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On δ-Small Pseudo Projective Modules

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Abstract

In this paper we give three new concepts, a concepts of δ-Small pseudo projective module, δ-small quasi projective and δ-small pseudo stable module, these concepts are generalization of Pseudo Projective modules, quasi projective module and pseudo stable module respectively we will study these concepts and give some results.

Keywords: δ-Small pseudo projective module; δ-small quasi projective module; δ-small pseudo stable module.

حول المقاساتالاسقاطية الزائفة من النمط δالصغير

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الخلاصة

في هذا البحث نعطي ثلاث مفاهيم جديده ومفمهوم المقاس الاسقاطي الزائف من النوع δ الصغير ومفمهوم المقاس الاسقاطي الظاهري من النوع δ الصغير, ومفمهوم المقاس الاسقاطي شبه المستقر من النوع δ الصغير . هذه المفاهيم هي تعاميم لمفاهيم المقاس الاسقاطي الزائف والمقاس الظاهريوالمقاس الاسقاطي شبه المستقر على التوالي. سندرس هذه المفاهيم ونعطي بعض النتائج.

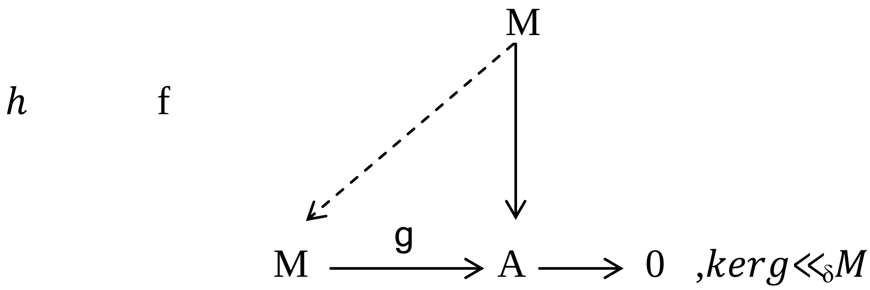
1. Introduction

All rings in this paper are commutative rings with identity, and all modules are unitary left R-modules. Let M be an R-module. A submodule A of M is called essential if every nonzero submodule of M has anonzero intersection with A, [1]. If A is a submodule of M, then the annihilator of $A$is defined as $Ann(A) = \{r \in R \mid rA = 0\}$, [1].If M is R-module, then $Z(M)$={x $\in M$: $Ann(A) \subseteq_e$ R} is called the singular submodule of M. If $Z(M) = M$, then$M$ is called the singular module, [1].A submodule A of M is called c-singular if M/A is a singular module [2]. A submodule N of a module M is called δ-small in M if for every c-singular submodule L of M, the equality $N + L = M$ implies $L = M$, [2].A non-zero module M is δ-hollow, if every proper submodule is δ-small in M, [3].Let P and M be an R-module.Then P is called M-projective, if for any epimorphism $g: M \to N$ and any homomorphism $f: P \to N$, there exists a homomorphism $h: P \to M$ such that $g \circ h = f$,[4].An R-module M is called quasi projective module if M is M-projective module, [5].An R-module M is called pseudo projective if for any given module A and epimorphisms$f: M \to A$and $g: M \to A$, there exists an h in $End\ (M)$ such that $f = g \circ h$, [6]. A submodule N of an R-module M is said to be pseudo stable if for every two epimorphisms$f, g: M \to A$, with $N \subseteq Ker\ g \cap Ker\ f$ there exists h in $End(M)$ such that $f = g \circ h$, then $h\,(N) \subseteq N$, [6]. A submodule A of an R-module M is called fully invariant if f (A) $\subseteq$ A for all f $\subseteq$ End(M). If every submodule of

M is fully invariant, then M is called a duo-module, [7].Let M and N bean R-modules. An epimorphism $g: M \rightarrow N$ is said to be δ-small epimorphism, if $kerg \ll_\delta M$, [8].

