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Optimal path of a moving service vehicle on network with probabilistic demands

Abdelaziz Foul

Department of Statistics and Operations Research, College of Science, King Saud University.

Abstract. In this paper, I examine twofold problem. The first one is concerned with finding the optimal location of a single facility in a network with demands randomly distributed over the edges. The second problem is about determining the optimal path between two specified nodes of the network of a moving vehicle that continuously interacts with randomly distributed requests for service over the edges. The problems are investigated using different performance measures and probability distributions of the demands.

Keywords: Facility location, Moving Service Vehicle, Random demand.

1Introduction:

Facility location has attracted much research in discrete and continuous optimization over nearly four decades. Investigators have focused on both algorithms and formulations in diverse settings. Facility location analysis refers to the modeling, formulation, and solution of a class of problems that can best be described as locating facilities in some given space. The expressions deployment, positioning, and locating are frequently used as synonyms. The location decisions must often be made considering different types of performance measures. Choices for the best location(s) differ for various types of objectives. For example for a company that wants to build a warehouse for its retailers, it may be important to find a location that minimizes the sum of the distances from the warehouse to the retailers. However, for the location of an emergency facility such as a fire station, the most suitable objective could be to minimize the maximum distance from the facility to the demand points in order for the fire station to respond quickly enough to the farthest point. Another example might be the location of a waste incinerator for a local municipality. Residents might want that the facility be located as far as possible from residential areas, while the municipality wants it to be close enough to transport the waste. In that case, an objective that maximizes the minimum distance of the facility from the residential areas would be more appropriate. Several models were presented to summarize the core components of the location problem in literature. Erkut et al. [7] has provided an illustrative study of location models containing *Continuous*, *Discrete* and *Network* Location models. This study states that in case of continuous location model, facilities can be located in some *d*-dimensional space while discrete location model shows that the facilities can be located at some specified points. On the other hand, network location model states that the facilities can be located on network.

They have also considered about forbidden zones, which represents restricted sites that cannot be candidate site for a facility. Labbe [13] have presented a voting approach to solve the obnoxious facility location problem on network. Labbe presented a comparison between the anti Condorcet points and anti-median points. Karkazis et al. [11] has proposed an algorithm for location of facilities causing atmospheric pollution in plane. The objective of this algorithm was to minimize the sum-weighted risk factors for each vertex summed over all possible wind directions. Giannikos [9] have presented a multi objective programming discrete model for locating treatment sites and routing of hazardous waste. Ben-Moshe et al [2] has proposed an algorithm for k-facilities, n-demand node and m regions. The objective of this algorithm was to maximize the minimum distance between demand nodes and facility. Cappanera [5] has proposed a model known as Obnoxious Facility Location and Routing (OFLR) model. He has implemented this using the Branch and Bound method. Chabini [6] have provided a study of all to one dynamic shortest paths problem. Chabini's algorithm has proven an optimal run time complexity that equals to the complexity of problem.

Depending on the application being modeled, the facilities and demand points may be nodes in a network or points in a planar region. Facility analysis involves as well the problem of determining a path of a moving service vehicle, which during its journey provides service to a set of demand points. Research work on moving vehicle on networks or the plane with deterministic or random demands is relatively very scarce. In the plane, Sherali and Kim [5] have introduced a new class of problems involving the determination of an optimal constrained path for a moving service vehicle that interacts with a set of fixed existing facilities. Using weighted-distance related cost function, they have analyzed both the total cost and the average cost problems. Later, Kim and Choi[8] extended the model of Sherali and Kim[19] to a larger class that includes a general cost structure. Foul [8] has determined an optimal straight-line route of a moving facility on the plane with random demand points. The problem of finding optimal paths on networks with deterministic demands have been investigated in ([1], [3], [4], and [12]).

In this paper, we focus on two fold problem. The first one is concerned with finding the optimal location of a single facility in a network with demands randomly distributed over the edges. The second problem is about finding the optimal travel path for a service vehicle, which moves between two specified nodes through a network and interacts with a number of requests for service that are generated randomly over the edges. Servicing the random demands is performed over all instants of time during the travel period. Consider for example, the case of determining a route in a transportation network for a patrol car maintaining radio contacts with a number of users and it unknown which one will request service. The problems are investigated using different performance measures and probability distributions of the demands.

