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Dependence and Value at Risk in the Stock Markets From the Americas: A Copula Approach

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Abstract

This work applies copula modeling to estimate the degree of dependence among the nine major equity markets from the Western Hemisphere, seven emerging markets from Latin America (Argentina, Brazil, Chile, Colombia, Peru, Venezuela, Mexico) and the two mature markets from North America (Canada, United States). The relevance of copula-measured dependence is assessed estimating Value at Risk for bilateral portfolio investments, comparing it with conventional VaR methodologies. The data encompass daily time series for the 1992-2009 period.

Keywords: Copula; Value at Risk; Stock Markets North America.

1. Introduction

Two important dimensions about equity markets concern their degree of association and its impact on risk. This paper analyzes these problems for the case of the stock markets from the Americas. It assesses and compares their levels of association, and estimates potential losses for bilateral investments in these markets.

Two important theories to deal with these issues are copula theory and Value at Risk. Their applications have been basically limited to examine the case of developed markets. Literature dealing with dependence and it is application to VaR analysis is practically absent for the case of the emerging markets concerning their relationship among themselves, as well as with their relationship with developed markets. This study is a contribution in this respect. It applies copula theory to determine the degree of dependence between nine stock markets from the Americas and makes use of this methodology to estimate potential losses via VaR analysis. The capital markets considered in this paper are from North America: United States, Canada and Mexico; and from South America: Argentina, Brazil, Chile, Colombia, Peru and Venezuela. Daily data for the 1992-2009 period is employed.

The work is structured in four sections in addition to this introduction. Section 2 presents a literature review stressing recent works dealing with applications of copula theory about stock markets and risk evaluation. Section 3 details the methodology detailing copula theory and four alternatives for Value at Risk: Delta Normal; Historical Simulation; Conditional Volatility VaR (VaR-GARCH), and Monte Carlo Simulation VaR (MCS- VaR). Section 4 advances the empirical analysis. Finally, Section 5 concludes the work.

2. Literature Review

Modeling with copulas has been used widely for multiple applications in actuarial, economic and financial studies. Copula theory was introduced over sixty years ago as a means to isolate the dependence structure among distribution functions. Partial solutions were fists advanced by Hoedfing (1940; 1941), Frétchet (1951), Dall'Aglio (1956) among others, Sklar (1959) consolidated those advances creating a new class of distributions whose margins are uniform in (0,1). He introduced the idea and the name of copula and its respective theorem now bears his name.

Concerning dependence between stock markets, copula theory has been applied widely. Hu (2003) examines the structure of dependence, rather than the degree of dependence for a sample of international markets and Kole, Koedijk and Verbeek (2005) analyze patterns of dependence between financial markets via copula. Samitas, Kenourgios and Paltalidis (2007) introduce a multivariate time-varying copula with Markov switching parameters to capture non-linear relationships for the BRIC countries plus United States and United Kingdom. Their evidence shows that during turbulent periods dependence increases. Chan-Lau, Mathieson and Yao (2004), Gonzalo and Olmo (2005) and Lopez (2006) and Messaud and Aloui (2015) have conducted contagion studies about financial markets through copula methodology. Ning (2012) explore the dependence structure between the equity market and exchange rates using several copulas with different dependence structures; her findings reveal a significant upper and lower dependence for these two markets. Boubaker and Sghaier (2014) determine dynamic dependence between the U.S. and other developed stock markets using an extreme value time-varying copula approach, for a period that includes the 2007-2009 crisis. Their empirical evidence suggest that dependence between the US and Japanese market is symmetric while dependence between the US and European markets is asymmetric.

Risk analysis, applying copula theory has also been dealt with in the financial literature. Nguyen and Huynh (2015) present two VaR estimation models for six currencies; each return series is assumed to follow an ARCH (1,1) - GARCH (1,1) model; innovations are simultaneously generated using Gaussian copula and student-t copula. Bob (2013) estimate VaR for a portfolio combining copula functions, Extreme Value theory, and GARH models; countries included in the portfolio are Germany, Spain, France, and Italy. Reyes and Ortiz (2013) analyze VaR for the stock markets from the North American Free Trade Agreement (NAFTA) bloc. Their analysis of trinational portfolios apply a multivariate GARCH model. Hsu, Huang and Chiou (2011) assess portfolio risk for six Asian markets using copula-extreme value based on semiparametric approaches. Monte Carlo VaR simulation suggests that the Clayton copula EVT yields the best results regardless of the shapes of the return distributions. Torres and Olarte (2009) employ copula modeling for VaR analysis. Embrechts, Höing and Pucceti (2005) used copula methodology to create diverse scenarios for VaR analysis, in order to identify the worse situation. Rank (2007) demonstrates the reliability of copula methodology for VaR analysis using the Monte Carlo approach. He applied copula theory to create various scenarios of VaR. Shim, Lee, MacMinn (2011) apply a copula approach to measure economic capital, Value-at-Risk and expected shortfall. Along similar research lines deserve to be mentioned some works about portfolio optimization. Krzemienowski and Szymczyk, (2016) optimize portfolios applying a copula based extension of conditional value at risk. Yingying, Pu, and Xang (2016) examine correlations and risk contagion between mixed assets and mixed-asset portfolio VaR measurements. Their methodology follow a dynamic view based on time varying copula models.

It should be noted that most studies analyze the case of developed financial markets; copula studies on emerging markets are still scarce. Some early studies include research by Hotta, Luke and Palaro (2008) and Ozun and Cifter (2011) who used copula theory in VaR valuation about Latin American emerging market portfolios. More recently, Chebbi, and Hedhli (2014) study comovements between the Tunisian, US French and Moroccan stock markets. Hussain and Li (2015) use extreme value theory to model dependence structures between the Chinese stock market and the US, UK, Japan, Hong Kong and Taiwan market. Finally, Lopez-Herrera, Santillan-Salgado and Cruz Ake (2016) examine volatility dependence structure between the Mexican and the world capital market. They employ a bivariate VECM in the mean, a VARMA-GARCH model with dynamic conditional correlation and finally fit a Clayton copula returns on two volatility regimes.

3. Methodology

Copula Theory

i. Definition

A function $C: [0,1]^n \rightarrow [0,1]$ is a n-copula with the following properties:

- $\forall\, u \in [0,1],\ C(1,\dots,1,u,1,\dots,1) = u$.
- $\forall\, u_i \in [0,1],\ C(u_1,\dots,u_n) = 0$ if at least one of the u_i's equals zero.
- positive. C is defined and n-increasing, i.e., C-volume of each box whose vertices are located in $[0,1]^n$ is

It follows from this definition that a copula is a multivariate distribution function over the range $[0,1]^n$ with uniform margins. In other words, copula is defined as a multivariate distribution function defined in the unitary cube $[0,1]^n \times [0,1]$ with margins uniformly distributed. This definition is very natural if taking into account how it is derived a copula from a multivariate continuous distribution; in fact, in this case the copula is simply the original multivariate distribution function with a transformation to a univariate margin.

Sklar's Theorem

Let F a n-dimensional distribution function with continuous margins$F_1, \dots, F_n$, exists a unique n- copula $C: [0,1]^n \rightarrow [0,1]$, such that

$$F(x_1, \dots, x_n) = C\big(F_1(x_1), \dots, F_n(x_n)\big) \qquad (1)$$

Therefore, copula combines the margins to form multivariate distribution function. This theorem provides a parameterization of multivariate distribution and construction scheme of copulas. In fact, given a distribution F with multivariate margins $F_1, \dots, F_n$, the function,

$$C(u_1, \dots, u_n) = F\left(F_1^{-1}(u_1), \dots, F_n^{-1}(u_n)\right) \qquad (2)$$

is automatically a n-copula. This copula is the copula of multivariate distribution$F$.

From the Sklar's theorem, is known that for any multivariate distribution function can be easily derived as a copula. Although there are a considerable number of copulas, only a few family copulas play an important role. Among copula families this work relies on the family of elliptical copulas.

ii. Elliptical Copulas

Elliptical copulas[1] are those have had ample application about financial markets. Peculiarity of these copulas is that they are associated with random variables whose multivariate distribution function is symmetric; this leads to level sets of elliptical shape. Within this family of copulas two of the most important are the normal copula (Gaussian copula) and t-Student copula; which derive multivariate distribution functions that have the same names.

Normal Copula. Normal copula is the copula derived from multivariate normal distribution. Normal copula provides a natural generalization of multivariate normal distributions.

Φ denotes the normal distribution (cumulative) and $\Phi_{\rho,n}$ denotes the n-dimensional standard normal distribution with correlation matrix ρ.

Normal n-copula with correlation matrix ρ is:

$$C_{\rho,n}(u_1, u_n) = \Phi_{\rho,n}\big(\Phi^{-1}(u_1), \Phi^{-1}(u_2)\big) \qquad (3)$$

Whose density is:

$$c_{\rho,n}(u_1, u_n) = \frac{1}{\sqrt{det\,\rho}}\, exp\left(-\frac{1}{2} y^t(u)(\rho^{-1} - Id)y(u)\right) \qquad (4)$$

$$\text{with } y^t(u) = \big(\Phi^{-1}(u_1), \Phi^{-1}(u_2)\big)$$

Normal copula is completely determined by knowledge of the correlation matrix coefficientρ.

t-Student Copula. t-Student copula is derived from the multivariate t-Student distribution. T-Student copula provides a natural generalization of multivariate t-Student distributions.

Let $T_{n,p,\nu}$ a n-dimensional t-Student distribution with ν degrees of freedom and correlation matrix ρ,

$$T_{n,p,\nu} = \frac{1}{\sqrt{det\,\rho}} \frac{\Gamma\left(\frac{\nu+n}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)(\pi\nu)^{n/2}} \int_{-\infty}^{x_1}\int_{-\infty}^{x_2} \frac{dx}{\left(1 + \frac{x^t\rho^{-1}x}{\nu}\right)^{\frac{\nu+n}{2}}} \qquad (5)$$

t-Student copula is:

$$C_{n,\rho,\nu}(u_1, u_n) = T_{n,p,\nu}\left(T_\nu^{-1}(u_1), T_\nu^{-1}(u_2)\right) \qquad (6)$$

where T_ν is the univariate t-Student distribution with ν degrees of freedom.