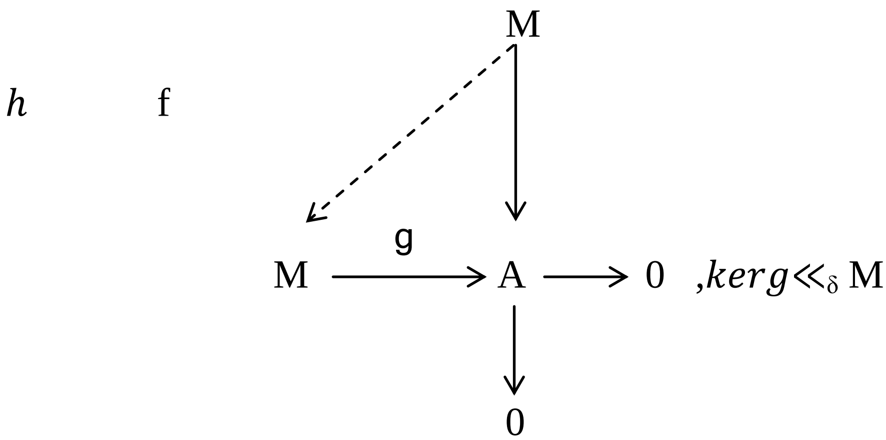
2. Preliminary Notes

Definition (2.1): An R-module M is said to be δ-small quasi projective if for any given module A, any δ-small epimorphism $g: M \rightarrow A$, any homomorphism $f: M \rightarrow A$ can be lifted to anendomorphism h of Msuch that the following diagram is commutative;



i.e. $g \circ h = f$.

Definition (2.2): An R-module M is said δ-small pseudo projective module if for any module A, with δ-small epimorphism$g : M \rightarrow A$ and epimorphism$f : M \rightarrow A$, there exists an $h$in$End\,(M)$ such that thefollowing diagram is commutative:



i.e.g ∘ h = f.

Definition (2.3): A submodule N of an R-module M is said to be δ-small pseudo stable module if for any epimorphism$f: M \rightarrow A$ and any δ-small epimorphism $g: M \rightarrow A$with $N \subseteq Ker\ g \cap Kerf$, there exists h in $End(M)$ such that $f = g \circ h$, then $h\,(N) \subseteq N$.

Examples (2.4):

1. Z_2as$Z$-module is δ-small quasi projective but not projective.
2. Qas$Z$-module is not quasi projective.
3. Z_6as$Z$-module is δ-small pseudo projective but not projective.
4.
5.

3. Main Results

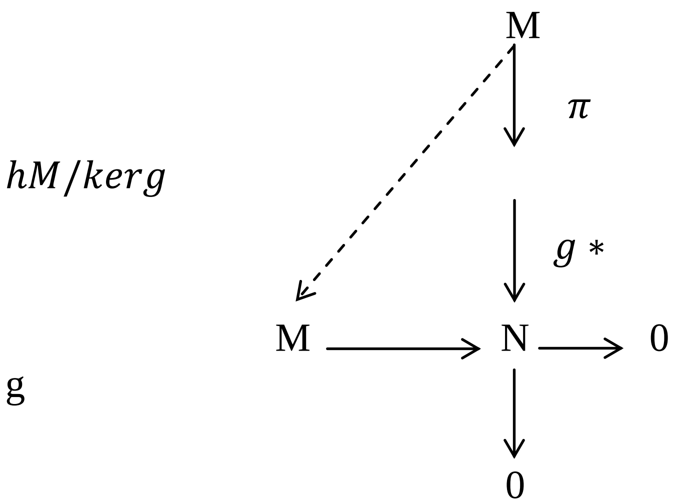
Proposition (3.1): Let$M$ be aδ-hollow module the following are equivalent:

(i) $M$is δ-small pseudo projective.

(ii)$M$is pseudo projective.

Proof:(i) $\Rightarrow$ (ii) Let M be a δ-small pseudo projective module and A be any module.and$f, g : M \to A$ are epimorphisms. Since M is δ-hollow module, then $kerg$ is δ-small submodule of M, So $g$is δ-small epimorphism. Thus, by (i) there exist ahomomorphism h in $End\ (M)$ such that $f = g \circ h$, therefore M ispseudo projective.
(ii) $\Rightarrow$ (i) clear from definition.

