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Quantile Function for Rayleigh and Scaled Half Logistic: Application in Missing Data

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ABSTRACT

In this research paper quantile Functions for Scaled half logistic and Rayleigh distributions has been constructed. Data generated through the quantile Functions and then different limits for the full and missing data set have been developed with scale parameter. A number of such mean control limits could be constructed through purposed method but for analysis purpose few of them have discussed. The missing data limits broadened than the full data in each case, which was expected to be. The average run length (ARL) was also calculated for different sample sizes (50,100,150). The general decreasing behavior of ARL according to increasing shifts was observed that shows a worthy sign, for two distributions, as the probability of detecting an out of control signal increased due to decrease in ARL.

Key words: Control charts; Rayleigh, and scaled half logistic distribution; average run length.

1. Introduction

The Scaled Half logistic and Rayleigh distribution are all well-known lifetime distributions.
Rao et al. (2013) jointly studied the exponential and half logistic models and presented many properties for the proposed model in terms of hypothesis testing Parameter estimation, and power of likelihood ratio. Rao and Kantam (2012) purposed the control charts constant for average and range are evaluated when the lifetime variate follows half logistic distribution and came to the conclusion that proposed method is better than existing of skewness correction method in terms of coverage probability. Panichkitkosolkul and Wattanachayakul (2012) proposed three types of confidence interval for half logistic distribution as standard bootstrap, percentile bootstrap, and, bias-corrected percentile bootstrap confidence intervals. Author(s) made comparison of all three confidence interval in terms of their coverage probability and concluded that bias-corrected percentile bootstrap confidence interval perform well as compare to the rest two if we assumed that location parameters are smaller than scale parameters.

Rayleigh distribution also used in life testing. Schick and Wolverton (1973) used Rayleigh as failure time of software system. Nadarajah and Kotz (2006) used incomplete beta ratio and obtained some generalization for the Rayleigh distribution. Mccool (2006) purposed two types of control charts one to monitor observed values of radial error and second based on ratio measure. Mccool (2006) mentioned that second approach is robust according to increased variability and can be used to distinguish the two source of out of control signals. Naqvi at el (2018) develop the Weibull quantile functions and its application for missing data.

Nair and Sankaran (2009) studied the role of quantile functions in reliability theory and presented the mutual relationship of mean, variance, percentile residual quantile functions also hazard function. Thomas et al. (2014) studied a class of reliability models named $q(u)$ using quantile function, their properties, and reliability characteristics and also discussed application of proposed model to the real data. Thomas et al. (2014) *stated that in life testing experiments researcher not interested till the last failure of the test items rather in a particular percentage*, Quantile Functions could provide the relevant percentage study for proposing useful estimates. Author(s) mentioned that incomplete beta function used to find their Quantile Functions; although we did not adopt their approach rather used Carter formula for it. According to Pearson and Hartley (1976) "Numerous approximate formulae for incomplete beta-function are valid in this range when $a$and b are moderately large, Carter's Formula(see Appendix) appears to be convenient to use.In industrial database studies researchers found the missing data problem especially when the nature of the data entered is manual rather than automated. One remedial is to avoid manual data when it is possible and use the bar-codes results but according to Lakshminarayan et al. (1999) some important information such as equipment environment, the person performing the maintenance of the equipment, results of manual test performed, the functions of the equipment etc. will still have to be manually entered, so the problem of missing data can be exit.
The significance of this research paper is the generation of random data through Quantile function for Rayleigh and the scaled half logistic distributions. The generated data utilized further for mean control limits also efficiency of the distributions checked by average run length for generated data.

1. GENERATION OF THE DATA

In this paper the limits for full and missing, random data generated, for three distributions named Exponential, Rayleigh, and Scaled Half-Logistic from their respective Quantile Functions (with known scale parameter) have been generated. The comparison of two limits in each case has been made, also the efficiency of the distribution checked through ARL(s).

