Journal of Progressive Research in Mathematics(JPRM)
ISSN: 2395-0218

SCITECH
RESEARCH ORGANISATION

Volume 11, Issue 3
Published online: April 07, 2017|

Journal of Progressive Research in Mathematics
www.scitecresearch.com/journals

New Iterative Method For Solving Nonlinear partial Differential Equations

A.M.S.Mahdy[1] and N.A.H.Mukhtar[2]
[1]Department of Mathematics, Faculty of Science, Zagazig University, Zagazig, Egypt
[2]Department of Mathematics, Faculty of Science, Benghazi University, Benghazi, Libya

Abstract

This paper presents an approximate analytical solution of the non-linear Benjamin-Bona-Mahony equation, Cahn Hilliard equation, Gardner equation, linear Klein Gordon equation.

Keywords: New Iterative Method; Nonlinear Benjamin-Bona-Mahony equation; Cahn Hilliard equation; Gardner equation; linear Klein Gordon equation; nonlinear Klein Gordon equation and nonlinear dispersive K (2, 2) equation.

1.Introduction

This equation was studied Benjamin-Bona and Mahony in 1972 using as model for the propagation of long waves in non-linear dispersive systems is of considerable interest in mathematical physics .This BBM equation does have some advantages ,They show the stability and uniqueness of solutions to the

BBM equation. further, has an finite number of integrals of motion, only has three integrals. the BBM equation also known as the regularized long-wave equation (RLWE) is the partial differential equation. The authors in [2] using Benjamin-Bona-Mahony equation, from Wikipedia, the free encyclopedia.

The cahn-Hiliard equation finds applications in diverse fields. In complex fluids and soft matter (inter facial fluid flow, polymer science and in industrial applications) we found some exact solutions of the equations by considering a modified extended tanh function method a numerical solution to a cahn-Hilliard equation is obtained using NIW method.

This equations is very crucial in materials many articles have investigated mathematically and numerically this equation The authors in[14] using solutions of the Cahn-Hilliard equation.

Gardner equation is a nonlinear partial differential equation set up by mathematician clifford Gardner in 1968.Gardner equation has application in hydrodynamics. plasma and quantum field theory The authors in [8] using Gardner equation, from Wikipedia, the free encyclopedia.

Searching for solitary solutions for nonlinear equation in mathematical physics is attractive in the solitary theory. For example, Wadati (1972,1973) developed the solitons for KdV and MKdV equation . In 1993, Rosenau and Hyman (1993) presented a family of fully nonlinear KdV equations K(m, n) and introduced a class of solitary waves with compact support that are solutions of a two parameter family of fully nonlinear dispersive partial different equations such as k(2, 2) equation The authors in [1] using Exact Solitary-Wave Special Solutions for the Nonlinear Dispersive K(m,n) Equations by Means of the Hompotopy.

Analysis Method.

The Klein. Gordon equation Nonlinear phenomena, that appear in many areas of scientific fields such as solid state physics, plasma physics, fluid dynamics, mathematical biology and chemical kinetics, can be modeled by partial differential equations. A broad class of analytical solution methods and numerical solution methods were used to handle these problems. The initial-value problem of the one-dimensional nonlinear Klein. Gordon equation, the authors in [12] using Numerical Solution of the nonlinear Klein-Gordon equation using radial basis functions.

Most of the natural events, such as chemical, physical, biological can be modeled by nonlinear differential equations. Besides exact solutions, we need their approximate solutions in terms of applicability. Therefore a lot of approximate numerical and analytical methods are developed and applied for nonlinear models. The authors in [13] using Numerical Solution of time-dependent Foam Drainage Equation(FDE).

2 Basic Idea of NIM[15]

To describe the idea of the NIM, consider the following general functional equation [3,4,6,7,9,15]:

$$u(x) = f(x) + N(u(x)), \qquad (1)$$

where N is a nonlinear operator from a Banach space B→B and f is a known function. We are looking for a solution u of (1) having the series form

$$u(x) = \sum_{i=0}^{\infty} u_i(x). \qquad (2)$$

The nonlinear operator N can be decomposed as follows

$$N\left(\sum_{i=0}^{\infty} u_i\right) = N(u_0) + \sum_{i=1}^{\infty}\left\{N\left(\sum_{j=0}^{\infty} u_j\right) - N\left(\sum_{j=0}^{\infty} u_j\right)\right\} \qquad (3)$$