Density of t-Student copula is:

[1] Elliptical copulas are derived from multivariate elliptical distributions (Johnson and Kotz, 1972).

$$c_{n,p,\nu}(u_1, u_n) = \frac{1}{\sqrt{det\,\rho}} \frac{\Gamma\left(\frac{\nu+n}{2}\right)\left[\Gamma\left(\frac{\nu}{2}\right)\right]^{n-1}}{\left[\Gamma\left(\frac{\nu+1}{2}\right)\right]^{n}} \frac{\prod_{k=1}^{n}\left(1+\frac{{y_k}^2}{\nu}\right)^{\frac{\nu+1}{2}}}{\left(1+\frac{y^t\rho^{-1}y}{\nu}\right)^{\frac{\nu+n}{2}}} \qquad (7)$$

$$\text{with } y^t = \left({T_\nu}^{-1}(u_1), {T_\nu}^{-1}(u_2)\right)$$

Description of t-Student copula is based on two parameters: the correlation matrix coefficient ρ, like in the case of normal copula, and also in the number of degrees of freedom ν.

iii. Dependence Measure with Copulas

Let X_1 and X_2 random variables with continuous margins distribution function F_1 and F_2 and joint distribution function F. It is interesting to examine whether traditional notions of dependence such as Pearson's correlation ρ, and Kendall τ and Spearman ρ_s rank correlations can be expressed in terms of copulas.

Pearson's correlation is given by:

$$\rho(X_1, X_2) = \frac{\iint_0^1 [C(u_1,u_2) - u_1u_2]\, dF_1^{\leftarrow}(u_1)\, dF_2^{\leftarrow}(u_2)}{SD(X_1)SD(X_2)} \qquad (8)$$

Kendall's correlation is given by

$$\tau\,(X_1, X_2) = 4\iint_0^1 C(u_1,u_2)\, dC(u_1,u_2) - 1 \qquad (9)$$

Spearman's correlation is given by

$$\rho_s\,(X_1, X_2) = 12\iint_0^1 [C(u_1,u_2) - u_1u_2] du_1\, du_2 \qquad (10)$$

We can see that Kendall and Spearman rank correlations are functions that depend on copula of X_1 and X_2, while the Pearson's linear correlation coefficient also depends on the margins. In the case of elliptical copulas exists a relationship between rank correlation and linear correlation. If (X_1, X_2) has a bivariate elliptical copula and continuous arbitrary margins. Rank correlations are:

$$\tau\,(X_1, X_2) = \frac{2}{\pi}\arcsin\rho \qquad (11)$$

and

$$\rho_s\,(X_1, X_2) = \frac{6}{\pi}\arcsin\frac{\rho}{2} \qquad (12)$$

Where ρ is the correlation between X_1 and X_2.

Relationship between Kendall τ and Pearson's ρ given in the above equation, is general for copulas of all elliptical distributions, such as normal copula and t-Student copula. This can be used to construct a linear correlation robust estimator. Simply replacing the empirical value of Kendall τ in the previous equation and solving for ρ, we have

$$\hat{\rho} = \sin\left(\frac{1}{2}\pi\hat{\tau}\right) \qquad (13)$$

iv. Estimated Copula Parameters

There are several methodologies for estimating the parameters associated to copula. Only two of them are mentioned, which are used in estimating the parameters of the elliptical copulas proposal.

a) Concordance Measures

This methodology is based on the relationship between concordance measures (rank correlation coefficients) and the copula parameters; in other words, between nonparametric correlation measures analyzed in this paper, which are Spearman and Kendall correlation coefficients, versus copula parameters.

b) Maximum Likelihood

This mechanism can be applied to estimate any copula family because can get the estimation of copula parameters by maximizing its log-likelihood function. The following explains in detail:

Let C a copula, such that,

$$F(x_1, \dots, x_n) = C\big(F_1(x_1), \dots, F_n(x_n)\big) \qquad (14)$$

with density function,

$$f(x_1, \dots, x_n) = c\big(F_1(x_1), \dots, F_n(x_n)\big) \cdot \prod_{j=1}^{n} f_j(x_j) \qquad (15)$$

Therefore, maximum likelihood estimation mechanism is defined as:

Let X a r.v.i.i.d. vector with multivariate distribution function F and continuous margins distribution function$F_1, \dots, F_n$; log-likelihood function is defined,

$$l(\theta) = \sum_{j=1}^{n} \ln c\left(F_1(x_{j,1}), \dots, F_n(x_{j,n})\right) + \sum_{j=1}^{n}\sum_{i=1}^{d} \ln f_i(x_{j,i}) \qquad (16)$$

where θ are the parameters set, both marginal and copula. Thus, given the marginal set and a copula, the log-likelihood function can be maximized obtaining the maximum likelihood estimator,

$$\hat{\theta}_{MLE} = \max_{\theta \in \Theta} l(\theta) \qquad (17)$$

Value at Risk

Value at Risk (VaR) is a concept developed in the financial framework of risk management. VaR comes from the need to quantify with a given significance level or uncertainty the amount or percentage of loss that a portfolio will face in a predefined period of time (Jorion 2010, Penza and Bansal 2001, Best 1998, Dowd 1998, Holton, 2016).

VaR concept incorporates three factors: a) a time horizon, i.e., a risk manager might be concerned about potential losses whether for a day, week, etc.; b) an associated probability, i.e., VaR represents the potential losses over a given period of time with a given probability; and c) an amount of money to investing.

i. Value at Risk Definition

VaR summarizes the expected maximum loss “or worse loss” over a target time horizon within a stated confidence interval.

ii. Value at Risk and Alternative Measures

In essence, VaR is just a number expressed in monetary units (or in percentages), which indicates the maximum expected loss for a time horizon and a confidence level. This quantity has the characteristic that can be specified using various methodologies, it is therefore not only necessary to specify the parameters mentioned but also the estimation method used in its calculation.

For VaR calculation, previously is required the estimate of portfolio returns to find the loss distribution that describes it; traditionally it has been assumed for simplicity that returns are normally distributed, but the empirical evidence indicates that their behavior follows other patterns so that more refined models have been created using other distributions and/or adjustments in volatility behavior for a better measurement of portfolio losses.

In this paper to estimate VaR we use widely accepted methodologies in financial environment: Delta-Normal VaR, Conditional Volatility VaR (VaR-GARCH), Historical Simulation VaR, and Monte Carlo Simulation VaR (MC-VaR), and subsequently referred as Copula VaR. Each VaR methodologies have their advantages and disadvantages; the efficiency of each depends on its statistical and financial assumptions. Different VaR estimating will be conducted for a portfolio with two assets (risk factors), using daily time series returns for each asset.

a) Delta-Normal VaR

In 1994, J.P. Morgan through his amazing Risk Metrics methodology introduced what is known as Delta-Normal Value at Risk. This methodology is based on Markowitz portfolio theory. Because of its easy implementation is the most common in financial practice; this method is based assuming normal distribution of the data series, i.e., for its estimation is assumed that prices or financial assets returns are independent and identically distributed, assuming a normal distribution. Along this linearity assumption is understood that volatility and correlation are stable during the time horizon analyzed. Delta-Normal VaR is determined as follows:

$$VaR = Z_c\sqrt{w^T\,\Sigma\,w} * \sqrt{\Delta t} \qquad (18)$$

where

Z_c = significance level (critical value) associated with normal distribution

w = asset weights vector

w^T = transposed w

Σ = asset portfolio variance-covariance matrix

Δt = time horizon

Resemblance to normal distribution makes that portfolio loss measurement easy because it is only necessary to estimate the percentile value previously selected. It should be noted that changes in portfolio can be interpreted either as gains or losses depending on whether it has a long or short position, i.e., which side of distribution is located. VaR estimation is located usually on the left side distribution, side corresponding to losses.

b) Conditional Volatility VaR (VaR-GARCH)

Delta-Normal VaR is based on the assumption that asset returns are normally distributed and therefore volatility of these returns is constant, i.e., homoscedastic; however, empirical evidence shows the opposite. To overcome this linearity assumption conditional volatility models we created. These models capture volatility that over time is not stable; in other words, capture heteroscedasticity of data series.

Actually, there are several econometric models about conditional volatility; this research uses only GARCH (Generalized Auto Regressive Conditional Heteroscedasticity) model, implemented by Bollerslev in 1986. GARCH models estimates that conditional variance depends on two factors: square residual innovations, ${\varepsilon^2}_{t-1}, {\varepsilon^2}_{t-2}, \dots, {\varepsilon^2}_{t-p}$, known as ARCH effect, with their respective autoregressive coefficients; and previous conditional variances, ${\sigma^2}_{t-1}, {\sigma^2}_{t-2}, \dots, {\sigma^2}_{t-q}$, known as GARCH effect, with their respective autoregressive coefficients; a intercept term γ_0, and an error term, u_t.

GARCH (p,q) model is expressed as follows:

$${\sigma^2}_t = \gamma_0 + \sum_{i=1}^{p}\alpha_i\,{\varepsilon^2}_{t-i} + \sum_{j=1}^{q}\beta_j\,{\sigma^2}_{t-j} + u_t \qquad (19)$$

where p and q determine the number of lags.

It should be mentioned that the sum of autoregressive coefficients of ARCH and GARCH effects should be less than or equal to 1, i.e., $\alpha_0 + \alpha_1 + \dots + \alpha_p + \beta_0 + \beta_1 + \dots + \beta_q \leq 1$; to ensure that the GARCH process is stationary and ergodic.

Conditional Volatility VaR estimation[2] is analogous to Delta-Normal VaR estimation with variance-covariance matrix decomposition; however here in VaR-GARCH stands the fact that volatilities matrix (standard deviations) is not based on historical volatilities as in Delta-Normal VaR, but this matrix is composed by GARCH volatilities, thus we have the following,

$$VaR = Z_c\sqrt{w^T\,\sigma_G\,C\,\sigma_G\,w} * \sqrt{\Delta t} \qquad (20)$$

where

Z_c = significance level (critical value) associated with normal distribution

w = asset weights vector

w^T = transposed w

σ_G = asset portfolio volatility matrix (GARCH conditional volatility matrix)

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[2] In this research we only specify GARCH methodology, There are several ARCH models. For a comprehensive description of these models see Hamilton (1994), Brockwell and Davis (2009) and Sanchez and Reyes (2005).

C = asset portfolio correlation matrix

$\sigma_G \, \mathrm{C} \, \sigma_G = \Sigma$ = asset portfolio variance-covariance matrix

Δt = time horizon

c) Historical Simulation VaR

Both Simulation VaR and Conditional Volatility VaR have properties more attractive than Delta-Normal VaR, the most significant property of these models is that they do not make any assumption about the structure of asset returns probability distribution.