1.1.Theoretical model

In order to develop the Quantile Functions for Rayleigh, and Scaled Half-Logistic we need the probability Density functions (pdf) and cumulative distribution functions (cdf) of the stated densities. The following 1-4 equations are the pdf and cdf for the Rayleigh, and Scaled Half-Logistic distribution respectively

$$f_{\lambda_r}(x) = \frac{2}{{\lambda_r}^2}\, x\ exp\left(-\left(\frac{x}{\lambda_r}\right)^2\right);\ for\ x\ \geq 0\ also\ \lambda_r > 0 \qquad (1)$$

$$F_{\lambda_r}(x) = 1 - exp\left(-\left(\frac{x}{\lambda_r}\right)^2\right) \quad (2)$$

$$f_{\lambda_{shl}}(x) = \frac{2exp\,(-x/\lambda_{shl})}{\lambda_{shl}\left(1+exp\,(-x/\lambda_{shl})\right)^2}; \qquad for\ x\ \geq 0, \lambda_{shl} > 0 \quad (3)$$

$$F_{\lambda_{shl}}(x) = \frac{(1-exp\,(-x/\lambda_{shl}))}{(1+exp\,(-x/\lambda_{shl}))} \quad (4)$$

One point is important to mention here that these two densities are scaled densities and the scale parameter(s) $\lambda_r, and\ \lambda_{shl}$ respectively, known as controlled factors

The Quantile Functions for this research paper developed with the help of order statistic. The probability density functions for order statistics given in the following equation (5).

$$f_{y_i}(Y) = \frac{n!}{(i-1)!(n-i)!}[F(y)]^{i-1}[1 - F(y)]^{n-i}\, f(y) \quad (5)$$

1.2.Methodology

In this section the working of the Quantile Function for Scaled Half-Logistic distribution has been discussed only just to avoid the repetition. Although the readers can get rest calculations from author by e-mail if needed.

The probability function and cumulative distribution function for Scaled Half-Logistic Distribution is given in equation (3) and (4) the pdf of order statistics is given in the equation (5), substitute the equation (3) and (4) in (5) accordingly and the following equation (6) will come up

$$g(y_{rshl}) = \frac{n!}{(r-1)!\,(n-r)!}\left(1 - \frac{1-\exp\left(-\frac{y_{rshl}}{\lambda_{rshl}}\right)}{1+\exp\left(-\frac{y_{rshl}}{\lambda_{rshl}}\right)}\right)^{n-r}\left(\frac{1-\exp\left(-\frac{y_{rshl}}{\lambda_{rshl}}\right)}{1+\exp\left(-\frac{y_{rshl}}{\lambda_{rshl}}\right)}\right)^{r-1}\left(\frac{2\exp\left(-\frac{y_{rshl}}{\lambda_{rshl}}\right)}{\left(1+\exp\left(-\frac{y_{rshl}}{\lambda_{rshl}}\right)\right)^2}\right);$$

$$0 \leq y_{rshl} \leq \infty \quad (6)$$

After applying some algebraic steps, distribution function of $y_{rshl}$is given below

$$F(y_{rshl}) = \frac{n!}{(r-1)!\,(n-r)!}\int_0^{\frac{1-\exp\left(-\frac{y_{rshl}}{\lambda_{rshl}}\right)}{1+\exp\left(-\frac{y_{rshl}}{\lambda_{rshl}}\right)}}(t)^{r-1}\,(1-t)^{(n-r+1)-1}\ dt;$$

$$0 \leq \frac{1-\exp\left(-\frac{y_{rshl}}{\lambda_{rshl}}\right)}{1+\exp\left(-\frac{y_{rshl}}{\lambda_{rshl}}\right)} \leq 1 \quad (7a)$$

Let us denote $x_r = \frac{1-\exp\left(-\frac{y_{rshl}}{\lambda_{rshl}}\right)}{1+\exp\left(-\frac{y_{rshl}}{\lambda_{rshl}}\right)}$