From Eqs.(2) and (3), Eq. (1) is equivalent to

$$\sum_{i=0}^{\infty} u_i = f + N(u_0) + \sum_{i=1}^{\infty}\left\{N\left(\sum_{j=0}^{\infty} u_j\right) - N\left(\sum_{j=0}^{\infty} u_j\right)\right\} \qquad (4)$$

We define the recurrence relation

$$u_0 = f, \qquad (5)$$

$$u_1 = N(u_0), \qquad (6)$$

$$u_{n+1} = N(u_0 + u_1 + ... + u_n) - N(u_0 + u_1 + ... + u_{n-1}), \quad n = 1, 2, 3... \qquad (7)$$

then

$$(u_0 + u_1 + ... + u_{n+1}) = N(u_0 + u_1 + ... + u_n), \quad n = 1, 2, 3...$$

$$u = \sum_{i=0}^{\infty} u_i = f + N\left(\sum_{i=0}^{\infty} u_i\right) \tag{8}$$

If N is a contraction, i.e.

$$\|N(x) - N(y)\| \leq k\,\|x - y\|\,, \quad 0 < k < 1,$$

then

$$\begin{aligned} \|u_{n+1}\| &= \|N(u_0 + u_1 + ... + u_n) - N(u_0 + u_1 + ... + u_{n-1})\| \\ &\leq k\,\|u_n\| \leq ... \leq k^n\,\|u_0\| \quad n = 0, 1, 2..., \end{aligned}$$

and the series $\sum_{i=0}^{\infty} u_i$ absolutely and uniformly converges to a solution of (1) [5] which is unique, in view of the Banach fixed point theorem [10]. The k-term approximate solution of(1) and (2) is given by $\sum_{i=0}^{k-1} u_i$

3. Applications of the NIM[11]:

In this section, we apply the new iterative method approach to study five examples

Example 3.1. We consider the BBM equation

$$u_t = u_{xxt} - u_x - u\,u_x$$

subject to the initial condition

$$u(x, 0) = \sec h^2(\frac{x}{4}), \tag{9}$$

therefore from (5) and (10) we obtain

$$u_0(x, t) = \sec h^2\left(\frac{x}{4}\right),$$

Therefore, the initial value problem (9) is equivalent to the following integral equations:

$$u(x, t) = \sec h^2\left(\frac{x}{4}\right) + I_t(u_{xxt} - u_x - u\,u_x - u_t), \tag{10}$$

Taking

$$N(u) = I_t\left(u_{xxt} - u_x - u\,u_x - u_t\right)$$

Therefore, from (5), (6) and (7) we can obtain easily the following first few components of the new iterative solution for the equation (9):

$$u_0(x,t) = \sec h^2\left(\frac{x}{4}\right),$$

$$u_1(x,t) = (\frac{1}{2}\sec h^2(\frac{x}{4})\tanh(\frac{x}{4}))t + (\frac{1}{2}\sec h^4(\frac{x}{4})\tanh(\frac{x}{4}))t,$$

$$\begin{aligned} u_2(x,t) \quad = \quad & -\frac{t}{4}\sec h^4(\frac{x}{4})\tanh(\frac{x}{4}) + \frac{t+t^2}{8}\sec h^2(\frac{x}{4})\tanh^2(\frac{x}{4}) \\ & +\frac{1}{16}(-7t - \frac{t^3}{3})\sec h^6(\frac{x}{4})\tanh\left(\frac{x}{4}\right) + \frac{1}{8}(t^3+4t)\sec h^4(\frac{x}{4})\tanh^3\left(\frac{x}{4}\right) \\ & -\frac{t}{8}\sec h^4\left(\frac{x}{4}\right) - \frac{t^2}{16}\sec h^6\left(\frac{x}{4}\right) + \frac{t^2}{4}\sec h^4\left(\frac{x}{4}\right)\tanh^2\left(\frac{x}{4}\right) - \frac{t^3}{24}\sec h^8\left(\frac{x}{4}\right)\tanh\left(\frac{x}{4}\right) \end{aligned}$$

and the rest of the components of iteration formula (8) are obtained. The approximate solution which involves few terms is given by