Historical Simulation VaR is based on generating risk factors (financial assets) scenarios by the observed data of a given period. Currently there are several methodologies for Historical Simulation VaR; however they are similar in their implementation mechanism and through empirical evidence has been found that hey yield similar results.[3]

Historical Simulation algorithm used in this paper is:

i) Collection of assets data that will make up the portfolio,

$$P_k = \begin{bmatrix} P_0 \\ P_1 \\ \vdots \\ P_n \end{bmatrix} \qquad (21)$$

where P_k is the asset prices vector (risk factors vector).

ii) Through the price vector we obtain the returns vector,

$$R_k = \begin{bmatrix} ln\dfrac{P_1}{P_0} \\ ln\dfrac{P_2}{P_1} \\ \vdots \\ ln\dfrac{P_n}{P_{n-1}} \end{bmatrix} = \begin{bmatrix} R_1 \\ R_2 \\ \vdots \\ R_n \end{bmatrix} \qquad (22)$$

where R_k is the asset returns vector.

iii) Obtain the latest price and fixed as seed. Through the last price are simulated the new prices. Seed price is multiplied by exponential of each returns and thus obtain a new data set, i.e.,

$$P_k{}^* = \begin{bmatrix} P_n * e^{R_1} \\ P_n * e^{R_2} \\ \vdots \\ P_n * e^{R_n} \end{bmatrix} = \begin{bmatrix} P_1{}^* \\ P_2{}^* \\ \vdots \\ P_n{}^* \end{bmatrix} \qquad (23)$$

where $P_k{}^*$ is the simulated asset prices vector.

iv) To obtain losses and profits vector requires three steps. Firstly it takes initial position value, W (if it has a portfolio comprised of only one asset) or initial positions, $W_1, W_2, \ldots, W_n$ (if it has a portfolio comprised of more than one asset). Then, are obtained price and weight vectors; this is done by diving initial position value versus ultimate price, and multiplied by each of simulated prices. If it has a portfolio with more than one asset to achieve price and weight vectors is necessary to obtain the sum of each ratio between initial positions and their respective final prices, multiplied by their corresponding simulated prices:

$$v_k = \begin{bmatrix} \dfrac{W}{P_n} * P_1{}^* \\ \dfrac{W}{P_n} * P_2{}^* \\ \vdots \\ \dfrac{W}{P_n} * P_n{}^* \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} \qquad (24)$$

where v_k is the price and weight simulated vector.

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[3] For a detailed knowledge about different mechanisms of Historical Simulation VaR, see De Lara (2009).

v) Next step forward is to obtain residual between prices and weights, and initial position value or initial position values to more than one asset. For the case of more than one asset is performed the residual sum of prices and weights vector versus their respective initial position values, thereby obtaining a losses and profits vector set.

$$V_k = \begin{bmatrix} v_1 - W \\ v_2 - W \\ \vdots \\ v_n - W \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix} \qquad (25)$$

where V_k is the price and weight simulated vector without order.

Finally, prices and weights vector values without order V_k are sorted out in ascending order, which yields the prices and weights vector, $V_k{}^*$. Through prices and weights vector, $V_k{}^*$ we estimate the value at risk, in others words, VaR estimate is obtained through c% percentile of prices and weights vector.

d) Monte Carlo Simulation VaR

Monte Carlo Simulation data from 1942 and is based on the use of random numbers to create future scenarios. Simulation scenario consist in create a sequence of values which together form a path of the interest variable (variable to be analyzed), i.e., the path is formed by simulating scenarios through a wide range of possible situations.

With respect to VaR, Monte Carlo Simulation serves to simulate scenarios on probable portfolio value for a specific date; in other words, the value of VaR is obtained by distributing values of portfolio simulation. This methodology does not take into account nonlinearity, incorporates changes over time, i.e., volatility; also captures heavy tails and extreme values.

Implementation of Monte Carlo Simulation VaR is as follows:

I) Initially is necessary to select a stochastic model that describes the behavior prices and specify the parameters involved. The most common model in economic and financial literature and we will use it in this paper is geometric brownian motion,

$$dS_t = \mu_t S_t \, dt + \sigma_t S_t \, dz_t \qquad (26)$$

where S_t represents the price of financial asset, dz_t is a normally random variable with mean zero and variance dt; and the parameters μ_t and σ_t represent the instantaneous trend and volatility at the moment t, for simplicity henceforth it assume that these parameters are constant.

To simulate trajectories, one must first find unique solution of stochastic equation above, which can be expressed as,

$$S_t = S_{t-1} \exp(\mu dt + \sigma dz_t) \qquad (27)$$

This last equation characterizes infinitesimal movements in financial asset prices. For practical reasons this last equation assumes discrete terms, having a small time intervalΔt, comprising the current time t and the time target T. The transformation is,

$$S_t = S_{t-1} \exp\left(\mu \Delta t + \sigma \varepsilon_t \sqrt{\Delta t}\right) \qquad (28)$$

Where ε_t is white noise, i.e., is a normally distributed random variable with mean zero and variance 1.

II) As a second step is necessary to generate a sequence of random numbers $\varepsilon_1, \varepsilon_2, \dots, \varepsilon_t$ to simulate the price path. Thus, the simulated prices set through time horizon objective is,

$$S_t = S_{t-1} \exp\left(\mu \Delta t + \sigma \varepsilon_1 \sqrt{\Delta t}\right)$$
$$S_{t+1} = S_t \exp(\mu \Delta t + \sigma \varepsilon_2 \sqrt{\Delta t}) S_{t+2} = S_{t+1} \exp(\mu \Delta t + \sigma \varepsilon_3 \sqrt{\Delta t}) \vdots S_{t+n} = S_{t+n-1} \exp\left(\mu \Delta t + \sigma \varepsilon_n \sqrt{\Delta t}\right) \qquad (29)$$

Expanding the previous methodology, for the case of a portfolio with more than one asset requires the following:

i) Firstly is necessary decomposing the variance-covariance matrix of asset prices, for it takes place the Cholesky decomposition mechanism.[4] Through Cholesky decomposition is obtained matrix A, which complies obtaining a lower triangular matrix, i.e.,

$$\Sigma = AA^T \qquad (30)$$

where A represents a square triangular matrix with dimension, $n \; x \; n$.

[4] For a detailed treatment of Cholesky decomposition mechanism, see De Lara (2009).

ii) Next step is to create a dimensional vector $n\,x\,1$ of random numbers, $\boldsymbol{\varepsilon} = \varepsilon_1, \varepsilon_2, \dots, \varepsilon_n$, with mean zero and variance 1. For the specific case of this research, are generated 100,000 random numbers.Generate a set with normally distributed returns,

$$Y = A^T \boldsymbol{\varepsilon} \qquad (31)$$

iii) where Y represents a vector of dimension $n\,x\,1$, that containing the transformed normal variables.

iv) Also, are generate a set with lognormal distributed prices,

$$Z = S_t e^Y \qquad (32)$$

where S_t represents a vector of dimension $n\,x\,1$, that containing the expected future prices.

v) Once the price vector Z, this is used to create the losses and gains vector V_k, through $V_k = \mathrm{Z} - \mathrm{W}$, where W is the total portfolio position.

vi) Similar to the case of Historical Simulation VaR, losses and gains vector values without order V_k, are sorted out in ascending order, resulting the losses and gains vector simulation $V_k{}^*$. Finally, employing losses and gains vectors VaR is estimated; in other words, the VaR estimate is obtained through the c% percentile of losses and gains vector.

e) Copula VaR

In recent years, research and implementations of VaR calculation using copula has increased, determining the dependence structure of portfolio and risky assets that comprise it, without starting from assumptions about their distributions and estimating heavy tails; more realistic results and avoiding overestimation or underestimation of VaR portfolio are obtained in this manner. Following Romano (2002), Rank (2007) and Fantazzini (2008), this research makes use of Monte Carlo Simulation VaR with adaptation to copula theory, in other words, Copula VaR.

To implement Copula VaR, initially must be determined the most suitable joint distribution function, i.e., the one that best describes the data. This is accomplished by selecting the specific marginal distribution functions for returns of individual risk and a copula function to develop a single marginal join distribution function.

Let $\mathrm{x_t}$ and $\mathrm{y_t}$ assets log-returns at time t, and $\beta \in (0,1)$ the assignment weight, therefore portfolio return is given by,

$$\mathrm{z_t} = \beta \mathrm{x_t} + (1-\beta)\mathrm{y_t} \qquad (33)$$

The joint conditional distribution function estimated at time $\mathrm{t}-1$, is given as,

$$\mathrm{H_t}(\mathrm{x},\mathrm{y}|\Phi_{\mathrm{t}-1}) = \mathrm{C_t}(\mathrm{F_t}(\mathrm{x}|\Phi_{\mathrm{t}-1}), \mathrm{G_t}(\mathrm{y}|\Phi_{\mathrm{t}-1})|\Phi_{\mathrm{t}-1}) \qquad (34)$$

thus, the density function is given as,

$$\mathrm{h_t}(\mathrm{x},\mathrm{y}|\Phi_{\mathrm{t}-1}) = \mathrm{c_t}(\mathrm{F_t}(\mathrm{x}|\Phi_{\mathrm{t}-1}), \mathrm{G_t}(\mathrm{y}|\Phi_{\mathrm{t}-1})|\Phi_{\mathrm{t}-1}) \qquad (35)$$

Detailed steps of procedure for estimating the p-th confidence level of Copula VaR for a holding period of one day are:

1. Let z a portfolio with two assets, which contains a position for each asset, whose value in time $\mathrm{t}-1$, is:

$$\mathrm{P_{z,t-1}} = \mathrm{P_{x,t-1}} + \mathrm{P_{y,t-1}} \qquad (36)$$

where $\mathrm{P_{x,t-1}}$ and $\mathrm{P_{y,t-1}}$ are the market prices of the two assets at time $\mathrm{t}-1$.

2. Simulate $\mathrm{j} = 100{,}000$ Monte Carlo scenarios for asset log-returns $\{\mathrm{x_{j,t}}, \mathrm{y_{j,t}}\}$, on time horizon $[\mathrm{t}-1, \mathrm{t}]$, using joint conditional distribution function.

a) First, it should simulate j random variables $(\mathbf{u_{j,x}}, \mathbf{v_{j,y}})'$ of copula $\mathbf{C_t}(\cdot)$.

i. If using a normal copula 2-dimensional[5] is used the following algorithm:

- Through Cholesky decomposition A, find the correlation matrix 2 x 2, Σ, i.e.,

$$\begin{bmatrix} 1 & \hat{\rho}_t \\ \hat{\rho}_t & 1 \end{bmatrix}$$

where $\hat{\rho}_t$ is the conditional correlation estimated by normal copula.

