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The effect of bias in relative performance appraisal

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Abstract.

This study examines how managers' biases affect the performance of their employees under relative performance appraisal. First, in case that performance cannot be measured quantitatively (as for administrative work), given a small reward, the findings suggest that the manager's bias increases the employees' level of effort and given a large reward, the bias decreases efforts. Examining cases over two periods, usually, bias toward an employee who performed better in the first period increases the total profit. Moreover, a bias leads to lower efforts in cases where performance can be measured quantitatively (as for sales). Furthermore, by establishing harsh conditions such as dismissing employees with lower evaluations, overall efforts may be increased by a bias against an underperforming employee. The overall findings suggest that the organizational performance improves through implementing a bias toward better performing employees.

.**Keywords:** Relative Performance Appraisal; Biased Contests; Appraisal For Administrative Work; Appraisal For Sales; Harsh Conditions; Two Periods.

Introduction

In organizations such as companies, employees strive to obtain higher evaluations, as rewards can depend on performance appraisals. The appraisals objectively recognize employees' strengths and weaknesses, which are then linked to their growth. Assuming that a manager will evaluate employees' abilities and performances properly, employees can then concentrate on the business. Further, a performance appraisal is an important tool for organizations where employees are valuable management resources for achieving organizational goals. As the resources for labor costs are limited, relative appraisals are common. Setting objective criteria is challenging, for it is difficult to reflect the circumstances accurately in the evaluation. Differences in the works or circumstances may appear as performance differences. Therefore, it is not easy for the manager to relatively evaluate the performance. There are reasons behind the results for each employee. A manager who evaluates employees also feels emotion and may be biased toward an employee with a favorable impression. The bias may then be implemented to obtain higher evaluation.

This study analyzes an organization's relative personnel evaluation system where the manager is partial in cases like administrative works, where performances cannot be strictly shown and cases like sales, where performances can be quantitatively grasped. In the former, when the reward is small, findings suggest that the employees' level of effort is higher when the manager has a recognizable bias. When the reward is large, the level of effort is higher if the employees are treated equally. Additionally, in cases over two periods, the manager may be biased to avoid making an employee who performs equally the inferior one across two consecutive periods, which can negatively affect their effects and organization's overall profit. The findings show that a bias toward the first-period winner increases the total profit in most cases. Furthermore, the total profit is maximized if the manager shows that the first-period superior will win when the performances of the two are equal in the second period.

Regarding the principal-agent model analyzing bias in evaluation, Prendergast and Topel (1996) found that manager's bias has both demerits and merits. Increasing the evaluation uncertainty and cost associated with the

wrong personnel placement are demerits and retaining rights to implement bias is merit. When the latter exceeds the former, equilibrium exists in situations where the manager is partial. Che and Gale (2003) showed that an auction that handicaps the most efficient firm might be desirable and Ederer (2010) analyzed the effect of intermediate feedback on the level of effort in a two-period model. Using a two-period model, Meyer (1992) testified that in a cost-minimization agreement that meets a certain level of effort, the principal's bias is zero in the first period, and the principal has a strictly positive bias toward the first-period winner in the second period. Ridlon and Shin (2013) showed that the bias toward the first-period winner raises total effort when the abilities are sufficiently similar, and the bias to the first-period loser increases effort if the ability difference is significant. Drugov and Ryvkin (2017) stated that bias to the winner in the first period can increase effort level for both the first and second periods.

In Sections 2–4 of this paper, it is assumed that the employees act on a manager's evaluation tendency to choose a superior employee when performances are equal, unlike Meyer (1992), who assumed additive bias on performance and unlike Ridlon and Shin (2013), who assumed multiplicative bias on performance. This assumption is suitable for analyzing bias on performances that are difficult to evaluate quantitatively, such as administrative work. The findings confirm that the bias toward the first-period winner increases the total profit, but there are cases where the total profit increases without bias. When comparing profit from the case where bias exists with profit from cases without bias for each period, there are differences from the existing research. These differences are likely caused by variances in the model's composition, such as determining compensation. Besides, it is found that the condition in which two consecutive inferior employees will lose their jobs has a positive effect on both the level of effort and the company's profit.