Proposition (3.2): Let M be a δ-small pseudo projective module and $g: M \to N$ be a δ-small epimorphism then there exists a homomorphism$h$ in $End(M)$ such that$Ker\ g = Ker(g \circ h)$ isδ-small pseudo stable under h.
Proof: Since $g: M \to N$ is δ-small epimorphism, then $M/kerg \cong N$. Let $g*: M/kerg \to N$ be an isomorphism, Let $\pi: M \to M/Kerg$be the natural epimorphism. Since M is δ-small pseudo projective, then there exists a homomorphism h in $End(M)$ such thatthe following diagram commutative:



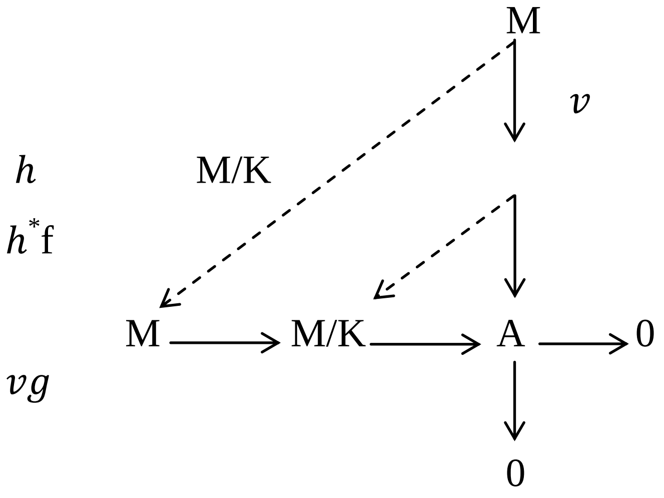
i.e. $g * \circ\ \pi = g \circ h$.
Now, let $x \in kerg \Rightarrow x \in ker\pi \Rightarrow \pi(x) = 0$

$$\Rightarrow g * \circ\ \pi\ (x) = 0 \Rightarrow g \circ h\ (x) = 0 \Rightarrow x \in kerg \circ h \Rightarrow kerg \subseteq kerg \circ h$$

On the other hand if $y \in kerg \circ h \Rightarrow g \circ h\ (y) = 0 \Rightarrow g * \circ\ \pi(y) = 0 \Rightarrow \pi(y) \in kerg *\ and\ since\ g *\ is\ one - one \Rightarrow \pi(y) = 0 \Rightarrow y \in ker\pi = kerg. Therefore\ kerg = kerg \circ h$.
Now, let $z \in kerg \Rightarrow g(z) = 0 \Rightarrow g \circ h(z) = 0 \Rightarrow g(h(z) - z) = 0 \Rightarrow h(z) - z \in kerg \Rightarrow h(z) \in kerg \Rightarrow h(kerg) \subseteq kerg$.

Proposition (3.3): Let M be a δ-small pseudo projective module, K be a δ-smallsubmodule of M if K is stable under $End\ (M)$, then M/K is δ-small pseudo projective.

Proof: Let $f : M/K \to A$ be an epimorphism, $g: M/K \to A$ be a δ-small epimorphism and $v : M \to M/K$ be the natural epimorphism where A is any R- module. Since M isδ-small pseudo projective Then there exist $h$in $End\ (M)$ such thatthe following diagramcommutatives:



i.e.$g \circ v \circ h = f \circ v$.Define $h^*: M/K \to M/K$ by$h^*(x+K) = h(x) + K$. Its clear that h^* is well define and homomorphism.

Now $h^* \circ v = v \circ h \Rightarrow g \circ h^* \circ v = g \circ v \circ h \Rightarrow g \circ h^* \circ v = f \circ v$

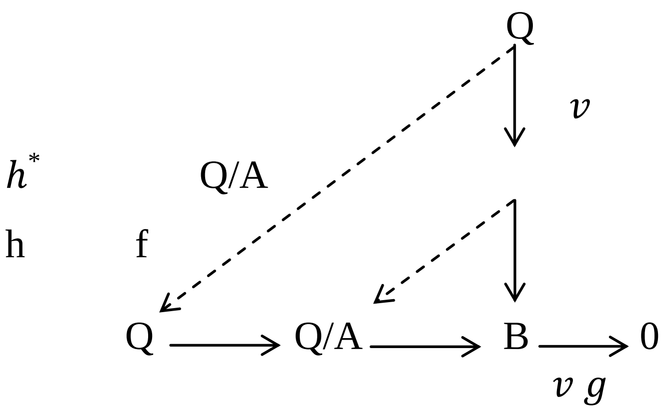
But v is onto $\Rightarrow g \circ h^* = f$, thus M/K is δ-small pseudo projective.