If we define

B_{Xr}(r,n-r+1) = $\int_0^{X_r} [t]^{(r)-1}\,[1-(t)]^{(n-r+1)-1}dt;$
And
B(r,n-r+1) = Γ(*r*) Γ(n-*r*+1) / Γ(n+1) = n! / (r-1)! (n-r)!
Then the above cdf is expressed by

$$F(y_{rshl}) = \frac{B_{Xr}(r, n-r+1)}{B(r, n-r+1)}$$

Which is also

$$I_x(a,b) = \frac{B_x(a,b)}{B(a,b)} \qquad (7b)$$

Then, the above cdf is expressed by

$$F(y_{rshl}) = I_{x_r}(r, n-r+1) \qquad (7c)$$

after some simple algebraic steps the following quantile function for Scaled Half-Logistic Distribution obtained

$$Q_x(\theta) = \lambda_{shl} ln\left(\frac{1+x(I/a,b)}{1-x(I/a,b)}\right) \qquad (8a)$$

Or

$$2\lambda_{shl} \tanh^{-1} x(I/a,b) \qquad (8b)$$

Where
$Q_x(\theta) = y_{rshl}$and$x(I/a,b)$ is percentage point of incomplete beta-function

Similarly the quantile functions for Rayleigh Distribution can calculated as result is given below:

For Rayleigh distribution:

$$Q_x(\theta) = \lambda_r \sqrt{\left[-ln\big(1 - x(I/a,b)\big)\right]} \qquad (9)$$

2. Results and discussions:

In this section we will discuss the results related to Rayleigh and Scaled Half-Logistic Distributions

Table 1: Random Numbers y_{rr} of Rayleigh Distribution (0.5) Percentage Point for $v_1 = n - r + 1$, $v_2 = r$ with Scale Parameter (768.1847)

r \ n		5	10	15	20	25	30	40	60	120
v_2	v_1	Y_r								
1	5	337.984	240.929	197.249	171.054	153.121	139.856	121.202	99.029	70.072
2	4	475.886	327.122	264.774	228.333	203.718	185.663	160.461	130.752	92.271

3	**3**	**639.556**	**421.515**	**337.414**	**289.493**	**257.528**	**234.255**	**201.986**	**164.215**	**115.629**
4	**2**	**821.441**	**509.351**	**402.453**	**343.355**	**304.482**	**276.411**	**237.759**	**192.850**	**135.489**
5	**1**	**1012.517**	**595.660**	**463.272**	**392.746**	**347.091**	**314.416**	**269.763**	**218.284**	**153.011**
6	**…**		**684.648**	**522.062**	**439.399**	**386.864**	**349.638**	**299.180**	**241.486**	**168.883**
7			**780.971**	**580.381**	**484.426**	**424.743**	**382.919**	**326.728**	**263.040**	**183.523**
8			**891.598**	**639.556**	**528.627**	**461.365**	**414.816**	**352.878**	**283.329**	**197.201**
9			**1029.680**	**700.928**	**572.640**	**497.201**	**445.728**	**377.958**	**302.617**	**210.104**
10			**1182.066**	**766.088**	**617.038**	**532.627**	**475.959**	**402.212**	**321.096**	**222.369**
11				**837.206**	**662.385**	**567.965**	**505.753**	**425.825**	**338.909**	**234.097**
12				**917.653**	**709.296**	**603.511**	**535.317**	**448.945**	**356.169**	**245.366**
13				**1013.408**	**758.500**	**639.556**	**564.835**	**471.695**	**372.965**	**256.238**
14				**1136.582**	**810.930**	**676.403**	**594.480**	**494.179**	**389.372**	**266.763**
15				**1274.334**	**867.872**	**714.390**	**624.421**	**516.488**	**405.450**	**276.982**
16					**931.243**	**753.910**	**654.830**	**538.706**	**421.251**	**286.930**
17					**1004.167**	**795.445**	**685.894**	**560.909**	**436.819**	**296.636**
18					**1092.326**	**839.617**	**717.817**	**583.171**	**452.194**	**306.124**
19					**1207.370**	**887.265**	**750.836**	**605.563**	**467.410**	**315.415**
20					**1336.904**	**939.589**	**785.232**	**628.159**	**482.499**	**324.529**
21						**998.412**	**821.351**	**651.032**	**497.489**	**333.481**
22						**1066.743**	**859.633**	**674.261**	**512.407**	**342.286**
23						**1150.092**	**900.661**	**697.932**	**527.277**	**350.956**
24						**1259.809**	**945.236**	**722.136**	**542.123**	**359.504**
25						**1383.886**	**994.511**	**746.980**	**556.968**	**367.938**
26							**1050.258**	**772.582**	**571.832**	**376.270**
27							**1115.409**	**799.083**	**586.739**	**384.507**
28							**1195.354**	**826.647**	**601.709**	**392.656**
29							**1301.220**	**855.476**	**616.765**	**400.726**
30							**1421.322**	**885.817**	**631.927**	**408.722**
40								**1478.745**	**795.098**	**485.812**
60									**1556.506**	**635.726**