$$\begin{aligned} u(x,t) \quad = \quad & \sum_{i=1}^{2} u_i = \sec h^2\left(\frac{x}{4}\right) + \left(\frac{1}{2}\sec h^2\frac{x}{4}\ \tanh\frac{x}{4}\right)t + \\ & \left(\frac{1}{2}\sec h^4\frac{x}{4}\tanh\frac{x}{4}\right)t - \frac{t}{4}\sec h^4\frac{x}{4}\tanh\frac{x}{4} + \frac{t+t^2}{8}\sec h^2\frac{x}{4}\tanh^2\frac{x}{4} \\ & \frac{1}{16}\left(-7t - \frac{t^3}{3}\right)\sec h^6\frac{x}{4}\tanh\frac{x}{4} + \frac{1}{8}\left(t^3+4t\right)\sec h^4\frac{x}{4}\tanh^3\frac{x}{4} \\ & -\frac{t}{8}\sec h^4\frac{x}{4} - \frac{t^2}{16}\sec h^6\frac{x}{4} + \frac{t^2}{4}\sec h^4\frac{x}{4}\tanh^2\frac{x}{4} \\ & -\frac{t^3}{24}\sec h^8\frac{x}{4}\tanh\frac{x}{4} + \ldots \end{aligned}$$

Example 3.2. We consider the Cahn-Hilliard equation

$$u_t = u_{xx} - u^3 + u \qquad (11)$$

subject the initial condition

$$u\left(x,0\right) = \frac{1}{1+e^{\frac{x}{\sqrt{2}}}} \qquad (12)$$

from (5) and (12) , we obtain

$$u_0 = (x,t) = \frac{1}{1+e^{\frac{x}{\sqrt{2}}}}$$

Therefore, The initial value problem (11) and (12) is equivalent to the following integral equations:

$$u\left(x,t\right)=\frac{1}{1+e^{\frac{x}{\sqrt{2}}}}+I_t\left(u_{xx}-u^3+u\right)$$

Taking

$$N\left(u\right)=I_t\left(u_{xx}-u^3+u\right)$$

Therefore from (5), (6) and (7), we can obtain easily the following first few components of the new iterative solution for the equation (11) and (12)

$$u_0\left(x,t\right)=\frac{1}{1+e^{\frac{x}{\sqrt{2}}}}$$

$$u_1\left(x,t\right)=e^{\sqrt{2}x}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-3}t-\frac{t}{2}e^{\frac{x}{\sqrt{2}}}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-2}-\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-3}t+\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-1}t$$

and the rest of the components of iteration formula (8) are obtained. The approximate solution which involves few terms is given by

$$\begin{aligned} u_2 \;=\;& \frac{-9t^2}{4}e^{\frac{3x}{\sqrt{2}}}+\frac{3t^2}{4}e^{\frac{x}{\sqrt{2}}}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-4}+3t^2e^{\frac{3x}{\sqrt{2}}}-3t^2e^{\sqrt{2}x}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-5}+\frac{15t^2}{8}e^{\sqrt{2}x}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-3}-\\ & \frac{t^4}{4}\left[e^{2\sqrt{2}x^3}-3e^{2x^2}+1+3e^{\sqrt{2}x}\right]\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-9}-\frac{3t^4}{8}\left[e^{\frac{2\sqrt{2}x^2+x}{\sqrt{2}}}-e^{\frac{x}{\sqrt{2}}}\right]\left(1+e^{\frac{x}{\sqrt{2}}}\right)-\\ & \frac{3t^4}{4}\left[e^{2x^2}+\frac{1}{4}e^{\frac{x^2+2\sqrt{2}x}{2}}+\frac{1}{4}e^{\frac{x}{2}}+1\right]\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-7}-\frac{3t^4}{4}e^{\frac{x}{\sqrt{2}}}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-4}-\frac{t^4}{4}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-3}+\\ & \frac{t^2}{2}e^{\sqrt{2}x}\left(1+e^{\frac{x}{\sqrt{2}}}\right)-\frac{t^2}{4}e^{\frac{x}{\sqrt{2}}}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-2}-\frac{t^2}{2}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-3}+\frac{t^2}{2}\left(1+e^{\frac{x}{\sqrt{2}}}\right) \end{aligned}$$