- Simulate two independent random variables distributed standard normal, $\mathrm{z_j} = (\mathrm{z_{j,1}}, \mathrm{z_{j,2}})'$.

- Define the vector, $\mathrm{b_j} = \mathrm{Az_j}$.

[5] It should be noted, the copula to estimate is a normal copula 2-dimensional; equation (3) denotes the general case of a normal copula n-dimensional.

- Determine $(u_{j,x}, v_{j,y})' = \left(\phi(b_{j,1}), \phi(b_{j,2})\right)'$, where $\phi(\cdot)$ is the standard normal cdf. $(u_{j,x}, v_{j,y})'$ vector is a random variable j of the 2-dimensional normal copula, $C_{Normal}(\cdot;\, \hat{\rho}_t | \Phi_{t-1})$.

ii. If using a t-student copula 2-dimensional[6] the following algorithm is applied:

- Through Cholesky decomposition A, find the correlation matrix 2 x 2, Σ, i.e.,

$$\begin{bmatrix} 1 & \hat{\rho}_t \\ \hat{\rho}_t & 1 \end{bmatrix}$$

where $\hat{\rho}_t$ is the conditional correlation estimated by t-student copula.

- Simulate two independent random variables distributed as standard normal, $z_j = (z_{j,1}, z_{j,2})'$.
- Simulate a random variable s_j, of $\chi^2_{\hat{v}_t}$ distribution, independent of z_j, where $\hat{v}_t$ is the degrees of freedom parameter estimated by t-student copula.
- Define the vector, $b_j = Az_j$.
- Construct the vector, $c_j = \frac{\sqrt{\hat{v}_t}}{\sqrt{s_j}} b_j$.
- Determine$(u_{j,x}, v_{j,y})' = \left(t_{\hat{v}_t}(c_{j,1}), t_{\hat{v}_t}(c_{j,2})\right)'$, where $t_{\hat{v}_t}(\cdot)$ is the t-student cdf, with degrees of freedom equal to $\hat{v}_t$. $(u_{j,x}, v_{j,y})'$ vector is a random variable j of the 2-dimensional t-student copula, $C_{t-Student}(\cdot;\hat{\rho}_t, \hat{v}_t | \Phi_{t-1})$.

b) The next step is to obtain the series on log-returns standardized assets by using the inverse function of the estimated marginal, which can be normal or t-student.

$$Q_j = (q_{j,x}, q_{j,y})' = \left(\hat{F}_t^{\,-1}(u_{j,x}), \hat{G}_t^{\,-1}(v_{j,y})\right) \qquad (37)$$

c) Finally, repeat this procedure to $j = 100{,}000$.

3. By using these 100,000 cases, the portfolio will be reevaluated at time t, i.e.,

$$P_{z,t}{}^{j} = P_{x,t-1} \exp(x_{j,t}) + P_{y,t-1} \exp(y_{j,t}), \qquad j = 1, \ldots, 100{,}000 \qquad (38)$$

4. The portfolio losses in each scenario j, are calculated by,

$$Loss_j = P_{z,t}{}^{j} - P_{z,t-1}, \qquad j = 1, \ldots, 100{,}000 \qquad (39)$$

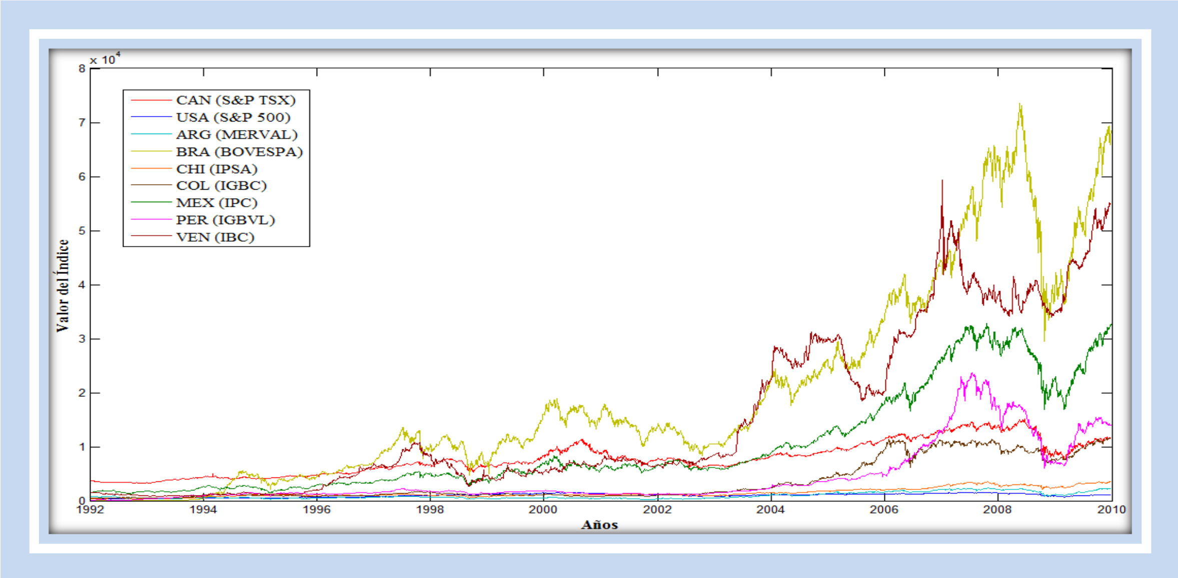
5. Then, VaR calculation at different confidence levels is easy. It takes into account the losses and gains vector without order, so as in the previous cases of VaR simulation, i.e., Historical Simulation VaR and Monte Carlo Simulation VaR, This vector is sorted in ascending order, resulting in the losses and gains vector whereby VaR calculation is done, via the c% percentile.

4. Empirical Evidence

We estimated the dependence between markets in the Western Hemisphere as well as potential losses via Value at Risk. This research covers the period 1992-2009, taking into account daily data time series for a total of 3,655 observations. The sample includes the major indexes of the nine American capital markets: S&P TX from Canada, S&P 500 from US; IPC from Mexico; MERVAL from Argentina; BOVESPA from Brazil; IPSA from Chile; IGBS from Colombia; IGBVL from Peru; and IBC from Venezuela. In order to have a similar sample size given some dissimilar calendar days, time series data were previously homogenized. Figure 1 gives an account of the behavior from American capital markets.

[6] It should be noted, the copula to estimate is a 2-dimensional t-student copula; equation (6) denotes the general case of a n-dimensional t-student copula.

Figure 1: Americas' Stock Market Indexes



Considering the importance of international portfolio investments and international research standards, the indexes and their returns are homogenized into a single currency, in this case the U.S. dollar. Figure 2 shows the returns of the American capital market via their respective stock market indices.

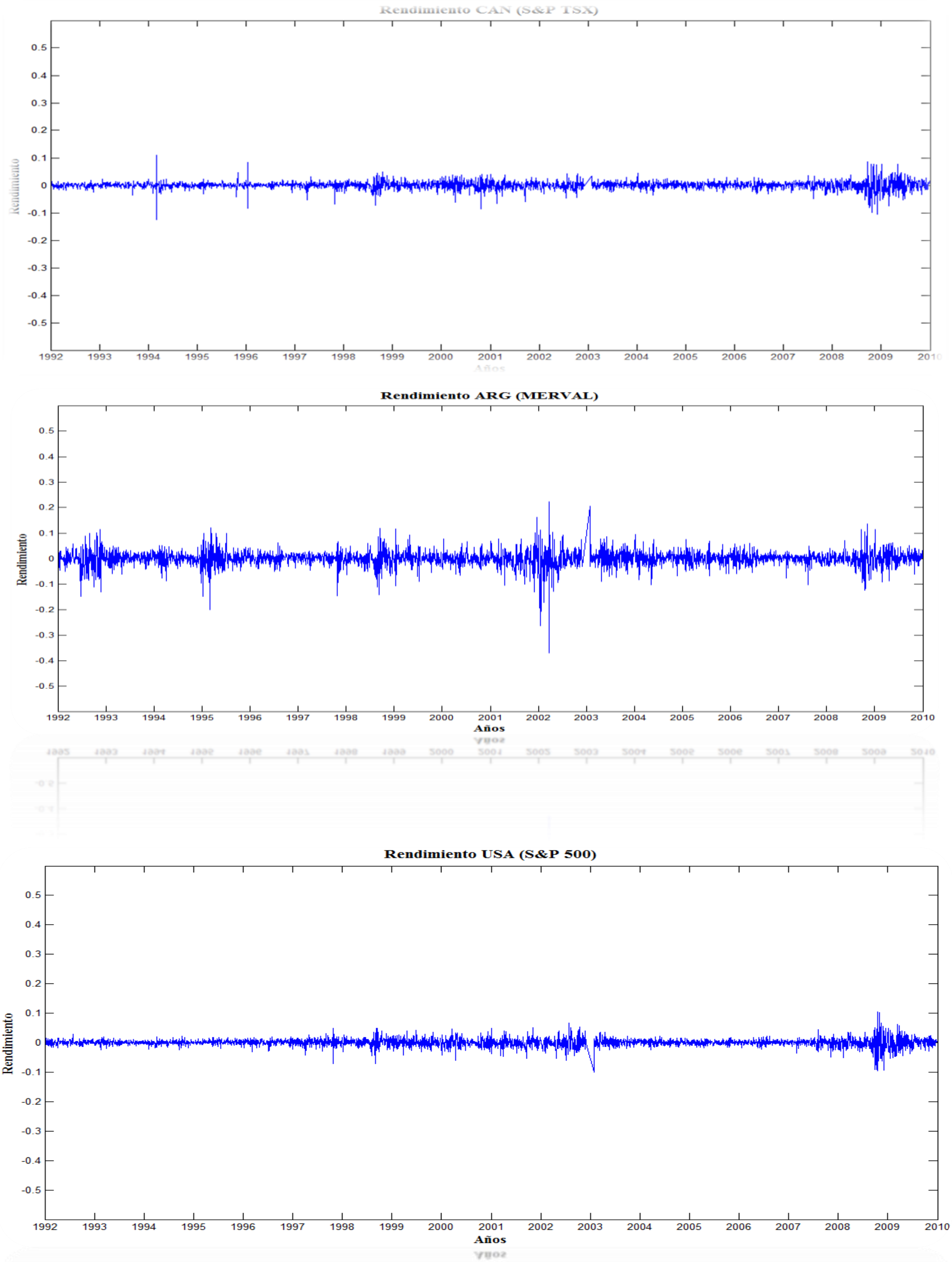
Returns summarized in Figure 2 show that the volatility of stock returns in the Western Hemisphere is unquestionable characterized by heteroscedasticity; therefore analyses based in the normality assumption is wrong, leading to spurious estimates.