120										**1681.810**
∞										

Table 2: Random Numbers $y_{rshl}$of Scaled Half-Logistic Distribution (0.5) Percentage Point for $v_1 = n - r + 1$, $v_2 = r$ with Scale Parameter (768.1847)

r \ n		5	10	15	20	25	30	40	60	120
v_2	v_1	Y_r								
1	**5**	**273.240**	**144.355**	**98.162**	**74.379**	**59.876**	**50.107**	**37.781**	**25.323**	**12.731**
2	**4**	**507.332**	**257.177**	**172.822**	**130.226**	**104.499**	**87.269**	**65.634**	**43.884**	**22.009**
3	**3**	**843.937**	**408.821**	**272.388**	**204.609**	**163.938**	**136.789**	**102.785**	**68.674**	**34.424**
4	**2**	**1277.507**	**571.523**	**376.107**	**281.324**	**224.961**	**187.504**	**140.739**	**93.957**	**47.073**
5	**1**	**1796.267**	**748.307**	**483.822**	**359.835**	**287.001**	**238.882**	**179.055**	**119.417**	**59.792**
6	**…**		**945.924**	**596.567**	**440.396**	**350.106**	**290.898**	**217.670**	**144.995**	**72.545**
7			**1175.983**	**715.897**	**523.467**	**414.458**	**343.635**	**256.601**	**170.678**	**85.321**
8			**1460.329**	**843.937**	**609.642**	**480.299**	**397.211**	**295.882**	**196.472**	**98.116**
9			**1846.159**	**983.650**	**699.655**	**547.918**	**451.769**	**335.564**	**222.386**	**110.929**
10			**2314.548**	**1139.368**	**794.420**	**617.653**	**507.470**	**375.701**	**248.432**	**123.761**
11				**1317.833**	**895.104**	**689.903**	**564.498**	**416.354**	**274.625**	**136.612**
12				**1530.457**	**1003.228**	**765.140**	**623.062**	**457.589**	**300.980**	**149.484**
13				**1798.844**	**1120.850**	**843.937**	**683.400**	**499.478**	**327.514**	**162.378**
14				**2169.839**	**1250.862**	**926.998**	**745.789**	**542.100**	**354.243**	**175.296**
15				**2621.538**	**1397.526**	**1015.207**	**810.550**	**585.541**	**381.186**	**188.239**
16					**1567.522**	**1109.697**	**878.066**	**629.896**	**408.361**	**201.209**
17					**1772.199**	**1211.963**	**948.794**	**675.270**	**435.790**	**214.208**
18					**2033.094**	**1324.037**	**1023.292**	**721.781**	**463.493**	**227.239**
19					**2396.926**	**1448.785**	**1102.249**	**769.560**	**491.492**	**240.302**
20					**2840.327**	**1590.449**	**1186.537**	**818.756**	**519.811**	**253.399**
21						**1755.686**	**1277.277**	**869.538**	**548.475**	**266.534**