$$\begin{aligned} u \;=\;& \sum_{i=0}^{2}u_i=\frac{1}{1+e^{\frac{x}{\sqrt{2}}}}+e^{\sqrt{2}x}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-3}t-\frac{t}{2}e^{\frac{x}{\sqrt{2}}}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-2}-\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-3}t+\\ & \left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-1}t-\frac{9t^2}{4}e^{\frac{3x}{\sqrt{2}}}+\frac{3t^2}{4}e^{\frac{x}{\sqrt{2}}}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-4}+3t^2e^{\frac{3x}{\sqrt{2}}}-3t^2e^{\sqrt{2}x}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-5}+\\ & \frac{15t^2}{8}e^{\sqrt{2}x}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-3}-\frac{t^4}{4}\left[e^{2\sqrt{2}x^3}-3e^{2x^2}+1+3e^{\sqrt{2}x}\right]\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-9}\\ & -\frac{3t^4}{8}\left[e^{\frac{2\sqrt{2}x^2+x}{\sqrt{2}}}-e^{\frac{x}{\sqrt{2}}}\right]\left(1+e^{\frac{x}{\sqrt{2}}}\right)-\frac{3t^4}{4}\left[e^{2x^2}+\frac{1}{4}e^{\frac{x^2+2\sqrt{2}x}{2}}+\frac{1}{4}e^{\frac{x}{2}}+1\right]\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-7}-\\ & \frac{3t^4}{4}e^{\frac{x}{\sqrt{2}}}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-4}-\frac{t^4}{4}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-3}+\frac{t^2}{2}e^{\sqrt{2}x}\left(1+e^{\frac{x}{\sqrt{2}}}\right)-\frac{t^2}{4}e^{\frac{x}{\sqrt{2}}}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-2}\\ & -\frac{t^2}{2}\left(1+e^{\frac{x}{\sqrt{2}}}\right)^{-3}+\frac{t^2}{2}\left(1+e^{\frac{x}{\sqrt{2}}}\right)\ldots \end{aligned}$$

Example 3.3. We consider the Gardner equation

$$u_t=6u^2u_x+6u+u_{xxx} \tag{13}$$

subject to the initial condition:

$$u\,(x,0) = -\frac{1}{2}\left(1 - \tanh(\frac{x}{2})\right) \tag{14}$$

from (5) and (14) we obtain

$$u_0(x,t) = -\frac{1}{2}\left(1 - \tanh(\frac{x}{2})\right)$$

Therefore, the initial value problem (13) and (14) is equivalent to the following integral equations:

$$u(x,t) = -\frac{1}{2}\left(1 - \tanh(\frac{1}{2})\right) + I_t\left(6u^2u_x + 6u + u_{xxx}\right)$$

Taking

$$N\,(u) = I_t\left(6u^2u_x + 6u + u_{xxx}\right)$$

Therefore from (5), (6) and (7), we can obtain easily the following first few components of the new iterative solution for the equation (13) and (14)

$$u_0(x,t) = -\frac{1}{2}\left(1 - \tanh(\frac{x}{2})\right)$$

$$u_1(x,t) = \frac{3t}{8}\sec h^2\frac{x}{2} - \frac{6t}{8}\sec h^2\frac{x}{2}\tanh\frac{x}{2} + \frac{7t}{8}\sec h^2\frac{x}{2}\tanh^2\frac{x}{2} + 3t\ \tanh\frac{x}{2} - \frac{t}{8}\sec h^4\frac{x}{2} - 3t$$