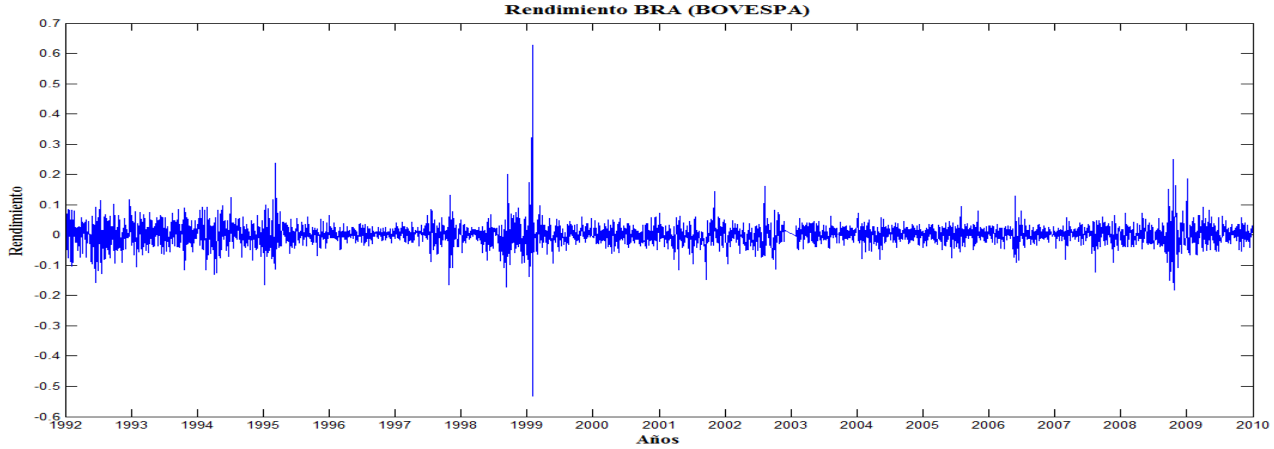
To determine the absence of linearity, various tests are presented, especially making use of various descriptive statistics: a histogram generated from each stock index returns and on each of these histograms is superimposed a normal curve. Also for each stock index returns are shown a Q-Q plot and various descriptive statistics, among which stands the Jarque-Bera normality test. In addition to the normal distribution curve superimposed on the histogram a t-student distribution curve is also shown, which is the most similar respect to the stock index returns distributions. Figure 3 details this results.

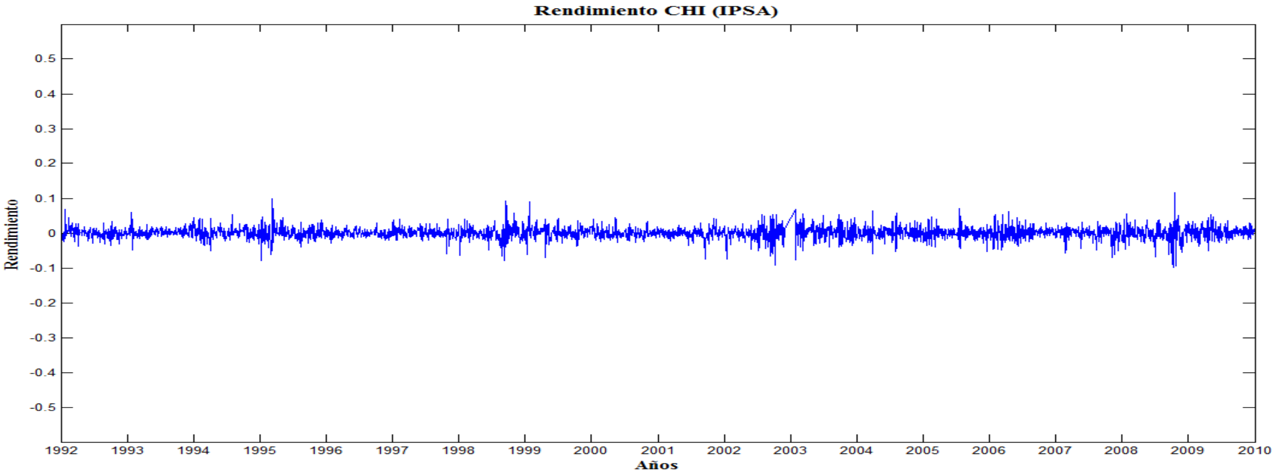
Regarding the linearity of returns it can be clearly seen that this assumption is not met. The behavior of the nine indexes returns is dissimilar to the normal distribution. Also the Q-Q plots confirm non-linearity, which is visible through S-shape behavior of the returns, which reveal the presence of heavy tails; this S-shape behavior differs from the straight line (normal distribution). Similarly the descriptive statistics reveal that the return series from the Western Hemisphere markets are far from a normal behavior: there are high levels of skewness and kurtosis in all stock index returns from the Americas, and with respect to statistical normality, the Jarque-Bera test shows that in all cases does not comply with the linearity assumption.

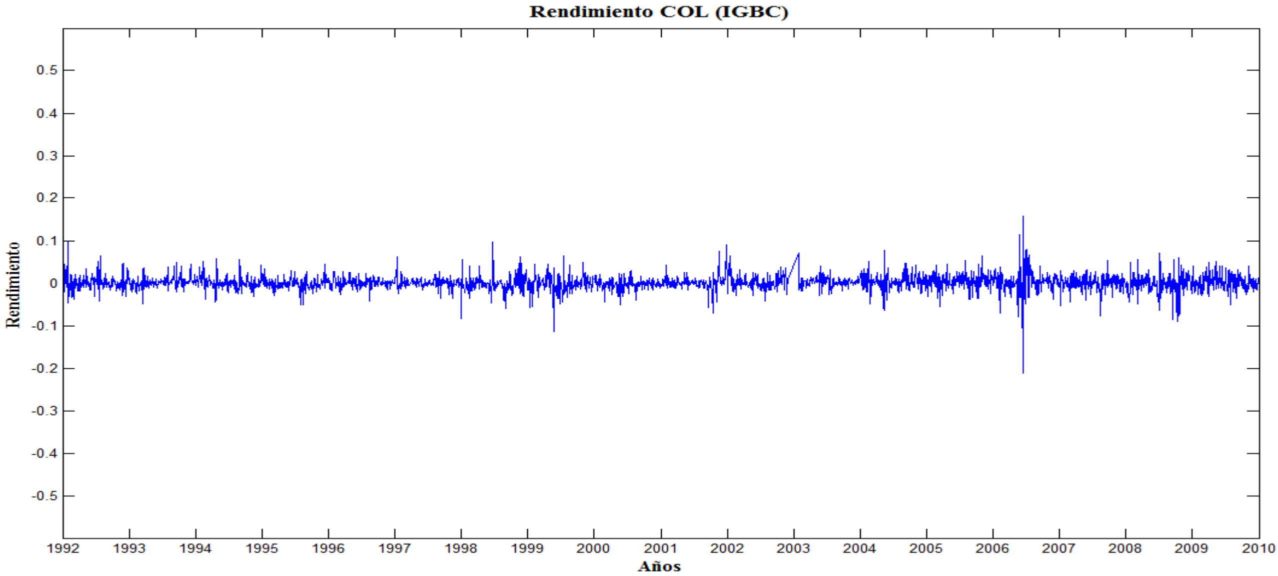
Non-linearity of stock index returns from the share markets from the Americas suggests that copula methodology is appropriate to optimize analyses of situations dissimilar to the normal distribution to obtain robust estimates. In particular, for estimating the degree of dependence this work uses elliptical copulas, i.e., normal copula and t-student copula; for estimating potential losses also uses these elliptical copulas coupled with the use of various alternative methodologies for estimating VaR.

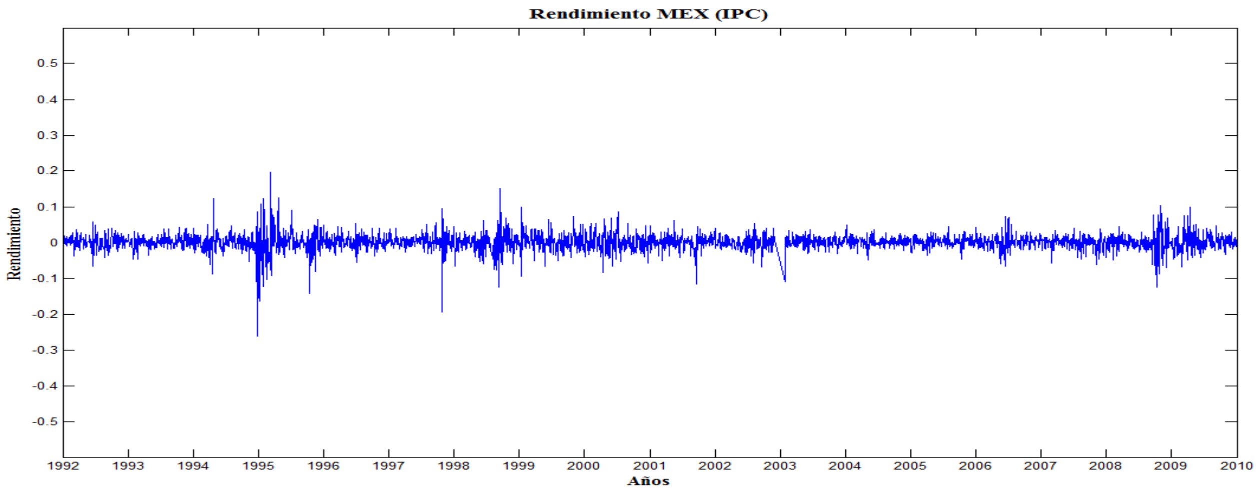
Figure 2: Stock Market Index Returns from the Americas