22						**1955.826**	**1375.952**	**922.102**	**577.511**	**279.708**
23						**2212.376**	**1484.586**	**976.671**	**606.946**	**292.922**
24						**2571.992**	**1606.035**	**1033.510**	**636.812**	**306.180**
25						**3010.431**	**1744.529**	**1092.929**	**667.140**	**319.483**
26							**1906.720**	**1155.297**	**697.967**	**332.833**
27							**2103.926**	**1221.064**	**729.329**	**346.234**
28							**2357.640**	**1290.775**	**761.267**	**359.686**
29							**2714.480**	**1365.112**	**793.827**	**373.193**
30							**3149.617**	**1444.936**	**827.055**	**386.756**
40								**3369.527**	**1211.097**	**526.100**
60									**3679.922**	**835.449**
120										**4211.314**
∞										

Note: rest of the tables can be provided on researcher's request.

Tables 1-2 are the random number tables generated for Rayleigh, and Scaled Half-Logistic distributions from equation (9),and (8a) with a known scale parameter 768.1847. These tables are generated for percentage point (0.5) of incomplete B-distribution for V_1=n-r+1, V_2= r, although such tables can be generated for any other values of percentage point as well as scale parameter. In Quantile functions the involvement of incomplete beta function x(I/a,b) which was not easy to tackle, although that problem was address through *Carter's Formula.* According to Pearson and Hartely (1976)*"Numerous formulae for incomplete beta function are valid in this range although when a and b are moderately large, Carter's formula appears to be convenient to use".* Hence we put restriction on a and b both larger than 40 here a = v_1 and b = v_2. There is an increasing trend into the values of the random number tables (1-2) which can easily be observed. Such random numbers are useful for further statistical analysis e.g as we used them for constructing full and missing data limits. Before defining the algorithm we will propose a control chart using quantiles for Rayleigh, and Scaled Half-Logistic distributions

2.1 Proposed control chart

It is assumed that in phase I the "n" in-control subgroups of three different sizes 50,100, and 150 are drawn, from each of (1-2) table for the given value of Scale Parameter (768.1847) taken from literature although any other value can be taken in this regard.
We take a random sample of say size 50 from Table 1, and 2 (certainly these tables can easily be extended for more values) and calculate its quantile say $\hat{Q}_p$= 0.10 or, 0.25 or, 0.5 and repeat this process 10,000 times in order to get the sampling distribution of $\hat{Q}_p$. Once we get the sampling distribution of pth quantile then we construct the mean limits for full and missing data.

The proposed control chart for monitoring $\hat{\theta}$ for two densities, Rayleigh and scaled half-logistic, proceed as follows:
Step-1 Draw a sample of size n at each subgroup and calculate the mean estimate $\hat{\theta}$. The process is declared to be in control if LCL $<\hat{\theta}<$ UCL, otherwise out of control for both stated densities. The algorithm for the construction of the quantile control limits for Rayleigh, and Scaled Half-Logistic distributions is as follows:

2.2. Algorithm

The proposed Steps for the construction of mean control limits for full data are as follows:

Step 1: Take a large random sample of size $n(=500)$ from $\boldsymbol{y_r}(y_{rr}, y_{rshl})$ generated for Rayleigh, and Scaled Half Logistic distributions with known scale parameter(s) 768.1847. Select the subgroup of size 50, 100, and 150 observations, from each generated table, $y_1, y_2, y_3, y_4, y_5, \cdots, y_i, \cdots, y_n$

Step 2: From generated sample we calculated $\hat{\theta}$ estimate of mean.

Step 3: In order to get the sampling distribution for $\hat{\theta}$ Steps 1-2 repeated for a large number of times, say 10,000.

Step 4: From the sampling distribution of $\hat{\theta}$ we calculated the two limits LCL (ɤ/2) and UCL (1-ɤ/2), the lower and upper control limits respectively. Here ɤ symbolize a given false alarm rate of 5%.

Step 5: Lastly in order to get more precise results repeat the Steps 1-4, 100 times and took the average LCL and average UCL which have been reported in the table.

For the Computational Strategy regarding missing data for Rayleigh, and Scaled Half Logistic Distributions readers are encouraged to see Naqvi *et.al.*(2018).