The remaining of the paper is organized as follows. In Section 2, the problem is analyzed and main results are described. Illustrative examples are provided.

2 Analysis:

2.1 Location of a single facility:

Let G = (N, E) be an undirected network, where N the set of nodes with, $N = \{1,2,..,n\}$ and

Ea set of undirected edges. The length of any edge (i, j) is denoted by $l(i, j)$and $d(i, j)$ is defined as the shortest distance between nodes $i, j \epsilon N$.Let w_{ij} a positive weight associated with each edge $(i, j) \in E$ and $\{Y_{ij} = ; (i, j) \in E\}$ be a set of independent randomly distributed demands over the edges $\{ (i , j): (i, j) \in E \}$. $f_{Y_{ij}}(y)$ is the density function and $F_{Y_{ij}}(y)$ is the cumulative distribution function of $Y_{ij}, (i, j) \in E$.

The problem we address in this section is of locating a single facility by considering the three following objectives functions:

- ***Minisum Problem:*** *Minimizes the weighted sum of the expected distances between the facility and the random demands distributed over the edges.*
- ***Minimax Problem:*** *Minimizes the maximum weighted expected distances between the facility and the random demands distributed over the edges.*
- ***Maximin Problem:*** *Maximizes the minimum weighted expected distances between the facility and the random demands distributed over the edges.*

Let $x \in E$, denotes the single facility. The problems can be stated respectively as:

$$min_{x \in E}(F_1(x) = \sum_{(i,j) \in E} w_{ij}\, E[d(x, Y_{ij})) \qquad (P_1)$$

$$\min_{x \in E}(F_2(x) = \max_{(i,j) \in E}\{ w_{ij}\, E[d(x , Y_{ij})]\} \qquad (P_2)$$

$$\max_{x \in E}(F_3(x) = \min_{(i,j) \in E}\{w_{ij}\, E[d(x, Y_{ij})]\} \qquad (P_3)$$

where $E[.]$ represents expected value of a random variable.

Definition 1 A point $x^*_{(p,q)}$on an edge(p, q) of $E$is a local optimum location, if for every $x \in (p, q)$,

$$F_1(x^*_{(p,q)}) \leq F_1(x) \text{ for problem}(P_1),$$
$$F_2(x^*_{(p,q)}) \leq F_2(x) \text{for problem}(P_2)$$

and $F_3(x^*_{(p,q)}) \geq F_3(x)$for problem$(P_3)$, respectively.

Definition 2 A point $x * \in E$ is global optimum location, if for every $x \in E$,

$$F_1(x^*) \leq F_1(x) \text{for problem}(P_1),$$
$$F_2(x^*) \leq F_2(x) \text{for problem}(P_2)$$

and $F_3(x^*) \geq F_3(x)$for problem(P_3) , respectively.

The objectives functions of problems (P_k) , $k = 1,2,3$ dependent on the expected value $E[d(x, Y_{ij})]$ and by definition, $[d(x, Y_{ij}) = \int_0^{l(i,j)} d(x, y) f_{Y_{ij}}(y) dy$. Therefore in order to express the objectives functions $F_k(x)$, $k = 1,2,3,$ as functions of x and y, we need to express explicitly $d(x, y)$as a function of $x$and y. Let's consider that $x$is in some edge (p, q) of E.We have two cases :

Case 1$(x, y) = (p, q)$. (see Figure 1)

In this case, x and y belong to the same edge, therefore $d(x, y) = |x - y|$ and

$$E[d(x, Y_{ij})] = \int_0^{l(i,j)} d(x, y) f_{Y_{ij}}(y) dy = \int_0^{l(i,j)} |x - y| f_{Y_{ij}}(y) dy$$

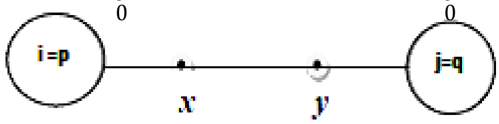

Figure 1.