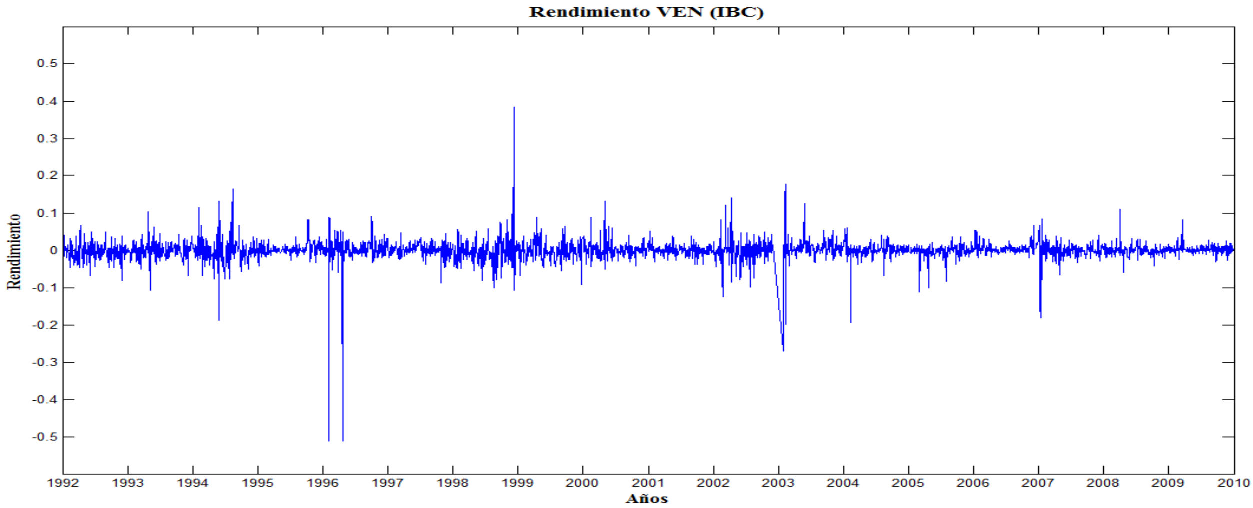



Rendimiento BRA (BOVESPA)
Rendimiento
Años
0.7 0.6 0.5 0.4 0.3 0.2 0.1 0 -0.1 -0.2 -0.3 -0.4 -0.5 -0.6
1992 1993 1994 1995 1996 1997 1998 1999 2000 2001 2002 2003 2004 2005 2006 2007 2008 2009 2010


Rendimiento CHI (IPSA)
Rendimiento
Años
0.5 0.4 0.3 0.2 0.1 0 -0.1 -0.2 -0.3 -0.4 -0.5
1992 1993 1994 1995 1996 1997 1998 1999 2000 2001 2002 2003 2004 2005 2006 2007 2008 2009 2010


Rendimiento COL (IGBC)
Rendimiento
Años
0.5 0.4 0.3 0.2 0.1 0 -0.1 -0.2 -0.3 -0.4 -0.5
1992 1993 1994 1995 1996 1997 1998 1999 2000 2001 2002 2003 2004 2005 2006 2007 2008 2009 2010


Rendimiento MEX (IPC)
Rendimiento
Años
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
1992 1993 1994 1995 1996 1997 1998 1999 2000 2001 2002 2003 2004 2005 2006 2007 2008 2009 2010


Rendimiento VEN (IBC)
Rendimiento
Años
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
1992 1993 1994 1995 1996 1997 1998 1999 2000 2001 2002 2003 2004 2005 2006 2007 2008 2009 2010

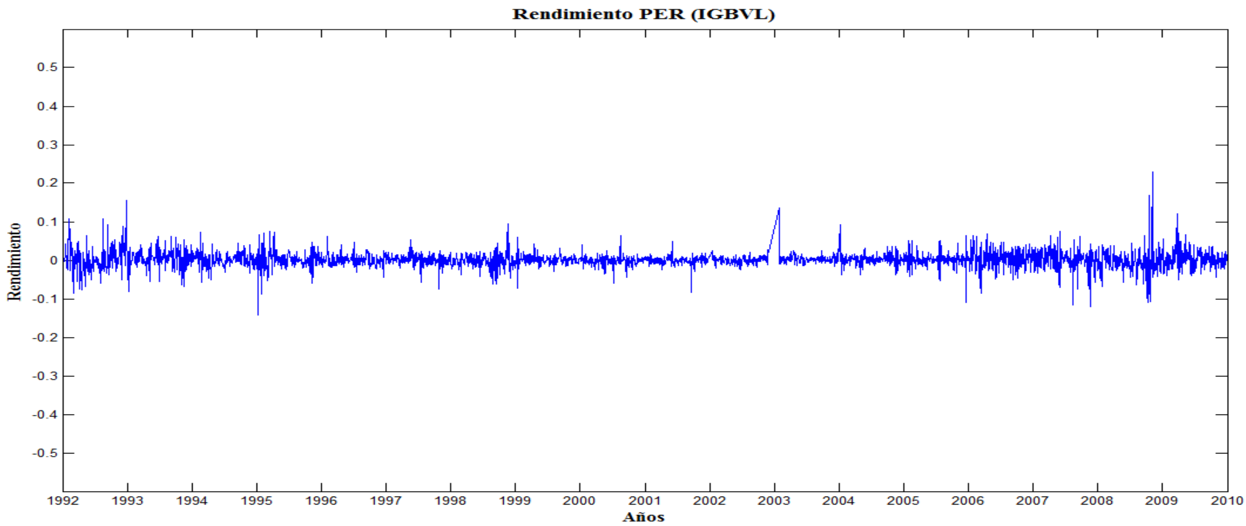
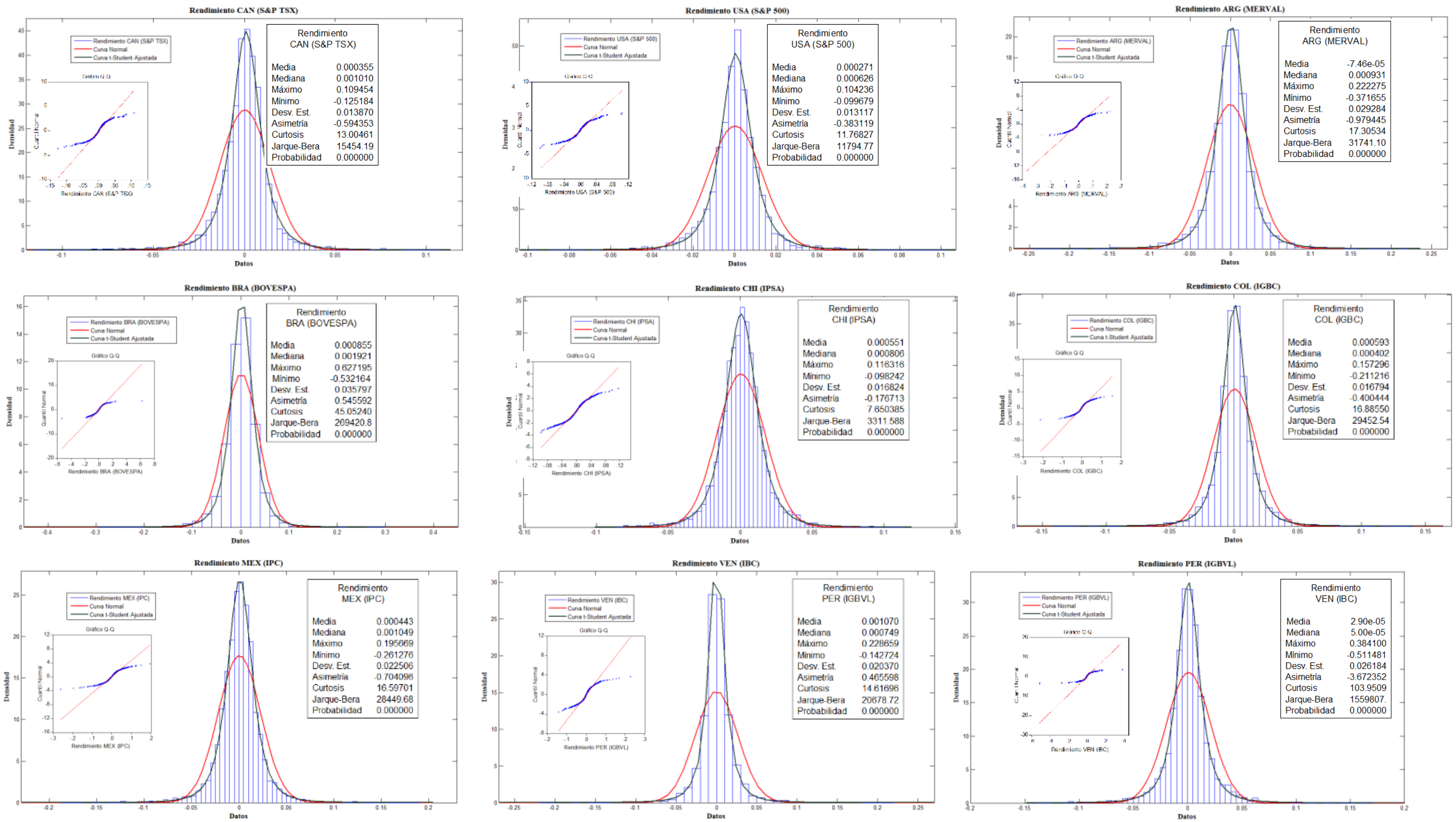

Rendimiento PER (IGBVL)
Rendimiento
Años
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
1992 1993 1994 1995 1996 1997 1998 1999 2000 2001 2002 2003 2004 2005 2006 2007 2008 2009 2010

Figure 3: Descriptive Statistics of the Stock Market Index Returns from the Americas



Analysis and Estimation of Dependence via Copulas

The main measure of copula dependence is estimated by its own estimate parameter. Parameter estimation of normal copula are made under the assumption of marginal normal, where the parameters of the marginal normal are those of a normal with mean μ, and standard deviation σ, being different means μ's, and standard deviation σ's, depending on each of different capital markets analyzed. In the case of t-student copula parameters, these are estimated under the assumption of marginal t-student, where the parameters of these marginal t-student i.e., mean μ, standard deviation σ, and degrees of freedom ν; are estimated by maximum likelihood, being different means μ's, standard deviations σ's, and degrees of freedom ν's, depending on each of the indices from American capital markets.

In addition to estimating the degree of dependence (association) of stock index returns via the estimation of their respective copulas, this paper also estimates dependence via copula using Kendall's tau and Sperman's rho parameters, dependence measures in terms of concordance. Comparing the three estimates reaffirms the importance and magnitude of the relationship between stock markets. Table 1 summarizes the results obtained.

The evidence shows that Kendall´s coefficient underestimates in all cases the degree of association among the stock markets from the Americas. The copula parameter and the Spearman coefficient yield very similar results, observing both the estimates for the Gaussian copula and the t-student copula. Taking a reference point the t-student copula parameter it is worth noting that markets with the highest dependence correspond to S&P 500 and S&P TSX (0.6426); and S&P 500 and IPC (0.5202) and S&P TSX and IPC (0.4522). This fact confirms the presence of financial integration trends between the members of the NAFTA. However, integration between these countries is still in process. Although dependence estimates were the highest in the Americas, mild market segmentation is present, considering that for that the degree of association between the Mexican market and the US market is above 0.50, but dependence between the financial markets from Canada and Mexico is below 0.50. Indeed, considering the rank correlation coefficients that help capturing the nonlinearity of returns, the Spearman correlation coefficient also shows dependence degrees above 0.5; however, the Kendall correlation coefficient is below 0.5.