$$
\begin{aligned}
u_2 \quad = \quad & -\frac{37314}{2048}t^4 \sec h^6\frac{x}{2}\tanh\frac{x}{2}+\frac{5598}{2048}t^4 \sec h^6\frac{x}{2}\tanh^3\frac{x}{2}-\frac{7770}{2048}t^4 \sec h^6\frac{x}{2}\tanh^5\frac{x}{2} \\
& -\frac{1188}{32}t^4 \sec h^2\frac{x}{2}\tanh^3\frac{x}{2}-\frac{306}{2048}t^4 \sec h^{10}\frac{x}{2}\tanh\frac{x}{2}-\frac{1458}{32}t^4 \sec h^2\frac{x}{2}\tanh\frac{x}{2}+ \\
& \frac{62604}{2048}t^4 \sec h^6\frac{x}{2}\tanh^2\frac{x}{2}-\frac{12096}{256}t^4 \sec h^4\frac{x}{2}\tanh^2\frac{x}{2}+\frac{4194}{2048}t^4 \sec h^8\frac{x}{2}\tanh\frac{x}{2} \\
& +\frac{18036}{256}t^4 \sec h^4\frac{x}{2}\tanh\frac{x}{2}+\frac{8640}{256}t^4 \sec h^4\frac{x}{2}\tanh^3\frac{x}{2}-\frac{4104}{2048}t^4 \sec h^8\frac{x}{2}\tanh^2\frac{x}{2} \\
& -\frac{9828}{256}t^4 \sec h^4\frac{x}{2}\tanh^4\frac{x}{2}+\frac{1296}{32}t^4 \sec h^2\frac{x}{2}\tanh^2\frac{x}{2}-\frac{1458}{2048}t^4 \sec h^8\frac{x}{2}- \\
& \frac{3906}{2048}t^4 \sec h^8\frac{x}{2}\tanh^4\frac{x}{2}-\frac{18}{2048}t^4 \sec h^4\frac{x}{2}+\frac{1674}{512}t^4 \sec h^6\frac{x}{2}-\frac{9072}{2048}t^4 \sec h^6\frac{x}{2}\tanh^4 \\
& \frac{5292}{2048}t^4 \sec h^6\frac{x}{2}\tanh^6\frac{x}{2}+\frac{1080}{32}t^4 \sec h^2\frac{x}{2}\tanh^4\frac{x}{2}+\frac{396}{2048}t^4 \sec h^{10}\frac{x}{2}\tanh^2\frac{x}{2} \\
& -\frac{2436}{256}t^4 \sec h^4\frac{x}{2}\tanh^5\frac{x}{2}+\frac{4116}{2048}t^4 \sec h^8\frac{x}{2}\tanh^3\frac{x}{2}+\frac{1470}{2048}t^4 \sec h^8\frac{x}{2}\tanh^5\frac{x}{2} \\
& +\frac{30}{2048}t^4 \sec h^{12}\frac{x}{2}\tanh\frac{x}{2}-\frac{2058}{2048}t^4 \sec h^6\frac{x}{2}\tanh^7\frac{x}{2}-\frac{378}{32}t^4 \sec h^2\frac{x}{2}\tanh^5\frac{x}{2} \\
& -\frac{42}{2048}t^4 \sec h^{10}\frac{x}{2}\tanh^3\frac{x}{2}-\frac{378}{32}t^4 \sec h^2\frac{x}{2}\tanh^3\frac{x}{2}+\frac{3276}{256}t^4 \sec h^4\frac{x}{2}\tanh^5\frac{x}{2} \\
& +\frac{18}{16}t^2 \sec h^2\frac{x}{2}-\frac{36}{16}t^2 \sec h^2\frac{x}{2}\tanh\frac{x}{2}+\frac{42}{16}t^2 \sec h^2\frac{x}{2}\tanh^2\frac{x}{2}+\frac{18}{2}t^2\tanh\frac{x}{2} \\
& -\frac{6}{16}t^2 \sec h^4\frac{x}{2}-\frac{18}{2}t^2+\frac{6}{16}t^2 \sec h^2\frac{x}{2}\tanh\frac{x}{2}-\frac{3}{16}t^2 \sec h^2\frac{x}{2}\tanh^3\frac{x}{2}+\frac{3}{8}t^2 \sec h^6\frac{x}{2} \\
& -\frac{15}{16}t^2 \sec h^4\frac{x}{2}\tanh^2\frac{x}{2}+\frac{6}{16}t^2 \sec h^2\frac{x}{2}\tanh^4\frac{x}{2}-\frac{81}{32}t^2 \sec h^4\frac{x}{2}\tanh\frac{x}{2} \\
& +\frac{82}{32}t^2 \sec h^4\frac{x}{2}\tanh^3\frac{x}{2}-\frac{7}{16}t^2 \sec h^2\frac{x}{2}\tanh^5\frac{x}{2}+\frac{14}{16}t^2 \sec h^4\frac{x}{2}\tanh^3\frac{x}{2}.
\end{aligned}
$$