Case 2$(x, y) \neq (p, q)$. (see Figure 2)

In this case, and according to Figure 2, the expression of $d(x, y)$ is given by

$$d(x, y) = min \begin{Bmatrix} x + y + d(p, i) \\ x - y + l(i, j + d(p, j) \\ -x + y + l(p, q) + d(q, i) \\ -x - y + l(p, q) + l(i, j) + d(q, j) \end{Bmatrix}$$

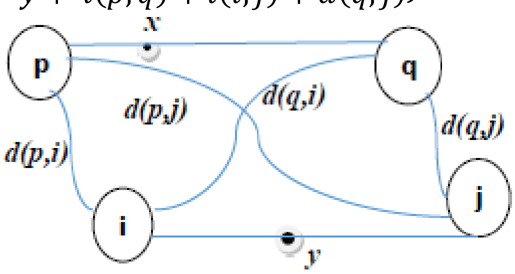

Figure 2.

$d(x, y)$can be written as

$$d(x, y) = min\{y + c_1(x) , -y + c_2(x) , y + c_3(x) , -y + c_4(x)\} \quad (1)$$

Where $c_1(x) = x + d(p, i)$, $c_2(x) = x + l(i, j + d(p, j)$,

$$c_3(x) = -x + l(p, q) + d(q, i) \ , \ c_4(x) = -x + l(p, q) + l(i, j) + d(q, j)$$

Taking into consideration that

$$\min\{c_1(x), c_3(x)\} = \min\{x + d(p,i), -x + l(p,q) + d(q,i)\} = d(x,i)$$

and $\min\{c_2(x), c_4(x)\} = \min\{x + l(i,j) + d(p,j), -x + l(p,q) + l(i,j) + (q,j)\}$

$$= \min\{x + d(p,j), -x + l(p,q) + d(q,j)\} + l(i,j) = d(x,j) + l(i,j)$$

From (1), $d(x,y)$ can then be written as

$$d(x,y) = min\{y + d(x,i), -y + d(x,j) + l(i,j)\}$$

or $d(x,y) = \begin{cases} y + d(x,i) & , \quad 0 \le y \le \bar{y} \\ -y + d(x,j) + l(i,j) & , \quad \bar{y} \le y \le l(i,j) \end{cases}$

where $\bar{y} = \frac{l(i,j)+d(x,j)-d(x,i)}{2}$. Clearly, $\bar{y}$ satisfies $0 \le \bar{y} \le l(i,j)$.

Hence $E[d(x,Y_{ij})] = \int_0^{l(i,j)} d(x,y) f_{Y_{ij}}(y)dy =$

$$= \int_0^{\bar{y}} (y + d(x,i)) f_{Y_{ij}}(y)dy + \int_{\bar{y}}^{l(i,j)} (-y + d(x,j) + l(i,j)) f_{Y_{ij}}(y)dy \qquad (2)$$

To integrate, we need to specify a distribution for the random variable Y_{ij}. Due to the difficulty to obtain an analytical form of the objective function $F_k(x), k = 1,2,3$ of problem $(P_k), k = 1,2,3$. In what follows, we will solve the problem $(P_k), k = 1,2,3$ numerically by considering some probability distributions of the random variable Y_{ij}.

Case when Y_{ij} follows the uniform distribution over $[0, l_{ij}]$, $(i,j) \in E$:

The probability density function is

$$f_{Y_{ij}}(y) = \begin{cases} \frac{1}{l(i,j)} & , \quad 0 \le y \le l(i,j) \\ 0 & , \quad otherwise \end{cases}$$

and from (2), we will have $E[d(x,Y_{ij})] = \int_0^{l(i,j)} d(x,y) f_{Y_{ij}}(y)dy =$