Proposition (3.4): Let M be a δ-small pseudo projective module and $g: M \to N$ be a δ-smallepimorphism, then N is δ-small pseudo projective.

Proof:Since $g: M \to N$ is an epimorphism, then $M/kerg \cong N$, but $M/kerg$is δ-small pseudo projective (Proposition 3.3), therefore N is δ-small pseudo projective.

Proposition (3.5): If T is a δ-small pseudo stable submodule of a δ-small quasi projectivemodule Q and A is a submodule of T, then T/A is a δ-small pseudo stable submoduleof Q/A.

Proof: Let $f : Q/A \to B$ be epimorphism, $g : Q/A \to B$ be a δ-small epimorphism with $T/A \subseteq kerf \cap kerg$ such that there exists h in $End(Q/A)$satisfying $f = g \circ h$. Let $v : Q \to Q/A$ be the natural epimorphism, then since Q is δ-small quasi projective, there exists a homomorphism h^* in $End(Q)$ such that the following diagramcommutatives:

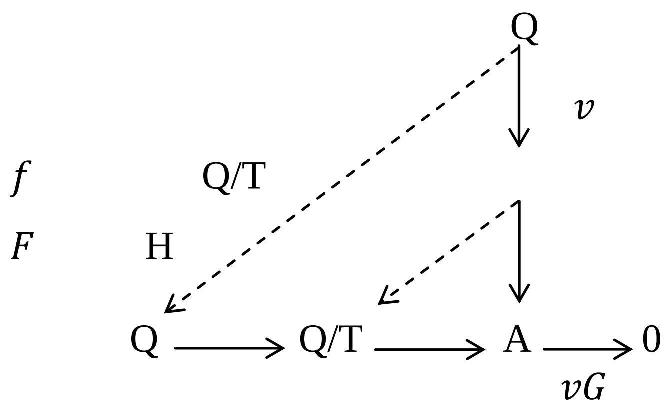


i.e. $h \circ v = v \circ h^* \Rightarrow f \circ v = g \circ h \circ v = g \circ v \circ h^*$.
Since we have $f \circ v(T) = f(T/A) = 0$
and $g \circ v(T) = g(T/A) = 0$.
Therefore $T \subseteq ker f \circ v \cap ker g \circ v$. But T is δ-small pseudo stable and hence $h^*(T) \subseteq T$. it follows that $h(T/A) = h \circ v(T) = v \circ h^*(T) \subseteq v(T) = T/A$.

Thus T/A is δ-small pseudo stable submodule of Q/A.

Proposition (3.6): Let Q be a δ-small quasi projective module and T be a δ-small pseudo stable submodule of Q. If C containing T, is not a δ-small pseudo stable submodule of$Q$, then C/T is not a δ-small pseudo stable submodule of Q/T.

Proof: Let $h : Q \to A$ be an epimorphism, $g : Q \to A$ be a δ-small epimorphism and Let Cbe not δ-small pseudo stable in Q, then there exists $f \in End(Q), C \subseteq kerg \cap kerh$ with following diagram is commutative:



i.e. $h = g \circ f$ such that $f(x) \notin C$for some $x \in C$.Let $v : Q \to Q/T$ be the natural epimorphism.

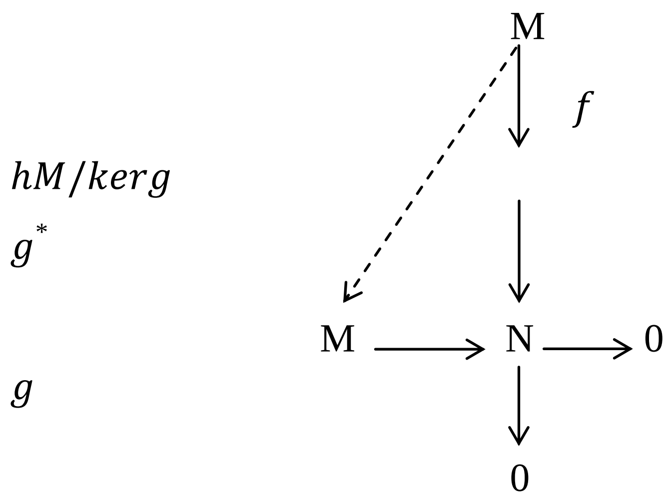
Define $F : Q/T \to Q/T$by $F(q + T) = f(q) + T$. so$F \circ v = v \circ f$.