$$
\begin{aligned}
u \;=\; & \sum_{i=0}^{2} u_i = -\frac{1}{2}\left(1-\tanh(\frac{x}{2})\right)+\frac{3t}{8}\sec h^2\frac{x}{2}-\frac{6t}{8}\sec h^2\frac{x}{2}\tanh\frac{x}{2}+\frac{7t}{8}\sec h^2\frac{x}{2}\tanh^2\frac{x}{2} \\
& +3t\ \tanh\frac{x}{2}-\frac{t}{8}\sec h^4\frac{x}{2}-3t-\frac{37314}{2048}t^4\sec h^6\frac{x}{2}\tanh\frac{x}{2}+\frac{5598}{2048}t^4\sec h^6\frac{x}{2}\tanh^3\frac{x}{2} \\
& -\frac{7770}{2048}t^4\sec h^6\frac{x}{2}\tanh^5\frac{x}{2}-\frac{1188}{32}t^4\sec h^2\frac{x}{2}\tanh^3\frac{x}{2}-\frac{306}{2048}t^4\sec h^{10}\frac{x}{2}\tanh\frac{x}{2}-\frac{1458}{32}t^4\sec h^2\frac{x}{2}\tanh\frac{x}{2} \\
& +\frac{62604}{2048}t^4\sec h^6\frac{x}{2}\tanh^2\frac{x}{2}-\frac{12096}{256}t^4\sec h^4\frac{x}{2}\tanh^2\frac{x}{2}+\frac{4194}{2048}t^4\sec h^8\frac{x}{2}\tanh\frac{x}{2} \\
& +\frac{18036}{256}t^4\sec h^4\frac{x}{2}\tanh\frac{x}{2}+\frac{8640}{256}t^4\sec h^4\frac{x}{2}\tanh^3\frac{x}{2}-\frac{4104}{2048}t^4\sec h^8\frac{x}{2}\tanh^2\frac{x}{2} \\
& -\frac{9828}{256}t^4\sec h^4\frac{x}{2}\tanh^4\frac{x}{2}+\frac{1296}{32}t^4\sec h^2\frac{x}{2}\tanh^2\frac{x}{2}-\frac{1458}{2048}t^4\sec h^8\frac{x}{2}- \\
& \frac{3906}{2048}t^4\sec h^8\frac{x}{2}\tanh^4\frac{x}{2}-\frac{18}{2048}t^4\sec h^4\frac{x}{2}+\frac{1674}{512}t^4\sec h^6\frac{x}{2}-\frac{9072}{2048}t^4\sec h^6\frac{x}{2}\tanh^4\frac{x}{2} \\
& \frac{5292}{2048}t^4\sec h^6\frac{x}{2}\tanh^6\frac{x}{2}+\frac{1080}{32}t^4\sec h^2\frac{x}{2}\tanh^4\frac{x}{2}+\frac{396}{2048}t^4\sec h^{10}\frac{x}{2}\tanh^2\frac{x}{2} \\
& -\frac{2436}{256}t^4\sec h^4\frac{x}{2}\tanh^5\frac{x}{2}+\frac{4116}{2048}t^4\sec h^8\frac{x}{2}\tanh^3\frac{x}{2}+\frac{1470}{2048}t^4\sec h^8\frac{x}{2}\tanh^5\frac{x}{2} \\
& +\frac{30}{2048}t^4\sec h^{12}\frac{x}{2}\tanh\frac{x}{2}-\frac{2058}{2048}t^4\sec h^6\frac{x}{2}\tanh^7\frac{x}{2}-\frac{378}{32}t^4\sec h^2\frac{x}{2}\tanh^5\frac{x}{2} \\
& -\frac{42}{2048}t^4\sec h^{10}\frac{x}{2}\tanh^3\frac{x}{2}-\frac{378}{32}t^4\sec h^2\frac{x}{2}\tanh^3\frac{x}{2}+\frac{3276}{256}t^4\sec h^4\frac{x}{2}\tanh^5\frac{x}{2} \\
& +\frac{18}{16}t^2\sec h^2\frac{x}{2}-\frac{36}{16}t^2\sec h^2\frac{x}{2}\tanh\frac{x}{2}+\frac{42}{16}t^2\sec h^2\frac{x}{2}\tanh^2\frac{x}{2}+\frac{18}{2}t^2\tanh\frac{x}{2} \\
& -\frac{6}{16}t^2\sec h^4\frac{x}{2}-\frac{18}{2}t^2+\frac{6}{16}t^2\sec h^2\frac{x}{2}\tanh\frac{x}{2}-\frac{3}{16}t^2\sec h^2\frac{x}{2}\tanh^3\frac{x}{2}+\frac{3}{8}t^2\sec h^6\frac{x}{2} \\
& -\frac{15}{16}t^2\sec h^4\frac{x}{2}\tanh^2\frac{x}{2}+\frac{6}{16}t^2\sec h^2\frac{x}{2}\tanh^4\frac{x}{2}-\frac{81}{32}t^2\sec h^4\frac{x}{2}\tanh\frac{x}{2} \\
& +\frac{82}{32}t^2\sec h^4\frac{x}{2}\tanh^3\frac{x}{2}-\frac{7}{16}t^2\sec h^2\frac{x}{2}\tanh^5\frac{x}{2}+\frac{14}{16}t^2\sec h^4\frac{x}{2}\tanh^3\frac{x}{2}\ldots
\end{aligned}
$$