Table 1: Dependence Measures Via Copulas the Americas' Capital Markets

		CAN (S&P TSX)		USA (S&P 500)		ARG (MERVAL)		BRA (BOVESPA)		CHI (IPSA)		COL (IGBC)		MEX (IPC)		PER (IGBVL)	
		Cópula Normal	Cópula t-Student	Cópula Normal	Cópula t-Student	Cópula Normal	Cópula t-Student	Cópula Normal	Cópula t-Student	Cópula Normal	Cópula t-Student	Cópula Normal	Cópula t-Student	Cópula Normal	Cópula t-Student	Cópula Normal	Cópula t-Student
USA (S&P 500)	Parámetro	0.6420	0.6426														
	Kendall	0.4438	0.4443														
	Spearman	0.6241	0.6247														
ARG (MERVAL)	Parámetro	0.3742	0.3746	0.3619	0.3639												
	Kendall	0.2442	0.2444	0.2357	0.2371												
	Spearman	0.3595	0.3598	0.3475	0.3495												
BRA (BOVESPA)	Parámetro	0.4226	0.4166	0.4385	0.4302	0.4442	0.4408										
	Kendall	0.2777	0.2735	0.3033	0.2973	0.2930	0.2908										
	Spearman	0.4066	0.4007	0.4418	0.4336	0.4277	0.4240										
CHI (IPSA)	Parámetro	0.3715	0.3717	0.3434	0.3437	0.3618	0.3612	0.3679	0.3593								
	Kendall	0.2423	0.2425	0.2232	0.2234	0.2357	0.2358	0.2398	0.2340								
	Spearman	0.3569	0.3571	0.3296	0.3298	0.3474	0.3468	0.3533	0.3450								
COL (IGBC)	Parámetro	0.2140	0.2136	0.1492	0.1495	0.1262	0.1270	0.1921	0.1864	0.1753	0.1763						
	Kendall	0.1373	0.1371	0.0953	0.0955	0.0806	0.0811	0.1230	0.1194	0.1122	0.1129						
	Spearman	0.2048	0.2044	0.1426	0.1429	0.1206	0.1214	0.1837	0.1783	0.1676	0.1686						
MEX (IPC)	Parámetro	0.4532	0.4522	0.5214	0.5202	0.4235	0.4263	0.4926	0.4875	0.3893	0.3915	0.1840	0.1838				
	Kendall	0.2994	0.2987	0.3491	0.3483	0.2784	0.2804	0.3279	0.3242	0.2545	0.2561	0.1178	0.1177				
	Spearman	0.4365	0.4356	0.5037	0.5025	0.4075	0.4103	0.4752	0.4703	0.3741	0.3763	0.1760	0.1757				
PER (IGBVL)	Parámetro	0.3488	0.3539	0.2605	0.2678	0.3109	0.3146	0.3239	0.3200	0.3594	0.3627	0.2192	0.2206	0.3251	0.3258		
	Kendall	0.2268	0.2303	0.1677	0.1726	0.2012	0.2037	0.2100	0.2073	0.2341	0.2363	0.1407	0.1416	0.2108	0.2112		
	Spearman	0.3348	0.3397	0.2494	0.2565	0.2981	0.3017	0.3107	0.3069	0.3451	0.3483	0.2097	0.2111	0.3119	0.3125		
VEN (IBC)	Parámetro	0.0739	0.0759	0.0807	0.0804	0.0956	0.0985	0.0962	0.0980	0.0826	0.0840	0.0343	0.0333	0.1277	0.1278	0.0579	0.0619
	Kendall	0.0471	0.0483	0.0514	0.0512	0.0609	0.0628	0.0614	0.0625	0.0526	0.0535	0.0218	0.0212	0.0815	0.0816	0.0369	0.0394
	Spearman	0.0706	0.0725	0.0771	0.0768	0.0913	0.0941	0.0919	0.0936	0.0789	0.0802	0.0328	0.0318	0.1220	0.1221	0.0553	0.0591

The remaining capital markets from the Americas show dependence with low levels of association: all below 0.5, most of them between 0.3 and 0.4. Remarkably, within this range of dependence the capital markets from Brazil and

Argentina, show relatively high dependence levels with the US markets, Additionally, the markets from Argentina, Brazil, and Mexico also show fair levels of dependence among themselves. The capital market from Venezuela shows the weakest dependence in relation to all other markets under study; its interaction with 7 of the 8 markets of the sample presents dependence ratios below 0.1; only in relation to the Mexican stock market, dependence is slightly above 0.1, and this only happens with estimations with copula parameters and the Spearman coefficients.

VaR Estimation and VaR Copula Estimation

Due to restrictions in the multivariate copula methodology value at Risk is estimated in this section for bilateral portfolios composed by indices from the capital markets under scrutiny. The methodologies applied are: Delta-Normal VaR, VaR-GARCH, Historical Simulation VaR, Monte Carlo simulation VaR and Copula VaR, Gaussian and t-student. Estimates are for a one day horizon and probability occurrence (significance level) of 99%, 97.5%, 95% and 90%; the investment amounts to US$ 100 000. Finally, diversification is made following a naïve strategy, i.e. 50 per cent of available resources to each market, to avoid potential concentration in one market in the allocation of resources.[7]

Table 2 shows the results of various Value at Risk methodologies proposals for each of the portfolios composed between the American capital markets. Potential losses are reported in percentage terms for better identification. The evidence confirms the effectiveness of non traditional methods over the conventional VaR alternatives. The former ones including copula based estimates capture appropriately the non linear behavior of the financial series, yielding more rigorous and realistic results vis a vis those from the other methodologies tested here.

Estimates of VaR Delta-Normal, VaR GARCH and Historical Simulation VaR, tend to underestimate potential losses. Estimates via Monte Carlo methods show the higher loss threshold, underlying that estimations carried out applying the t-student copula presents the greatest losses. As expected, exemplifying this situation, the differential between losses estimated by the Delta Normal approach and the Copula Monte Carlo Simulation increase the stricter is the confidence level. In the case of a US and Canadian portfolio potential losses at 90% confidence level potential losses are 1.54% and 2.58% for the Delta Normal and Copula Monte Carlo Simulation, respectively; at a 99.0% confidence levels are 2.81% and 10.90% for the same alternative estimates. In the case of a S&P 500 and IPC portfolio at a 90.0% confidence level losses are estimated at 1.98% and 3.45% for the Delta Normal and Copula Monte Carlo Simulation methodologies, respectively; for the stricter 99.0% confidence level those estimates are 3.91% and 14.032%. Finally for Canadian-Mexican portfolio losses at a 90% confidence level losses are estimated at 1.97% and 2.67% according to de Delta-Normal a Copula t-student Monte Carlos Simulation, respectively; at the stricter 99% confidence level losses amount to 3.60% and 11.01%, applying the same methodologies, respectively. Thus, the US-Canadian Portfolio is less risky in the North American Countries.

A higher degree of dependence should lead to smaller losses. However, regarding investments between Latin American stock market pairs, their measures of association do not necessarily reflect in the estimation of potential losses. It is possible to find fairly relatively important dependence levels such as the case of the capital markets from Argentina and Brazil which do not reflect lower potential losses. This could be attributed to the characteristics and dynamics of co-movements between these markets. In other words, in this situation although moderate dependence levels are present, possibly there are of more negative co-movements than positive co-movements, asymmetrical effects reflected in potential losses.

[7] Naïve diversification is not optimal, but is used in this research because the identical investment allocations in all bilateral cases, facilitates comparison between the obtained aggregate results.