Table 3
Mean Control Limits (0.025 and 0.975), for Rayleigh Distribution with Scale Parameter =768.1847

Quantile	Nature of the Data	Sample size=50		Sample size=100		Sample size=150	
		LCL_r	UCL_r	LCL_r	UCL_r	LCL_r	UCL_r
0.10	Full	563.094	702.3496	583.2881	681.8755	592.1294	672.6136
0.25		575.8086	716.0977	595.9949	695.5354	604.9897	686.2275
0.50		590.0029	732.0917	610.174	710.8309	619.355	701.4992
0.10	Missing	555.0784	710.8017	577.4141	687.7251	587.5137	677.3582
0.25		567.543	724.9156	590.1112	701.5141	600.2203	691.1371
0.50		581.385	740.306	604.322	716.7349	614.6435	706.3453

Table 4
Mean Control Limits (0.025 and 0.975), for Scaled Half-Logistic Distribution with Scale Parameter =768.1847

Quantile	Nature of the Data	Sample size=50		Sample size=100		Sample size=150	
		LCLshl	UCLshl	LCLshl	UCLshl	LCLshl	UCLshl
0.10	Full	738.1902	1049.902	781.7744	1002.986	801.7525	982.1404
0.25		764.2764	1083.571	809.1995	1035.081	829.5495	1013.906
0.50		794.0336	1121.614	840.2665	1072.137	860.9948	1050.182
0.10	Missing	720.042	1068.919	769.387	1016.364	791.3036	993.1405
0.25		746.0989	1103.374	796.091	1049.297	819.0283	1025.183
0.50		775.3174	1141.722	827.2626	1086.484	850.1792	1061.791

It can be observed that in above Tables (3-4) the mean control limits at different quantile distributions having the wider control limits for missing data as compare to the full data with no matter of the change in sample size and the quantile. Although if we consider the difference between the limits with the known scaled parameter 768.1847 for all three lifetime distributions then Rayleigh performs better than exponential and Scaled Half logistic as showing. Whatever the sample size and quantile the contracted difference in full and missing mean limits can be easily observed in each case for Rayleigh distribution. Although this is the observational discussion part of many constrain of control variables as well as restrictions the efficiency of the distribution can better assess through the ARL(s).

2.3.Average Run Length Study Table 5,6

The ARL study conducted for different sample sizes (50,100,150) and for different quantile (10%, 25% and 50%). Regardless of the sample sizes and the quantiles, It can be observed that, ARL having opposite relation with the shifts as shift increases ARL decreases for three distributions. The decreasing trend according to increasing shifts shows a worthy sign, as the probability of detecting an out of control signal increases due to decrease in ARL. One important point is mentioned here that scale parameter is varying from 768.1847(1.5) to 772.6847 i.e. the shift took place into the scale parameter for three densities. ARL(s)study conducted for Full data set only.

Table 5: ARL when Scale Parameter = 768.1847 for Rayleigh Distribution

N	50	100	150	Shifts (Scale Parameter)
k	2.992615	2.998058	2.988728	768.1847
	298.3800	295.8240	295.3240	
Percentile 0.10	296.3900	287.1447	295.2007	768.1847
	287.5193	282.5567	285.4333	769.6847
	281.8040	280.7500	281.3633	771.1847
	271.7347	264.8800	276.8140	772.6847
k	2.981802	2.998058	2.991453	768.1847
	301.0720	297.6980	297.1600	
Percentile 0.25	294.2680	290.4767	290.0053	768.1847
	288.4720	287.8953	283.5820	769.6847
	281.0060	285.5560	281.9820	771.1847
	268.7573	281.2153	278.1067	772.6847
K	2.997268	2.994900	2.984980	768.1847
	304.0980	295.7300	300.3420	
Percentile 0.50	299.1820	291.2527	293.3100	768.1847
	292.0227	286.9727	286.1587	769.6847
	286.2200	284.8260	281.9493	771.1847
	280.8080	279.7053	273.8200	772.6847

