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SEMIGLOBAL TOTAL DOMINATION IN GRAPHS

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ABSTRACT

A subset D of vertices of a connected graph G is called a semiglobal total dominating set if D is a dominating set for G and G^{sc} and < D > has no isolated vertex in G, where G^{sc} is the semi complementary graph of G. The semiglobal total domination number is the minimum cardinality of a semiglobal total dominating set of G and is denoted by $\gamma_{sgt}(G)$. In this paper exact values for $\gamma_{sgt}(G)$ are obtained for some graphs like cycles, wheel and paths are presented as well.

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KEYWORDS: Semicomplete grap; global total domination number; semicomplementary graph; semiglobal total domination number.

1. INTRODUCTION

For a comprehensive introduction to theoretical and applied facts of domination in graphs the reader is directed to the book [4].A set D of vertices is called a dominating set of G if each vertex not in D is joined to some vertex in D. The domination number $\gamma(G)$ is the minimum cardinality of the dominating set of G [4].

Many variants of the domination number have been studied. For instance a dominating set D is called a Global dominating set of G if D is a dominating set of both G and G^c. The global domination number of G, denoted by $\gamma_g(G)$ is the minimum cardinality of the global dominating set of G. This concept was introduced independently by Brigham and Dutton [1] (the term factor domination number was used) and Sampathkumar [7]. A dominating set D of a graph G is a total dominating set if the induced sub graph $< D >$ has no isolated vertices. The total domination number $\gamma_t(G)$ is the minimum cardinality of a total dominating set of G. This concept was introduced by Cockayne, Dawes and Hedetniemi in [2].

A dominating set D of a connected graph is called a independent dominating set of G if the induced subgraph $< D >$ is a null graph [4].G be a connected graph, then the semi complementary graph of G, denoted by G^{sc}, has the same vertex set as that of G and has edge set {uv / u,v ∈ V(G), uv $\notin E(G)$ and there is w ∈ V(G) such that uw,wv ∈ E(G)} [6]. In this paper, we introduce a new graph parameter, the semiglobal total domination number, for a connected graph G. We call D ⊆ V(G) a semiglobal total dominating set, if D is a dominating set for G and G^{sc} and $< D >$ has no isolated vertices in G, where G^{sc} is the semicomplementary graph of G. The semiglobal total domination number is the minimum cardinality of a semiglobal total dominating set of G and is denoted by $\gamma_{sgt}(G)$.

2. MAIN RESULTS

Theorem 2.1 For any graph of order n, $2 \leq \gamma_{sgt}(G) \leq n$.

Proof: By definition a semiglobal total dominating set needs at least 2 vertices and so

$\gamma_{sgt}(G) \geq 2$. The set of all vertices of G is clearly a semiglobal total dominating set of G so that $\gamma_{sgt}(G) \leq n$. Thus $2 \leq \gamma_{sgt}(G) \leq n$. ■

In the following theorems we give the exact values of some graphs.

Theorem 2.2 $\gamma_{sgt}(K_{m,n}) = 2$ m, n ≥ 2.

Proof: Let V_1 and V_2 be the partite sets of $K_{m,n}$ with $|V_1|$ = m , $|V_2|$ = n. Every vertex in a partite set dominates every other vertex of the other. Then D = {u_i, v_j } is a minimal dominating set for $K_{m,n}$ where $u_i \in V_1$, $v_j \in V_2$, for some i and j. Then the induced subgraph $< D >$ has no isolated vertex. The semicomplementary graph of the complete bipartite graph $K_{m,n}$ is a disconnected graph $K_m \cup K_n$, where $<V_1> = K_m$ and $<V_2> = K_n$. Any two vertices in V_1 or that of V_2 are adjacent in $K_{m,n}^{sc}$. Hence D

= { u_i, v_j } is a dominating set for the semicomplementary graph of $K_{m,n}$. Thus $\gamma_{sgt}(K_{m,n}) = 2$, m, n ≥ 2. ■

Corollary 2.3 $\gamma_{sgt}(K_{1,n}) = 2$, n ≥ 2. ■

The crown graph $C_n \odot K_1$ is the graph obtained from cycle C_n by attaching a pendant edge to each vertex of the cycle.

