v^2a_1(x)), x(u^2v^2a_1(x)), x^{n-r_7-1}(u^2v^2a_1(x))\Big\}$$

Similarly, it suffices to show that $u^2v^2x^{r_6-r_7}a_1(x) \in \text{span}\ \gamma.$

$u^2v^2x^{r_6-r_7}a_1(x) = u^2v^2(g_2(x)+up_2(x)+vg_3(x)+uvp_3(x)+u^2l_1(x)+v^2s_1(x)+u^2v^2l_2(x)) + u^2v^2m(x)$, where $u^2v^2m(x)$ is a polynomial in C of degree less than r_6.

Since any polynomial in C must have degree greater than or equal to $\deg(u^2v^2a_1(x)) = r_7$, then $r_7 \le deg(m(x)) < r_6$. Hence $u^2v^2m(x) = \alpha_0u^2v^2a_1(x) + \alpha_1xu^2v^2a_1(x) + \ldots + \alpha_{r_6-r_7-1}x^{r_6-r_7-1}u^2v^2a_1(x)$. Hence, β is a generating set. By comparing the coefficients, we get that none of the elements in β is a linear combination of the others. Therefore β is a minimal generating set. □

4 Lee weight and the Gray map of linear codes over $F_2+uF_2+vF_2+uvF_2$ $+u^2F_2+v^2F_2+u^2v^2F_2$

In this section we will define the Lee weight and the Gray map for linear codes over $F_2+uF_2+vF_2+uvF_2+u^2F_2+v^2F_2+u^2v^2F_2$.

Definition 4.1 *Let* $\phi:(F_2+uF_2+vF_2+uvF_2+u^2F_2+v^2F_2+u^2v^2F_2)^n \to F_2^{7n}$ *be the map given by:*

$$\phi(a+ub+vc+uvd+u^2e+v^2f+u^2v^2t)=(a+b+c+d+e+f+t,\ f+t, e+t, d+t, c+d, b+t, t).$$

We note from the definition that ϕ is a linear map that takes a linear code over $F_2+uF_2+vF_2+uvF_2+u^2F_2+v^2F_2+u^2v^2F_2$ of length n to a binary code of length $7n$. By using this map, we can define the Lee weight w_L as follows:

Definition 4.2 *For any element* $a+ub+vc+uvd+u^2e+v^2f+u^2v^2t \in F_2+uF_2+vF_2+uvF_2+u^2F_2+v^2F_2+u^2v^2F_2$, *we define*

$$w_L(a+ub+vc+uvd+u^2e+v^2f+u^2v^2t)=w_H(a+b+c+d+e+f+t,\ f+t, e+t, d+t, c+d, b+t, t)$$

, where w_H denotes the ordinary Hamming weight for binary codes.

Note that ϕ extends to a distance preserving isometry:

$$\phi:((F_2+uF_2+vF_2+uvF_2+u^2F_2+v^2F_2+u^2v^2F_2)^n, Lee\ weight) \to (F_2^{7n}, Hamming weight)$$

By observing the linearity of the map ϕ, we obtain the following theorem:

Theorem 4.1 *If* C *is a linear code over* $F_2+uF_2+vF_2+uvF_2+u^2F_2+v^2F_2+u^2v^2F_2$ *of length* n, *size* 2^k *and minimum Lee weight* d , *then* $\phi(C)$ *is a binary linear code with parameters* $\{7n,k,d\}$.

5 Conclusion

In this paper, we studied the structure of cyclic codes of length n over the ring $F_2+uF_2+vF_2+uvF_2+u^2F_2+v^2F_2+u^2v^2F_2$. The rank, the minimal spanning set and Gray images of this family of codes are studies as well.

Questions appear about constacyclic codes over the ring $F_2+uF_2+vF_2+uvF_2+u^2F_2+v^2F_2+u^2v^2F_2$. Also it will be interesting to construct a decoding algorithm for these codes that work for any length n.

References

[1] T. Abualrub and I. Siap, Cyclic codes over the rings Z_2+uZ_2 and $Z_2+uZ_2+u^2Z_2$, Designs Codes and Cryptography, 42(2007) pp. 273-287.

[2] M. Al-Ashker and M Hamoudeh, Cyclic codes over $Z_2+uZ_2+u^2Z_2+\ldots+u^{k-1}Z_2$, Turk. .J.Math, 35(2011), pp. 737-749.

[3] G. Bini and F. Flamini, Finite commutative rings and their applications, University of Michigan, Universita degli Studi Roma Tre, U.S.A and Italy, (2002).

[4] T. Blackford, Cyclic codes over Z_4 of oddly even length, Discrete Applied Mathematics, 128(2003), pp. 27-46.

[5] A. Bonnecaze and P. Udaya, Cyclic codes and self-dual codes over F_2+uF_2, IEEE Trans. Inform. Theory, 45(1999), pp. 1250-1255.

[6] A. Calderbank, E. Rains, P. Shor, J. Neil, and N.J.A. Sloane, Quantum error corrections via

codes over GF(4), IEEE Transactions on Information Theory, 4(1998), pp. 1369-1387.

[7] A. Calderbank and N.J.A. Sloane, Modular and P-adic cyclic codes, Des. Codes Crypt, 37(1995), pp. 21-35.

[8] J.H. Conway and N.J.A. Sloan, Self-dual Codes over the Integers modulo 4, Journal of Combinatorial Theory Series 62 (1993) pp. 30-45.

[9] M. Grasst, Linear Codes Bounds, IAKS, Fakultat Fur Informatik, Universitat Karlsurhe (TH), Availiable Online at http://www.codetables.de/(grassl@ira.uka.de).

[10] A.R. Hammons, The Z_4 -Linearity of Kerdock, Preparata, Goethals, and Related Codes, IEEE Transactions on Information Theory. 40(1994), pp. 301-319.

[11] T. Hurley, Group rings and rings of matrices, Inter.J.Pure App.Math, 3(2006), pp. 319-335.

[12] J. Wolfmann, Binary Images of Cyclic Codes over Z_4, Transactions on Information Theory, 47(2001), pp. 1773-1779.

[13] B. Yildiz. and S. Karadeniz, Cyclic Codes over $F_2 + uF_2 + vF_2 + uvF_2$, Des. Codes Crypt., 55(2010), pp. 1-11.

[14] B. Yildiz and S. Karadeniz, Linear Codes over $F_2 + uF_2 + vF_2 + uvF_2$, Des. Codes Crypt., 54(2010), pp. 61-81.