Table 2: Value at Risk Measures of Capital Markets from the Americas

		CAN (S&P TSX)						USA (S&P 500)						ARG (MERVAL)						BRA (BOVESPA)					
	VaR	D-N	GARCH	S-H	S-MC	S-MC C-N	S-MC C-t	D-N	GARCH	S-H	S-MC	S-MC C-N	S-MC C-t	D-N	GARCH	S-H	S-MC	S-MC C-N	S-MC C-t	D-N	GARCH	S-H	S-MC	S-MC C-N	S-MC C-t
USA (S&P 500)	99%	2.81	3.07	3.73	9.19	9.30	10.90																		
	97.5%	2.36	2.59	2.47	4.94	5.07	5.91																		
	95%	1.98	2.16	1.84	3.55	3.61	4.19																		
	90%	1.54	1.68	1.24	2.15	2.20	2.58																		
ARG (MERVAL)	99%	4.27	5.45	5.66	9.41	9.67	11.55	4.20	5.36	5.27	10.01	10.48	12.26												
	97.5%	3.59	4.59	3.81	5.11	5.20	6.01	3.53	4.51	3.77	5.66	5.85	7.01												
	95%	3.01	3.85	2.82	3.62	3.78	4.45	2.96	3.79	2.81	4.08	4.16	4.89												
	90%	2.34	2.99	1.90	2.23	2.25	2.69	2.31	2.95	1.82	2.38	2.49	2.99												
BRA (BOVESPA)	99%	4.98	6.27	5.67	18.32	19.18	21.42	4.97	6.25	5.71	21.47	22.05	26.01	6.37	8.37	7.70	22.30	22.32	26.49						
	97.5%	4.19	5.27	4.35	10.86	10.75	12.63	4.18	5.25	4.20	12.36	12.48	14.85	5.36	7.04	5.48	12.59	12.77	15.04						
	95%	3.51	4.42	3.26	7.47	7.58	8.98	3.50	4.40	3.19	8.94	8.98	10.77	4.49	5.91	4.09	9.09	9.11	10.74						
	90%	2.72	3.43	2.26	4.63	4.72	5.67	2.71	3.42	2.25	5.45	5.67	6.62	3.49	4.59	2.83	5.64	5.69	6.75						
CHI (IPSA)	99%	2.92	3.09	3.83	9.56	9.80	11.48	2.82	2.97	3.71	8.64	8.98	10.91	4.48	5.57	5.57	18.67	18.97	22.26	5.11	6.31	6.03	22.73	22.76	27.81
	97.5%	2.45	2.60	2.51	5.23	5.25	6.34	2.37	2.49	2.41	4.87	4.95	5.83	3.77	4.69	4.00	10.47	10.55	12.86	4.30	5.31	4.42	12.70	12.81	15.34
	95%	2.05	2.17	1.87	3.68	3.74	4.40	1.98	2.09	1.84	3.41	3.49	4.16	3.16	3.93	2.89	7.51	7.61	9.08	3.60	4.44	3.21	9.41	9.43	10.81
	90%	1.59	1.68	1.27	2.30	2.37	2.69	1.54	1.62	1.26	2.05	2.13	2.52	2.46	3.06	2.01	4.63	4.71	5.49	2.79	3.45	2.19	5.64	5.71	6.74
COL (IGBC)	99%	2.74	3.17	3.47	9.38	9.42	11.37	2.61	3.01	3.43	7.90	9.11	10.87	4.11	5.34	5.38	18.61	18.79	22.33	4.84	6.18	5.67	22.04	22.61	26.59
	97.5%	2.30	2.66	2.34	5.14	5.23	6.16	2.19	2.53	2.27	4.73	4.86	5.84	3.46	4.49	3.71	10.39	10.57	12.68	4.07	5.20	3.98	12.77	12.81	15.63
	95%	1.92	2.23	1.70	3.62	3.68	4.35	1.83	2.11	1.66	3.25	3.43	4.17	2.90	3.77	2.69	7.49	7.56	8.99	3.40	4.35	3.00	9.21	9.26	11.13
	90%	1.49	1.72	1.17	2.25	2.28	2.67	1.42	1.64	1.13	2.13	2.16	2.47	2.25	2.93	1.81	4.57	4.60	5.51	2.63	3.37	2.03	5.65	5.67	6.96
MEX (IPC)	99%	3.60	3.91	4.64	9.32	9.35	11.01	3.61	3.91	4.59	12.16	12.21	14.23	5.08	6.26	6.57	13.79	14.14	16.66	5.81	7.11	6.96	21.83	22.00	25.65
	97.5%	3.03	3.29	3.30	4.98	5.13	6.14	3.04	3.28	3.26	6.44	6.71	7.97	4.28	5.28	4.38	7.54	7.67	9.26	4.89	5.98	4.85	12.09	12.14	14.74
	95%	2.53	2.75	2.32	3.56	3.64	4.33	2.55	2.75	2.27	4.59	4.76	5.66	3.59	4.42	3.31	5.42	5.65	6.57	4.09	5.01	3.67	8.76	8.81	10.74
	90%	1.97	2.12	1.56	2.13	2.23	2.67	1.98	2.14	1.56	2.66	2.96	3.45	2.79	3.44	2.21	3.38	3.43	4.07	3.17	3.89	2.48	5.34	5.59	6.52
PER (IGBVL)	99%	3.23	3.66	4.02	8.21	9.14	10.63	3.08	3.45	3.83	11.47	11.87	14.17	4.67	5.14	5.70	12.77	13.13	15.49	5.30	5.65	6.40	20.93	20.96	24.07
	97.5%	2.71	3.09	2.90	3.99	5.00	5.84	2.58	2.91	2.69	6.25	6.38	7.67	3.93	4.34	4.11	7.05	7.29	8.37	4.45	4.76	4.37	11.51	11.58	13.87
	95%	2.27	2.60	1.98	3.41	3.47	4.21	2.15	2.45	1.89	4.40	4.55	5.43	3.29	3.65	3.03	5.01	5.07	6.08	3.72	3.99	3.29	8.41	8.43	9.83
	90%	1.75	2.03	1.33	2.11	2.13	2.59	1.66	1.91	1.26	2.47	2.83	3.40	2.55	2.85	1.92	3.07	3.13	3.81	2.88	3.11	2.20	5.11	5.19	6.06
VEN (IBC)	99%	3.53	5.67	4.13	9.38	9.44	11.22	3.50	5.64	4.01	8.80	8.95	10.67	4.78	7.27	5.89	18.09	18.88	21.88	5.34	7.84	6.18	22.73	23.03	26.97
	97.5%	2.97	4.77	2.76	5.10	5.13	6.18	2.94	4.75	2.66	4.74	4.82	5.75	4.02	6.13	4.08	10.24	10.31	12.42	4.49	6.60	4.55	13.06	13.12	14.95
	95%	2.49	4.00	1.96	3.64	3.67	4.32	2.47	3.98	1.88	3.23	3.36	4.11	3.38	5.14	2.95	7.25	7.33	8.92	3.76	5.53	3.29	9.18	9.40	11.15
	90%	1.94	3.11	1.30	2.11	2.26	2.70	1.92	3.10	1.28	2.05	2.09	2.47	2.63	4.01	1.98	4.45	4.50	5.32	2.92	4.30	2.24	5.72	5.88	6.86

		CHI (IPSA)						COL (IGBC)						MEX (IPC)						PER (IGBVL)					
	VaR	D-N	GARCH	S-H	S-MC	S-MC C-N	S-MC C-t	D-N	GARCH	S-H	S-MC	S-MC C-N	S-MC C-t	D-N	GARCH	S-H	S-MC	S-MC C-N	S-MC C-t	D-N	GARCH	S-H	S-MC	S-MC C-N	S-MC C-t
USA (S&P 500)	99%																								
	97.5%																								
	95%																								
	90%																								
ARG (MERVAL)	99%																								
	97.5%																								
	95%																								
	90%																								
BRA (BOVESPA)	99%																								
	97.5%																								
	95%																								
	90%																								
CHI (IPSA)	99%																								
	97.5%																								
	95%																								
	90%																								
COL (IGBC)	99%	2.94	3.28	3.79	8.75	8.82	10.68																		
	97.5%	2.47	2.75	2.58	4.78	4.86	5.71																		
	95%	2.06	2.30	1.79	3.32	3.43	4.17																		
	90%	1.59	1.78	1.25	2.07	2.09	2.48																		
MEX (IPC)	99%	3.78	4.01	4.74	14.84	15.16	17.64	3.49	3.92	4.53	15.04	15.08	17.56												
	97.5%	3.18	3.37	3.31	8.08	8.35	9.75	2.93	3.30	2.97	8.23	8.26	9.89												
	95%	2.66	2.82	2.35	5.87	5.94	7.21	2.45	2.76	2.17	5.83	5.87	7.09												
	90%	2.06	2.18	1.63	3.53	3.57	4.30	1.90	2.14	1.48	3.56	3.64	4.38												
PER (IGBVL)	99%	3.75	3.86	4.37	13.40	13.63	16.34	3.30	3.72	4.09	13.45	13.69	16.04	3.99	4.45	5.06	11.59	11.93	14.13						
	97.5%	3.15	3.25	2.92	7.51	7.56	8.83	2.77	3.13	2.72	7.38	7.49	9.05	3.35	3.75	3.53	6.40	6.53	7.60						
	95%	2.63	2.73	2.19	5.24	5.33	6.35	2.31	2.63	2.07	5.34	5.36	6.42	2.80	3.15	2.39	4.57	4.64	5.49						
	90%	2.03	2.13	1.43	3.29	3.34	3.95	1.78	2.06	1.33	3.26	3.33	4.01	2.16	2.46	1.64	2.76	2.83	3.29						
VEN (IBC)	99%	3.72	5.76	4.66	13.41	14.23	16.76	3.63	5.80	4.04	14.05	14.13	17.07	4.23	6.26	5.05	14.73	14.79	17.44	3.90	6.00	4.83	13.56	13.85	16.08
	97.5%	3.13	4.85	2.96	7.75	7.78	9.25	3.06	4.88	2.80	7.79	8.09	9.66	3.56	5.27	3.36	8.13	8.37	9.88	3.28	5.01	3.28	7.37	7.48	8.94
	95%	2.62	4.06	2.02	5.51	5.58	6.62	2.56	4.09	1.99	5.67	5.81	6.73	2.99	4.42	2.37	5.68	5.89	7.14	2.74	4.19	2.17	5.26	5.42	6.32
	90%	2.03	3.16	1.42	3.42	3.45	4.01	1.99	3.18	1.36	3.50	3.52	4.12	2.32	3.44	1.67	3.55	3.58	4.32	2.13	3.28	1.48	3.26	3.30	3.95

VaR: Value at Risk; D-N: Delta-Normal; GARCH: GARCH-Copula; S-H: Historical Simulation; S-MC: Monte Carlo Simulation Copula (C-N: Gaussian, C-t-student)

Therefore, although it is feasible to suggest that the degree of dependence influences the estimation of potential losses, it is possible to find situations such as in the case from Argentina and Brazil. Thus, it cannot be affirmed beyond question that with greater dependence degree between markets, the lower the potential losses, or vice versa.

In relation to lower and higher estimated potential losses, it is possible to conclude that the portfolio consisting of the developed markets from North America, Canada and United States, is which presents minor potential losses, confirming their high dependence ratios previously estimated. Dependence between these markets results in a smaller risk threshold. Finally it is worth noting that all bilateral portfolios which include the market from Brazil. Chile, Colombia and Peru produce large potential losses. The Mexican market is seemingly the less risky in terms of bilateral investments with other emerging markets from the Americas.

5. Conclusions

The first objective of this paper was to verify the degree dependence between a representative set from capital markets from the Americas. The evidence indicates that there is a limited degree of dependence degree between the capital markets from Latin America. In relation to the capital markets from North America a fairly high degree of dependence was found. Based on the empirical evidence it can be concluded that the dependence degree between the Latin American capital markets is rather low, particularly for the case of Venezuelan market, which shows very low levels of dependence. The markets of North America show medium to high degrees of dependence which can be attributed to their economic integration schemes, although it can be concluded the presence of mild segmentation. It is therefore possible to conclude that financial integration seen through dependence degrees is low and dissimilar for the case of the Latin American capital markets and relatively high between the markets of North America.

Concerning the second objective of this paper, the estimation of potential losses of bilateral portfolios, estimates were carried out applying various techniques of Value at Risk. Estimates obtained by the copula VaR methodology proved to be more restrictive, estimating greater losses than other methodologies. Also, it was found that the degrees of association among the capital markets of the Western Hemisphere American capital markets are reflected in the estimation of potential losses. Higher degrees of dependence lead to lower potential losses and vice versa. Exceptions are present. Relatively high levels of dependence, such as in the case of the markets from Argentina and Brazil did not reflect in lower potential losses. This could largely be attributed to the dynamics and structure of co-movements between these markets.

In summary, it be also concluded that because the dependence degree is rather low for the Latin American capital markets, this signals the potential for international diversification, constructing of portfolios combining assets from these markets. Finally, results from this research also suggest that copula theory leads to more realistic estimates, than traditional methodologies, in the estimation of dependence degrees, as well as in the estimation of Value at Risk.

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