Theorem 2.4 $\gamma_{sgt}(C_n \odot K_1) = n$

Proof: Let $G = C_n \odot K_1$. $V(G) = \{ v_0, v_1, v_2, \ldots, v_{n-1}\} \cup \{ u_0, u_1, u_2, \ldots, u_{n-1} \}$.

$E(G) = \{ v_i v_{i+1} / i = 0, 1, 2, \ldots, n-1$, subscript modulo n$\} \cup \{ u_i v_i / i = 0, 1, 2, \ldots, n-1\}$. Let D be a minimal semiglobal total dominating set of G. Then D must contain n vertices of the cycle in $C_n \odot K_1$. Since $< D > = C_n$ and D dominate the vertices of G as well as G^{sc},

$\gamma_{sgt}(C_n \odot K_1) = n$ ■

Theorem 2.5 Let G be a complete graph, then $\gamma_{sgt}(K_n) = n$.

Proof: Let D be a minimal semiglobal total dominating set of G. Obviously $|D| \leq n$ by Theorem 1.4. If $|D| < n$, then D can only dominate vertices of G and D does not dominate G^{sc}, since G^{sc} is totally disconnected. ■

Remark 2.6 The bounds in Theorem 1.4 are sharp. For the complete graph K_n **(**$n \geq 2$), $\gamma_{sgt}(K_n) = n$. For the complete bipartite graph $K_{m,n}$, ($m \geq 2$, $n \geq 2$), $\gamma_{sgt}(K_{m,n}) = 2$. Thus K_n **(**$n \geq 2$) has the largest possible semiglobal total domination number n and the complete bipartite graph have the smallest semiglobal total domination number 2.

Theorem 2.7 For any wheel W_n, $\gamma_{sgt}(W_n) = 3$, $n \geq 4$.

Proof: $V(W_n) = \{w, v_0, v_1, v_2, v_3, \ldots, v_{n-1}\}$, be the vertex set of the wheel W_n.

Let $D = \{w, v_0, v_{n-1}\}$ be the semiglobal total dominating set of W_n . The vertex w is the only in the semi complementary graph of W_n. Hence D must contain w. The vertices v_0, v_{n-1} that is the initial and end vertices of the cycle of the wheel dominate all the other vertices in G^{sc}. Hence D is the minimal semiglobal total dominating set of W_n . Hence $\gamma_{sgt}(W_n) = 3$ if $n \geq 4$. ■

Theorem 2.8 For a path P_n on n vertices,

$$\gamma_{sgt}(P_n) = \begin{cases} \frac{n}{2} \; if \; n \equiv 0 \; (mod\ 4); \\ \frac{n+1}{2} \; if \; n \equiv 1,3 \; (mod\ 4); \\ \frac{n}{2} + 1 \; if \; n \equiv 2 \; (mod\ 4). \end{cases}$$

Proof: Let P_n be the path of order n. $V(P_n) = \{v_0, v_1, v_2, v_3, \ldots, v_{n-1}\}$.

If $G = P_n$ $(n \geq 3)$ then $G^{sc} = P_{\frac{n}{2}} \cup P_{\frac{n}{2}}$ if n is even,

$= P_{\frac{n+1}{2}} \cup P_{\frac{n-1}{2}}$ if n is odd.

Let D be a minimal semiglobal total dominating set in G, which must contain v_{n-1}, the end vertex of P_n.

If $v_0, v_1, v_2, v_3, \ldots, v_{n-1}$ are the vertices of G then $v_0, v_2, \ldots, v_{n-2}$ induce $P_{\frac{n}{2}}$ and vertices $v_1, v_3, \ldots, v_{n-1}$ induce another $P_{\frac{n}{2}}$,when n is even. When n is odd, $P_{\frac{n+1}{2}}$ and $P_{\frac{n-1}{2}}$ are respectively induced by $\{v_0, v_2, \ldots, v_{n-1}\}$ and $\{v_1, v_3, \ldots, v_{n-2}\}$. Let $v_0 \notin D$. If $v_1 \in D$ then v_2 should be in D to ensure the totality property. Now we consider four cases.

Case i) $n \equiv 0 \; (mod\ 4)$.

Then $D = \{v_{4i+1}, v_{4i+2} \,/\, i = 0, 1, \ldots, \frac{n}{4} - 1\}$.