Its clear that F is well define and homomorphism. And $F(x + T) = f(x) + T \notin C/T$. Now since $T \subseteq kerg \cap kerh$, then there exists G and H in $Hom\,(Q/T, A)$ such that $g = G \circ v$ and $h = H \circ v$. since$g$ and h are epimorphism, so G and H are epimorphism.

Now $h = g \circ f \Rightarrow H \circ v = G \circ v \circ f = G \circ F \circ v$. And since v is epimorphism therefore $H = G \circ F. So\ we\ have\ H(C/T) = H \circ v(C) = h(C) = 0$, similary $G(C/T) = 0$. Therefore $C/T \subseteq kerH \cap kerG$. But$F(x + T) \notin C/T$. And hence C/T is not δ-small pseudo stable.

Proposition (3.7): Let Mbe aδ-small pseudo projective module, $g : M \to N$ is any δ-smallepimorphism, then $Kerg$ is a δ-small pseudo stable submodule of M.

Proof: Since $g : M \to N$ be a δ-small epimorphism, then $M/kerg \cong N$, Let g^*: $M/kerg \to N$ be an isomorphism, $f : M \to M/kerg$ be the natural epimorphism, then $kerg \subseteq kerg \cap ker\, g^* \circ f$. since M is δ-small pseudo projective, there exists h in $End\,(M)$ such that the following diagram commutatives:

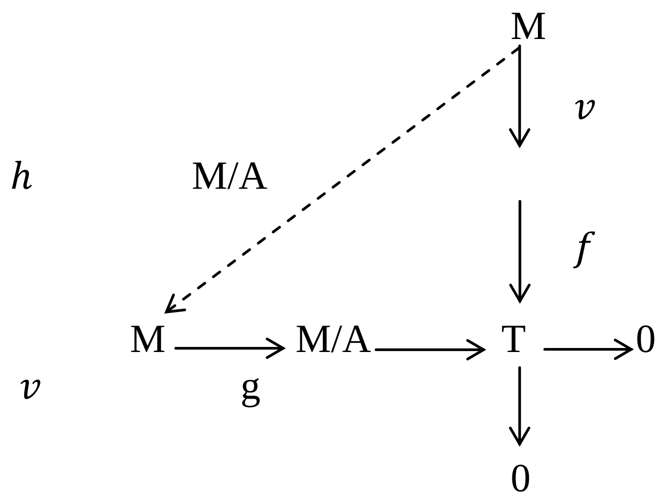


i.e. $g^{*} \circ f = g \circ h$.

Now,if $h(kerg) \not\subset kerg$, then there exists $x \in kerg$ such that $h(x) \in h(kerg)$ and $h(x) \notin kerg$.
Now $0 \neq g \circ h(x) = g^{*} \circ f(x) = 0$, wich is a contradiction, since $kerg \subseteq ker\, g^{*} \circ f$, thus $kerg$ is δ-small pseudo stable.

Proposition (3.8): Let M be a δ-small pseudo projective module, A and B be invariant submodulesof M. Then$A \cap B$ is a δ-small pseudo stable submodule of M if either Aor B is δ-small in M.

Proof: Let A be δ-small in M, $g: M/A \to T$ be any δ-small epimorphism, $f: M/A \to T$. be any epimorphism, where T is any R-module and $v : M \to M/A$ be the natural epimorphism. Then $\cap B \subseteq Ker\, g \circ v \cap Ker\, f \circ v$. Since M is δ-small pseudo projective, then there exists h in $End\,(M)$ such that the following diagram commutatives:



i.e. $f \circ v = g \circ v \circ h$.now since $h(A \cap B) \subseteq h(A) \subseteq A$ and $h\,(A \cap B) \subseteq h(B) \subseteq B$ since A and B are invariant submodules, Thus $h(A \cap B) \subseteq A \cap B$. Hence $A \cap B$ is δ-small pseudo stable.

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