Example 3.4. We consider the non-linear dispersive K(2,2) equation

$$u_t = -2u\ u_x - 6u_x\ u_{xx} - 2u\ u_{xxx} \tag{15}$$

subject the initial condition

$$u(x,0) = \frac{4}{3}\cos^2\frac{x}{4} \tag{16}$$

from (5) and (16), we obtain

$$u_0(x,t) = \frac{4}{3}\cos^2\frac{x}{4}$$

Therefore, the initial value problem (15) and (16) is equivalent to the following integral equations:

$$u(x,t) = \frac{4}{3}\cos^2\frac{x}{4} + I_t(-2u\ u_x - 6u_x u_{xx} - 2u\ u_{xxx}$$

Taking

$$N(u) = I_t(-2u\ u_x - 6u_x\ u_{xx} - 2u\ u_{xxx}$$

Therefore from (5), (6) and (7), we can obtain easily the following first few components of the new iterative solution for the equation (15) and (16)

$$u_0(x,t)=\frac{4}{3}\cos^2\frac{x}{4}$$

$$u_1(x,t)=\frac{16t}{9}\cos^3\frac{x}{4}\sin\frac{x}{4}-\frac{2t}{3}\cos\frac{x}{4}\sin\frac{x}{4}$$

$$\begin{aligned}u_2 \;=\;& \frac{1327104}{3359232}t^3\cos^7\frac{x}{4}\sin\frac{x}{4}+\frac{6801408}{3359232}t^3\cos^5\frac{x}{4}\sin^3\frac{x}{4}+\\ & \frac{3936}{1296}t^3\cos^3\frac{x}{4}\sin^3\frac{x}{4}+\frac{16}{54}t^3\cos\frac{x}{4}\sin\frac{x}{4}+\frac{16320}{3888}t^3\cos^3\frac{x}{4}\sin^5\frac{x}{4}\\ & -\frac{2304}{2592}t^3\cos^5\frac{x}{4}\sin\frac{x}{4}+\frac{480}{1296}t^3\cos^3\frac{x}{4}\sin^3\frac{x}{4}+\frac{288}{1296}t^3\cos^3\frac{x}{4}\sin^2\frac{x}{4}-\frac{96}{1296}t^3\cos\frac{x}{4}\sin\frac{x}{4}\end{aligned}$$

and the rest of the components of iteration formula (8) are obtained. The approximate solution which involves few terms is given by

$$\begin{aligned}u \;=\;& \sum_{i=0}^{2}u_i=\frac{4}{3}\cos^2\frac{x}{4}+\frac{16t}{9}\cos^3\frac{x}{4}\sin\frac{x}{4}-\frac{2t}{3}\cos\frac{x}{4}\sin\frac{x}{4}+\frac{1327104}{3359232}t^3\cos^7\frac{x}{4}\sin\frac{x}{4}\\ & +\frac{6801408}{3359232}t^3\cos^5\frac{x}{4}\sin^3\frac{x}{4}+\frac{3936}{1296}t^3\cos^3\frac{x}{4}\sin^3\frac{x}{4}+\frac{16}{54}t^3\cos\frac{x}{4}\sin\frac{x}{4}+\frac{16320}{3888}t^3\cos^3\frac{x}{4}\sin^5\frac{x}{4}\\ & -\frac{2304}{2592}t^3\cos^5\frac{x}{4}\sin\frac{x}{4}+\frac{480}{1296}t^3\cos^3\frac{x}{4}\sin^3\frac{x}{4}+\frac{288}{1296}t^3\cos^3\frac{x}{4}\sin^2\frac{x}{4}\\ & -\frac{96}{1296}t^3\cos\frac{x}{4}\sin\frac{x}{4}\ldots\end{aligned}$$

Example 3.5. We consider the non-linear Klein-Gordon equation

$$u_t=u_{xx}-u+u^3 \tag{17}$$

subject the initial condition

$$u(x,0)=-\sec hx \tag{18}$$

from (5) and (18) we obtain

$$u_0(x,t)=-\sec hx$$

Therefore, the initial value problem (17) and (18) is equivalent to the following integral equations:

$$u(x,t)=-\sec hx+I_t(u_{xx}-u+u^3)$$

Taking

$$N(u)=I_t(u_{xx}-u+u^3)$$

Therefore from (5), (6) and (7) ,we can obtain easily the following first few components of the new iterative solution for the equation (17) and (18)