$\int_0^{\bar{y}} (y + d(x,i)) f_{Y_{ij}}(y)dy + \int_{\bar{y}}^{l(i,j)} (-y + d(x,j) + l(i,j)) f_{Y_{ij}}(y)dy =$

$$= \frac{[(l(i,j) + d(x,i) + d(x,j))^2 - 2(d(x,i)^2 + d(x,j)^2)]}{4l(i,j)}$$

In this case, the objective functions $F_k(x), k = 1,2,3$ can then be written as

$$F_1(x) = \sum_{\substack{(i,j)\in E, \\ (i,j)\neq(p,q)}} w_{ij} \frac{\left[\left(l(i,j)+d(x,i)+d(x,j)\right)^2 - 2\left(d(x,i)^2 + d(x,j)^2\right)\right]}{4l(i,j)} + w_{pq}\left(x^2 - xl(p.q) + \frac{l(p,q)^2}{2}\right).$$

$$F_2(x) = max\left\{max_{\substack{(i,j)\in E,\\(i,j)\neq(p,q)}}\left\{w_{ij}\frac{\left[\left(l(i,j)+d(x,i)+d(x,j)\right)^2-2\left(d(x,i)^2+d(x,j)^2\right)\right]}{4l(i,j)}\right\},\ w_{pq}(x^2-\right.$$
xlp.q+lp,q22).

$$F_3(x) = min\left\{min_{\substack{(i,j)\in E,\\(i,j)\neq(p,q)}}\left\{w_{ij}\frac{\left[\left(l(i,j)+d(x,i)+d(x,j)\right)^2-2\left(d(x,i)^2+d(x,j)^2\right)\right]}{4l(i,j)}\right\},\ w_{pq}(x^2-\right.$$
xlp.q+lp,q22).

Case when Y_{ij} follows the exponential distribution with parameter $\boldsymbol{\lambda_{ij}}, (\boldsymbol{i},\boldsymbol{j}) \in \boldsymbol{E}$:

The probability density function is

$$f_{Y_{ij}}(y) = \begin{cases}\lambda_{ij}\, e^{-\lambda_{ij}\, y} & , \quad 0 \le y \le +\infty \\ 0 & , \quad otherwise\end{cases}$$

In this case, the objective functions $F_i(x)$, $i = 1,2,3$,are given by

$$F_1(x) = \frac{1}{\lambda_{ij}{}^2}\sum_{\substack{(i,j)\in E,\\(i,j)\neq(p,q)}} w_{ij}\,[\lambda_{ij}\left(x-d(x,j)\right)e^{-\lambda_{ij}\,l(i,j)} - \left(\lambda_{ij}\,l(i,j)+2\right)e^{-\lambda_{ij}\frac{l(i,j)+d(x,j)-d(x,i)}{2}} +$$
λijdx,i+1eλijli,j+dx,j−dx,i2]+(2e−λijx+λijx−1).

$$F_2(x) =$$
$$max\left\{max_{\substack{(i,j)\in E,\\(i,j)\neq(p,q)}}\left\{w_{ij}\,[\lambda_{ij}\left(x-d(x,j)\right)e^{-\lambda_{ij}\,l(i,j)} - \left(\lambda_{ij}\,l(i,j)+2\right)e^{-\lambda_{ij}\frac{l(i,j)+d(x,j)-d(x,i)}{2}} +\right.\right.$$
λijdx,i+1eλijli,j+dx,j−dx,i2], (2e−λijx+λijx−1).

$$F_3(x) =$$
$$min\left\{min_{\substack{(i,j)\in E,\\(i,j)\neq(p,q)}}\left\{w_{ij}\,[\lambda_{ij}\left(x-d(x,j)\right)e^{-\lambda_{ij}\,l(i,j)} - \left(\lambda_{ij}\,l(i,j)+2\right)e^{-\lambda_{ij}\frac{l(i,j)+d(x,j)-d(x,i)}{2}} +\right.\right.$$
λijdx,i+1eλijli,j+dx,j−dx,i2], (2e−λijx+λijx−1).

Due to the complicated forms of the objectives functions, finding closed forms of the optimal solution of problem (P_k) , $k = 1,2,3$ is quite an unreachable task. For illustration, we give an example.

Example 1

Let us consider the network in Fig. 3. The number next to each edge (i,j) in the network represents the length $l(i,j)$. Y_{ij} is taken to be uniformly distributed over the interval $[0, l(i,j)]$ for every$(i,j) \in E$ and $w_{ij} = 1, (i,j) \in E$.

