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On the analytical study of Mellin moments of parton distribution functions

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Abstract.

Mellin moments of parton distribution functions and structure functions are used for obtaining the solution of the scale evolution equation of parton densities as well as for the evaluation of scattering cross sections.They are obtained as integrals of the distribution or structure functions over the Bjorken-*x* variable. Mellin moment is indeed an important tool in the study of structure functions. In this paper, we review the evolution equations of Mellin moments of parton distribution functions. We also study their applications in QCD analysis. Without making any assumption, scaling violation can be studied based on the shape of input parameterization of parton densities.

Keywords: Mellin moments, Parton distribution functions, QCD.

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Introduction

Studies of parton distribution functions (PDFs) open a new way to a better understanding of the partonic quark-gluon structure of the nucleon. PDFs [1] are the most important quantities in Quantum Chromodynamics (QCD) to study the structure function of proton in deep inelastic scattering (DIS). They are among the most important sources of information on hadron structure at the level of quarks and gluons. The quest for a sound understanding of the dynamics behind parton interactions, particularly in the era of LHC, has driven the interest of many high-energy physicists. Our current knowledge of such dynamics is very limited, and it should be interesting to investigate them in approaches that have been successfully applied to single parton densities. One possible avenue is to take Mellin moments of two-parton distributions [2]. This will naturally give limited information about the dependence of the distributions on their momentum fractions, but it has the potential to quantify parton correlations in a genuinely non-perturbative framework. Mellin moment is not only used for obtaining the solution of the scale evolution equation of the parton densities but also for the evaluation of scattering cross sectionsand the gluon density could be extracted in next-to-leading order QCD [3]. It is also an important tool in the derivation of asymptotic expansions. The preliminary studies have revealed that such a technique can be used in situations where numerical evaluation of scattering cross sections for varying PDFs is required.

PDFs and DIS structure functions often encounter Mellin moments [4-6]. Mellin moments of the parton distribution functions and structure functions are obtained as integrals of the distribution or structure functions over the Bjorken-*x* variable [7]. Mellin moment is indeed an important tool in the study of structure functions.The standard Dokshitzer-Gribov-Lipatov-Altarelli-Parisi(DGLAP) evolution equation [8-11] is the most familiar resummation technique that describes scaling violations of parton densities.

In other words, the Mellin transformation method is one of the popular evolution methods [12]. It is used because Mellin transformation of the DGLAP equation becomes a simple multiplication of two moments, viz., moments of the splitting function and the distribution function. In this paper, we review the evolution equations of moments of parton distribution functions and their applications in QCD.

Methodology

According to the well-known DGLAP evolution equation [8-11], the quark and gluon distribution functions change with Q^2. The DGLAP equation can be solved with the help of either Mellin transform or the polynomial expansion in *x*-space.

The Q^2 evolution of quark and gluon densities is described by the DGLAP equation, which has the form

$$\frac{dq\,(x,t)}{dt} = \frac{\alpha_s(t)}{2\pi}(P \otimes q)(x,t) \tag{1}$$

where $q\ (x,\ t)$ denotes the parton distribution function, $\alpha_s\ (t)$ is the running coupling constant and $\otimes$ denotes the Mellin convolution.

$$t = ln\frac{Q^2}{\Lambda^2_{QCD}} \tag{2}$$

and

$$(A \otimes B)(x) \equiv \int_x^1 \frac{dz}{z} A\left(\frac{x}{z}\right) B(z) \tag{3}$$

The splitting function $P\ (z,\ t)$ can be expanded in the perturbative series of $\alpha_s\,(t)$.

The *n*-moment of the function $f\,(x)$ is defined as follows:

$$\tilde{f}_n = \int_0^1 x^{n-1} f(x)dx \tag{4}$$

Taking into account the relation, the convolution can be factorized into simple products as

$$\int_0^1 dx x^{n-1}(A \otimes B)(x) = \tilde{A}_n \tilde{B}_n \tag{5}$$

The moment of parton distribution function thus obeys the evolution equation

$$\frac{d\tilde{q}_n(t)}{dt} = \frac{\alpha_s(t)}{2\pi}\gamma_n(t)\tilde{q}_n(t) \tag{6}$$

where the anomalous dimension $\gamma_n(t)$ is a moment of the splitting function $P(z, t)$ and is given as

$$\gamma_n(t) = \int_0^1 dz z^{(n-1)} P(z,t) \tag{7}$$

It is interesting to note that Eq. (6) can be solved analytically to find the parton distribution function$q(x, t)$ via inverse Mellin transform as

$$q(x,t) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dn x^{-n}\tilde{q}_n(t) \tag{8}$$

Results and discussion

The DGLAP evolution equations for the truncated Mellin moments of parton distributions functions have been derived in Ref. [13]. The determination of parton densities is shown from the first truncation[14]; which is useful both from the theoretical and experimental points of view. The perturbative QCD analysis is based on such evolution equations satisfied by the parton distribution functions. In Ref. [5], the different small-x behaviour of the initial parametrization of the first moment of non-singlet spin dependent structure function has been discussed.

Conclusion

The literature survey has shown that Mellin moment has been used in asymptotic analysis for estimating asymptotically the moments of Drell-Yan cross sections [15]. The Mellin transformation method is used to obtain the analytical solution in moment space. The integro-differential equations become very simple using this transformation method. The need to study this is also based on the fact that it is used in the analytical calculation of moments of structure functions and the longitudinal structure function on perturbative QCD up to second order corrections and in leading twist approximation.

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