Table 6: ARL when Scale Parameter = 768.1847 for Scaled Half Logistic Distribution

N	50	100	150	Shifts (Scale Parameter)
K	2.986028	2.995203	2.988728	768.1847
	304.0060	296.7840	297.8440	
Percentile 0.10	297.2340	290.8467	297.7520	768.1847
	287.4147	288.0407	296.1173	769.6847
	284.0133	273.6100	289.8647	771.1847
	279.7073	272.9213	286.2640	772.6847
K	2.991388	2.99409	2.981306	768.1847
	295.7880	295.9220	300.9720	
Percentile 0.25	292.0313	293.1000	298.7260	768.1847
	288.8520	283.1507	283.4387	769.6847
	287.3027	281.6247	281.0920	771.1847
	285.4573	273.6513	266.5500	772.6847
k	2.991388	2.997981	2.98759	768.1847
	295.3520	295.0280	299.196	
Percentile 0.50	294.2940	289.3433	298.6593	768.1847
	293.0027	288.4133	296.1573	769.6847
	289.7853	282.6540	280.6627	771.1847
	284.9100	280.9927	277.0753	772.6847

3. Discussion and Conclusion

The significance of this research paper is the generation of random data through Quantile function(s). We utilized generated random data for further investigations i-e construction of the mean limits for full and missing data at false alarm rate ɤ and for different sample sizes (50,100,150). It can be observed from Tables (3-4) that we had wider control limits for missing data as compare to the full data with no matter of the change in sample size, quantile, and distribution(s) which really demanded in practice and authenticity of proposed strategy as well.

The efficiency of the distribution(s) Rayleigh and the scaled half logistic checked by average run length. It can be observed from Tables (5-6) that regardless of the change in sample sizes, quantile, and distribution(s) the ARL having opposite relation with the shifts as shift increases ARL decreases. The decreasing trend according to increasing shifts which needed in actuality, as the probability of detecting an out of control signal increases due to decrease in ARL, is a well-meaning signal. One important point is mentioned here that the shift took place into the scale parameter for two densities 768.1847(1.5) to 772.6847. ARL(s) study conducted for Full data sets only. Both distributions are efficient in accordance with ARL The general decreasing behavior of ARL according to increasing shifts was observed, that shows a worthy sign, for two distributions as the probability of detecting an out of control signal increased due to decrease in ARL.

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APPENDIX: CARTER'S FORMULA

If X denotes the standardized normal deviate corresponding to $P = 1 - I$ and if $\lambda = 1/6\left(\chi^{2} - 3\right)$, $\tau = \left[\left(1/2\right) + \left(5/12\right)\right]$, we compute in turn

$$\mathbf{A} = \frac{\mathbf{1}}{\mathbf{12}}\left(\frac{\mathbf{1}}{\mathbf{a} - {}^{1}\!/_{2}} + \frac{\mathbf{1}}{\mathbf{b} - {}^{1}\!/_{2}}\right), \mathbf{h} = \frac{\mathbf{1}}{\mathbf{3A}} \text{ also } \mathbf{z} = \frac{\mathbf{X}\sqrt{(\mathbf{h} + \boldsymbol{\lambda})}}{\mathbf{h}} - \left(\frac{\mathbf{1}}{\mathbf{a} - {}^{1}\!/_{2}} - \frac{\mathbf{1}}{\mathbf{b} - {}^{1}\!/_{2}}\right)(\boldsymbol{\tau} - \mathbf{A}), \mathbf{x}(\mathbf{I} \mid \mathbf{a}, \mathbf{b}) = \frac{\mathbf{a}}{(\mathbf{a} + \mathbf{b}e^{2z})}$$

P	0.50	0.25	0.10	0.05	0.025	0.01	0.005
X	0	0.6745	1.2816	1.6449	1.9600	2.3263	2.5758
λ	-0.5000	-0.4242	-0.2263	-0.0491	0.1402	0.4020	0.6058
T	0.1667	0.2046	0.3035	0.3921	0.4868	0.6177	0.7196