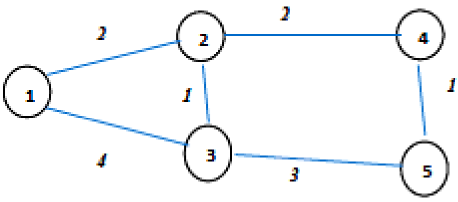

Figure 3.

An algorithm for finding the global optimum of an undirected graph G can be simply described as follows. (Apply the algorithm for each problem $P_k, k = 1,2,3.$).

Step 1 : For each edge (p, q) of E. find the local optimum$x^*_{(p,q)}$.

Step 2 : Among all the local optimums $x^*_{(p,q)}$, $(p, q) \in E$, choose the one with the smallest$F_k\left(x^*_{(p,q)}\right)$for $k = 1,2$ and greatest $F_k\left(x^*_{(p,q)}\right)$for $k = 3$. That local optimum is also the global optimum $x^*$of G.

Mathematica software was used to find the local optimums for each problem which are illustrated in the following tables:

$Edge\ (p, q)$	(1,2)	(1,3)	(2,3)	(2,4)	(3,5)	(4,5)
$x^*_{(p,q)}$	2	0	0	0	0	0
$F_1(x^*_{(p,q)})$	9.85	17.41	9.85	9.85	11.18	13.85

$Edge\ (p, q)$	(1,2)	(1,3)	(2,3)	(2,4)	(3,5)	(4,5)
$x^*_{(p,q)}$	2	4	0	0.031	0.75	0
$F_2(x^*_{(p,q)})$	2.5	3.25	2.5	2.47	2.75	4.43

$Edge\ (p, q)$	(1,2)	(1,3)	(2,3)	(2,4)	(3,5)	(4,5)
$x^*_{(p,q)}$	2	4	0.5	0	0	0.5
$F_3(x^*_{(p,q)})$	0.5	0.5	0.25	0.5	0.5	0.26

Summary of results:

- The global optimum location of problem P_1 is at node 2 with objective function value equal to 9.85.
- The global optimum location of problem P_2 is at node 2 with objective function value equal to 2.5.
- The global optimum location of problem P_3 is at node 2 or node 3 with objective function value equal to 0.5.

2.2 Location of a moving facility:

Let G = (N, E) be an undirected network, where $N = \{1,2,..,n\}$ is the set of nodes and E is a set of undirected edges. The length of any edge (i,j) is denoted by $l(i,j)$ and $d(i,j)$ is defined as the shortest distance between nodes $i, j\epsilon N$.

Let $\{Y_{ij} = ; (i,j) \in E\}$ be a set of independent randomly distributed demands over edges $\{(i,j): (i,j) \in E\}$. $f_{Y_{ij}}(y)$ is the density function and $F_{Y_{ij}}(y)$ is the cumulative distribution function of $Y_{ij}, (i,j) \in E$. Let w_{ij} a positive weight associated with each edge $(i,j) \in E$, and $\{S, D\}$ two specified nodes of N.Suppose that there is a vehicle moving in the network along a path at constant velocity *v*, starting from some origin *S* and arriving at a destination located at *D*.

The problem we address here is to determine the optimal path from *S* to *D* to this moving vehicle that continuously gives service over some time framework *T* to requests generated randomly over the edges. The problem is studied by considering three objectives functions as follows

- *minimizes the weighted sum of the expected travel times between the moving facility and the random demands distributed over the edges.*
- *minimizes the maximum weighted expected travel times between the moving facility and the random demands distributed over the edges.*
- *maximizes the minimum weighted expected travel times between the moving facility and the random demands distributed over the edges.*

If we denote by

X(t) : Position of the moving facility at time *t* .

P : Set of all paths from *S* to *D*.

p : a path of the set *P*.