$$u_0(x,t) = -\sec hx$$

$$u_1 = \left(-\sec hx \ \tan h^2 x\right)t + (\sec hx)\,t$$

$$\begin{aligned} u_2 \;=\; & -t^2 \sec h^5 x + \frac{9}{2}t^2 \sec h^3 x \ \tanh^2 x - t^2 \sec hx + \frac{t^2}{2}\tanh^4 x \\ & -\frac{t^2}{2}\sec h^3 x + t^2 \sec hx \ \tanh^2 x - \frac{t^2}{2}\sec hx \\ & -\frac{t^4}{4}\sec h^3 x \ \tanh^6 x + \frac{3t^4}{4}\sec h^3 x \ \tanh^4 x - \frac{3t^4}{4}\sec h^3 x \ \tanh x \end{aligned}$$

and the rest of the components of iteration formula (8) are obtained. The approximate solution which involves few terms is given by

$$\begin{aligned} u \;=\; & \sum_{i=0}^{2} u_i = -\sec hx - (\sec hx \ \tan h^2 x)t + (\sec hx)\,t - t^2 \sec h^5 x + \frac{9}{2}t^2 \sec h^3 x \ \tanh^2 x \\ & -t^2 \sec hx + \frac{t^2}{2}\tanh^4 x - \frac{t^2}{2}\sec h^3 x + t^2 \sec hx \ \tanh^2 x - \frac{t^2}{2}\sec hx \\ & -\frac{t^4}{4}\sec h^3 x \ \tanh^6 x + \frac{3t^4}{4}\sec h^3 x \ \tanh^4 x - \frac{3t^4}{4}\sec h^3 x \ \tanh x \ldots \end{aligned}$$

4. Conclusions

In this paper, the NIM were successfully applied for finding the approximate solutions of the nonlinear with initial conditions. The fact that the NIM solve nonlinear problems without using Adomian's polynomials and He's polynomials is a clear advantage of this technique over the decomposition method. The results show that the method are powerful and efficient techniques in finding exact and approximate solutions for nonlinear differential equation.

References

[1] Ahmet Yildirim, Canan Unin and Syed Tauseef Mohyad.(2010), Exact Solitary-Wave Special Solutions For the Nonlinear Dispersive K(2,2) Equations by Method, Appl.Appl.Math, ISSN: 1932-9466, Special lssue No.1,pp 82-93.

[2] Benjamin-Bona-Mahony equation, from Wikipedia, the free encyclopedia.

[3] Bhalekar S,Daftardar-Gejji V.(2008). New iterative method: application to partial differential equations,Applied Mathematics and Computation,203(2):778-783.

[4] Bhalekar S,Daftardar-Gejji V,(2010). Solving evolution equations using a new iterative method, Numerical Methods for partial Differential Equations,26(4):906-916

[5] Cherruauit Y,(1989),Convergence of Adomian's method.Kybernetes,18(2):31-38.

[6] Daftardar-Gejji Bhalekar VS.(2010). solving fractional boundary value problems with dirichlet boundary conditions using a new iterative method, Computers & Mathematics with Applications,59(5):1801-1809.

[7] Daftardar-Gejji V,Jafari H.(2006),An iterative method for solving nonlinear functional equations, Journal of Mathematical Analysis and Applications,316(2):753-763.

[8] Gardner equation, from Wikipedia, the free encyclopedia.

[9] Hemeda AA.(2012).Formulation and solution of nth-order derivative fuzzy integro-differential equation using new iterative method with a reliable algorithm, Journal of Applied Mathematics, Article ID 325473,17 pages.

[10] Jerri AJ.(1999).Introduction to Integral Equations with Applications. second Ed.Wiley.Interscience

[11] Marwan Taiseer Alquran. (2012). Applying Differential Transform Method to Nonlinear Partial Differential Equations: A Modified Approach, Applications and Applied Mathematics,vol.7, Issue 1,pp.155-163.

[12] Mehdi Dehghan, Ali Shokri.(2009). Numerical solution of the non-linear Klein-Gordon equation using radial basis function, Journal of computational and Applied mathematics, Vol. 230, Issue. 2,pp 400-410..

[13] Murat Gubes,Yildiray Keskin and Galip Oturanc.(2015),Numerical Solution of time-dependent Foam Drainage Equation(FDE),Computational Methods for Differential Equations,vol.3,No. 2,pp.111-122.

[14] Yavuz Ugurlu and Dogan Kaya,(2008), Solutions Of The Cahn-Hilliard Equation, Computers & Mathematics With Applications, vol 56, Issue 12,pp 3038-3045.

[15] Mohamed A. Ramadan and Mohamed S. Al-luhaibi, (2014), New iterative method for solving the Fornberg-Whitham equation and comparison with Homotopy Perturbation transform method, British journal of Mathematics and Computer Science, 4(9) 1213-1227