Case ii) $n \equiv 1 \; (mod\ 4)$.

Then $D = \{v_{4i+1}, v_{4i+2} \,/\, i = 0, 1, \ldots, \frac{n-1}{4} - 1\} \cup \{v_{n-2}\}$.

Case iii) $n \equiv 2 \; (mod\ 4)$.

Then $D = \{v_{4i+1}, v_{4i+2} \,/\, i = 0, 1, \ldots, \frac{n-2}{4} - 1\} \cup \{v_{n-3}\} \cup \{v_{n-2}\}$.

Case iv) $n \equiv 3 \; (mod\ 4)$.

Then D = $\{v_{4i+1}, v_{4i+2}$ / $i = 0, 1, \ldots, \frac{n}{4}\}$.

Hence the result follows. ■

Theorem 2.9 For a cycle C_n on n vertices,

$$\gamma_{sgt}(C_n) = \begin{cases} \frac{n}{2} & if \quad n \equiv 0 \ (mod\ 4); \\ \frac{n+1}{2} & if \quad n \equiv 1,3 \ (mod\ 4); \\ \frac{n}{2}+1 & if \ n \equiv 2 \ (mod\ 4). \end{cases}$$

Proof: The result follows from theorem 1.11. ■

Theorem 2.10 [5] G be a connected graph with vertex set V. Then $G^c = G^{sc}$ if and only if the distance between any pair of nonadjacent vertices is 2.

The following theorem relates the global domination and the semiglobal total domination number of C_n

Theorem 2.11 For C_n where n = 4, 5, $\gamma_g(C_n) = \gamma_{sgt}(C_n)$.

Proof: Consider the graph C_4 and C_5. The distance between any pair of nonadjacent vertices is 2 in C_4 and C_5. Hence $G^c = G^{sc}$ by the above theorem. Let D be minimal global dominating set and D_1 be minimal semiglobal total dominating set of C_4 and S be minimal global dominating set and S_1 be minimal semiglobal total dominating set of C_5 . Hence $D = D_1$ and $S = S_1$. Therefore

$\gamma_g(C_n) = \gamma_{sgt}(C_n)$ where n = 4, 5. ■

Theorem 2.12 Let G be a graph of order $n \geq 3$. If $G \cong K_n - e$ then $\gamma_{sgt}(G) = n-1$.

Proof: Let $G \cong K_n - e$, where $e = uv \in E(K_n)$. So $uv \neq E(G)$ and hence $uv \in E(G^{sc})$. The G^{sc} contains n-2 isolated vertices, say $v_2, v_3, \ldots, v_{n-1}$. Hence every minimal semiglobal total dominating set D must contain all the n-2 isolated vertices .Thus $D = \{v_2, v_3, \ldots, v_{n-1}\} \cup \{u\}$ (or $\{v\}$). Thus $\gamma_{sgt}(G) = n-1$. ■

The following theorem relates the semiglobal total domination number and the minimum degree of G.

Theorem 2.13 Let G be a graph with diam(G) $\leq$ 2. Then $\gamma_{sgt}(G) \leq \delta(G) + 1$.

Proof: Let x be a vertex of minimum degree in G. Since $1 \leq d(G) \leq 2$, then N(x) is a dominating set for G. Now {x} ∪ N(x) is a dominating set for G^{sc} and also a total dominating set for G. Thus we have D = {x} ∪ N(x) is a semiglobal total dominating set for G and

$|D| = \delta(G) + 1$. Hence the result. ■

Theorem 2.14 Let T be a non trivial tree where $T \neq K_{1,n}$ or $T \neq P_n$ then $\gamma_{sgt}(T) = n - |L|$, where $|L|$ denotes the number of pendant vertices of T.

Proof: Let T be a non trivial tree where $T \neq K_{1,n}$ or $T \neq P_n$. Let D = {v ∈ $V(G)$/ v is not a pendant vertex}. Let u ∈ D be any vertex. Let deg(u) = k (say).Since G is not a star there exists a non pendant vertex v ∈ D adjacent to u. Then u dominates the vertices of N(u) in G and u dominates them in G^{sc} and vice versa. Hence D is the minimal semiglobal total dominating set.

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