The problems can be stated as:

$$\min_{p\in P}(F_1(x) = \sum_{(i,j)\in E}\int_{t\in T_p} w_{ij}\, E[d(X(t)\,,Y_{ij})]\, dt \tag{3}$$

$$\min_{p\in P}(F_2(x) = \max_{(i,j)\in E}\{\int_{t\in T_p} w_{ij}\, E\left[d(X(t)\,,Y_{ij})\right] dt\} \tag{4}$$

$$\max_{p\in P}(F_3(x) = \min_{(i,j)\in E}\{\int_{t\in T_p} w_{ij}\, E[d(X(t)\,,Y_{ij})]\, dt\} \tag{5}$$

whereT_p represents the time spent by the moving facility to travel from S to D along the path p. Since $T_p = \sum_{(k,l)\in p} T_p(k,l)$, where$T_p(k,l)$ is the time spent by the moving facility to travel from node $k$to node $l$along the path p, then

$$\int_{t\in T_p} w_{ij}\, E[d(X(t)\,,Y_{ij})]\, dt = \sum_{(k,l)\in p}\int_{t\in T_p(k,l)} w_{ij}\, E\left[d(X(t)\,,Y_{ij})\right]dt$$

Let $x = X(t)$ and suppose that facility is moving at constant speed v = 1 that is:

$v = \frac{dx}{dt} = 1 \qquad\qquad dx = dt \quad \Leftrightarrow$

By using a change of variable from $t$to$x$, we have

$$E\left[d(X(t),Y_{ij})\right] = \int_0^{l(i,j)} d(x,y) f_{Y_{ij}}(y)dy$$

and therefore Problems (3) – (5) become

$$\min_{p\in P}\sum_{(k,l)\in p}\int_0^{l(k,l)}\left[\sum_{(i,j)\in E} w_{ij}\int_0^{l(i,j)} d(x,y) f_{Y_{ij}}(y)dy\right]dx \tag{6}$$

$$\min_{p\in P}\sum_{(k,l)\in p}\int_0^{l(k,l)}\max_{(i,j)\in E}\{\left(\int_0^{l(i,j)} w_{ij}\, d(x,y) f_{Y_{pq}}(y)dy\right)dx\} \tag{7}$$

$$\max_{p\in P}\sum_{(k,l)\in p}\int_0^{l(k,l)}\min_{(i,j)\in E}\{\left(\int_0^{l(i,j)} w_{ij}\, d(x,y) f_{Y_{pq}}(y)dy\right)dx\,\} \tag{8}$$

Consider problem (6). The following steps can be similarly applied to problems (7) and (8).

Let$f_{(k,l)}(x) = \sum_{(i,j)\in E} w_{ij}\int_0^{l(i,j)} d(x,y) f_{Y_{ij}}(y)dy$, for $x \in (k,l)$,

and let $L(k,l) = \int_0^{l(k,l)} f_{(k,l)}(x)dx$. Then Problem (6) can then be written as

$$\min_{p\in P}\sum_{(k,l)\in p} L(k,l) \tag{9}$$

If $L(k,l)$ is considered as the new length of edge (k,l), $(k,l)\epsilon E$, then problem (9) represents the shortest path problem from node S to node D.

The lengths $L(k,l), (k,l)\epsilon E$depend on the distance $d(x,y)$, and according to section 2.1, are given by

$$L(k,l) = \int_0^{l(k,l)} \Bigg[\sum_{\substack{(i,j)\in E\\(i,j)\neq(k,l)}} w_{ij} \left(\int_0^{\bar{y}}(y + d(x,i))f_{Y_{ij}}(y)dy + \int_{\bar{y}}^{l(i,j)}(-y + d(x,j) +$$

$$li,j)fYijydydx+wkl0l(k,l)x-yfYijydy \qquad (10)$$

In problem (7), if we let $f_{(k,l)}(x) = \int_0^{l(k,l)} max_{(i,j)\in E}\{\left(\int_0^{l(i,j)} w_{ij}\, d(x,y) f_{Y_{pq}}(y)dy\right)\}$, $x \in (k,l)$and $L'(k,l) = \int_0^{l(k,l)} f_{(k,l)}(x)dx$, then Problem (7) can then be written as

$$min_{p\in P} \sum_{(k,l)\in p} L'(k,l) \qquad (11)$$

If $L'(k,l)$ is considered as the new length of edge $(k,l), (k,l)\epsilon E$, then problem (11) represents the shortest path problem from node S to node D.

The lengths $L'(k,l), (k,l)\epsilon E$are given by

$$L'(k,l) = \int_0^{l(k,l)} max\{ \max_{\substack{(i,j)\in E\\(i,j)\neq(k,l)}} \{\Bigg(w_{ij} \left(\int_0^{\bar{y}}(y + d(x,i))f_{Y_{ij}}(y)dy + \int_{\bar{y}}^{l(i,j)}(-y + d(x,j) +$$

$$li,j)fYijydy\} ,wkl0l(k,l)x-yfYijydy\}dx \qquad (12)$$

In problem (8), if we let $f_{(k,l)}(x) = \int_0^{l(k,l)} min_{(i,j)\in E}\{\left(\int_0^{l(i,j)} w_{ij}\, d(x,y) f_{Y_{pq}}(y)dy\right)\}$, $x \in (k,l)$and $L''(k,l) = \int_0^{l(k,l)} f_{(k,l)}(x)dx$, then Problem (6) can then be written as

$$max_{p\in P} \sum_{(k,l)\in p} L''(k,l) \qquad (13)$$

If $L''(k,l)$ is considered as the new length of edge $(k,l), (k,l)\epsilon E$, then problem (13) represents the longest path problem from node S to node D.

The lengths $L''(k,l), (k,l)\epsilon E$are given by

$$L''(k,l) = \int_0^{l(k,l)} min\{ \min_{\substack{(i,j)\in E\\(i,j)\neq(k,l)}} \{\Bigg(w_{ij} \left(\int_0^{\bar{y}}(y + d(x,i))f_{Y_{ij}}(y)dy + \int_{\bar{y}}^{l(i,j)}(-y + d(x,j) +$$

$$li,j)fYijydy\} ,wkl0l(k,l)x-yfYijydy\}dx \qquad (14)$$

Example 2

Let us consider the network in Fig. 4. The number next to each edge (i,j) in the network represents the length $l(i,j)$. The origin node of travel is node 1 and the destination is node 6.

Y_{ij} is taken to be uniformly distributed over the interval $[0, l(i,j)]$ for every$(i,j) \in E$ and $w_{ij} = 1, (i,j) \in E$.

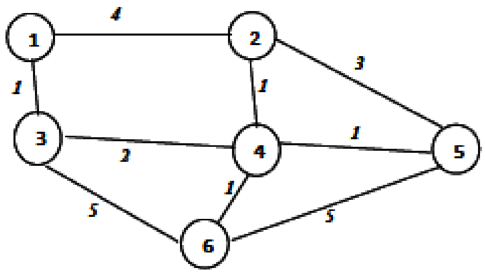

Figure 4

Mathematica software was used to compute the new lengths $L(i,j)$, $L^{'}(i,j)$, and $L^{''}(i,j)$, given by (10), (12), and (14), for every edge$(i,j)$of E. The following table gives the new lengths.

$Edge\ (i,j)$	(1,2)	(1,3)	(2,4)	(2,5)	(3,4)	(3,6)	(4,5)	(4,6)	(5,6)
$L(i,j)$	101.1	21.46	14.17	70.72	35.4	136.08	16.01	16.14	38.54
$L^{'}(i,j)$	17.66	4	4.01	14.22	6.1	23.94	3.25	3.25	8.01
$L^{''}(i,j)$	4.12	0.33	0.5	2.46	0.99	4.7	0.33	0.33	1.2

Consequently,

- The optimal minisum path is$p^* =$(1 , 3 ,4, 6) and the objective value is 71.
- The optimal minimax path is $p^* =$(1 , 3 , 4 , 6) and the objective value is 13.35.
- The optimal maximin path is $p^* =$(1 , 2 , 5 , 4 , 3 , 6) and the objective value is 12.67.

Case of maximin location of a moving facility with fixed demands :

The problem we address here is to determine the optimal path from S to D of a moving vehicle that continuously gives service over some time framework T to fixed demand nodes.

The problem is

maximizes the minimum weighted travel times between the moving facility and the demand nodes.

Following the same assumptions and analysis as in section 2.2, the problem can be stated as follows:

$$max_{p\in P}(\sum_{(i,j)\epsilon p}[\int_0^{l(i,j)} min_{k\epsilon N}\{w_k\, d(x,k)\}dx])$$

w_i is a fixed demand from node i and $d(x,k) = min\{x + d(i,k), -x + l(i,j) + d(j,k)$.

If we let $\bar{L}(i,j) = \int_0^{l(i,j)} min_{k\epsilon N}\{w_k\, d(x,k)\}dx$, then problem (13) becomes

$$max_{p\in P}\sum_{(k,l)\epsilon p}\bar{L}(i,j)$$

If $\bar{L}(i,j)$ is considered as the new length of edge (i,j), $(i,j)\boldsymbol{\epsilon}E$, then problem (14) represents the longest path problem from node S to node D.

Suppose w.o.l.o.g that $w_k = 1$ for all $k \in N$, then

$\bar{L}(i,j)= \int_0^{l(i,j)} \min_{k\epsilon \bar{N}}\{d(x,k)\}ds$, where $\bar{N} = N_1 \cup N_2, N_1 = N\backslash\{i\}$, $N_2 = N\backslash\{j\}$

Let $d_i = min_{k\epsilon N_1}\{d(i,k)\}$ and $d_j = min_{k\epsilon N_2}\{d(j,k)\}$, then

$$\min_{k\epsilon N}\{d(x,k)\} = \min\{x + d_i, -x + l(i,j) + d_j\}, \qquad 0 \le x \le l(i,j)$$

Let $a_{ij} = \frac{l(i,j)+d_j-d_i}{2}$. $\bar{L}(i,j)$is given according by

$$\bar{L}(i,j) = \begin{cases} \frac{l^2(i,j)}{2} + d_j l(i,j), & if \;\; a_{ij} \le 0 \\ \frac{l^2(i,j)}{2} + d_j l(i,j) - a_{ij}^2, & if \;\; 0 < a_{ij} < l(i,j) \\ \frac{l^2(i,j)}{2} + d_i l(i,j), & if \;\; a_{ij} \ge l(i,j) \end{cases}$$

Example 3 Let us consider the network in Fig. 5. The number next to each edge (i,j) in the network represents the length $l(i,j)$.The origin node of travel is node 1 and the destination is node 7.

Excel solver is used to compute the new lengths $\bar{L}(i,j)$ for the following network :

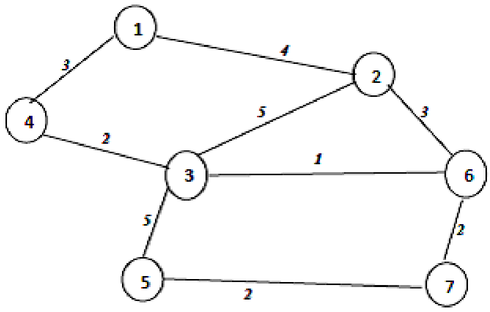

Figure 5

$Edge\ (i,j)$	(1,2)	(1,4)	(2,3)	(2,6)	(3,4)	(3,5)	(3,6)	(5,7)	(6,7)
$\bar{L}(i,j)$	16	9.5	15.25	7.25	3.75	13.5	1.25	5	3.75

The following table gives the new lengths.

The maximin optimal path for the moving facility is given by the longest path from node 1 to node 7, that is $p^* = (1\ ,2\ ,3\ ,5,7)$ with total length 50.

3The conclusion:

In this paper, I studied a twofold problem. The first one is concerned with finding the optimal location of a single facility in a network with demands randomly distributed over the edges. The second problem is about finding the optimal travel path for a service vehicle, which moves between two specified nodes through a network and interacts with a number of requests for service that are generated randomly over the edges. Servicing the random demands is performed over all instants of time during the travel period. I have determined optimal solutions to the stated problems numerically. Future work might be, the possibility of determining optimal solutions to the problems analytically and under set restriction.

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