Next, when the performance differences can be grasped, the impact of bias is examined. A game is analyzed where two people compete for performance based on the employee's effort and random shocks. The relationship between effort and bias is analyzed using the cumulative distribution function and density function of the difference between the two random shocks. Bias is found to lower the level of effort in cases where employees are aware of it; however, when harsh conditions are imposed only on an employee, such as significant disadvantages or penalties given if the performance of the employee becomes inferior, the equilibrium effort level increases. Both efforts may increase by implementing bias to those without harsh conditions. Moreover, by implementing bias toward the employee with harsh conditions, the equilibrium effort level declines and becomes a decreasing function of bias. When the bias is unknown in advance and the manager judges by checking the performance of two employees, the manager may implement bias toward the employee with the harsh conditions, but the higher the severe conditions are, the lower the level of equilibrium effort becomes. The organization's performance improves by implementing bias toward an excellent employee, not toward a nonperforming employee. Tsoulouhas et al. (2007) and Ridlon and Shin (2013) analyzed the handicap settings for performance or ability. This paper analyzes how negative rewards, such as handicaps on each side, affect an employee's effort and company's overall profit.

The remainder of the paper proceeds as follows.

Section 2 introduces and explains the basic model.

Section 3 demonstrates how managers' bias affects employees' efforts. The results show that when the reward is small, the manager should show bias and not treat the employees equally, but when the reward is large, employees should be treated equally.

In Section 4, the model is analyzed for two periods. First, the effects on the employees' efforts and the organization's profit are shown through the bias that a manager does not want to make an employee inferior for two consecutive periods. Consequently, there are negative effects on the efforts and on the profit. Generally, the bias based on the result of the first period is examined. For example, in the case of a tie for the second period, when the manager is supposed to evaluate the first-period inferior as the superior of the second period, total profit is greater without bias and when the manager is inferred to evaluate the superior in the first period as the superior, total profit becomes greater with bias.

In Section 5, the bias is shown when the performance difference can be grasped strictly. Basically, bias has negative effects on efforts. When the condition is set upon an employee, such as dismissing the inferior, the equilibrium effort rises, and a bias for an employee without harsh conditions may increase both efforts and profit. Section 6 concludes the paper.This study examines how managers' biases affect the performance of their employees under relative performance appraisal. First, in case that performance cannot be measured quantitatively (as for

administrative work), given a small reward, the findings suggest that the manager's bias increases the employees' level of effort and given a large reward, the bias decreases efforts. Examining cases over two periods, usually, bias toward an employee who performed better in the first period increases the total profit. Moreover, a bias leads to lower efforts in cases where performance can be measured quantitatively (as for sales). Furthermore, by establishing harsh conditions such as dismissing employees with lower evaluations, overall efforts may be increased by a bias against an underperforming employee. The overall findings suggest that the organizational performance improves through implementing a bias toward better performing employees.

2. Basic model

The basic settings are as follows.

Imagine games played by two employees (A vs. B). If one becomes higher in the relative evaluation, they get Λ for a reward. The reward for the lower employee is zero. As it is a relative evaluation, there are no ties. Even if both employees perform well, one gets no reward. Even if neither employee performs well, one will still receive Λ.

Each employee chooses the degree of effort. Let $d_a(0 \le d_a \le 1)$ and

$d_b(0 \le d_b \le 1)$ represent the efforts exerted by employee A and employee B.

The cost of effort is kd^2.

The employees act on the manager's evaluation tendency to choose which employee is superior if their performances are the same. The manager's evaluation tendency is common recognition among employees. An employee who can achieve a satisfactory level of performance for a given business is defined as a good performer; otherwise, the employee is defined as a bad performer. The degree of effort to become a good performer for A is ${d_a}^*$ and that for B is ${d_b}^*$, A and B do not know the value and regard the degree of effort as the probability of being a good performer. The manager who evaluates them can only know whether the employee is a good or a bad performer. Table 1 shows the payoff.

Table 1: Basic payoff matrix in case of good performer or bad performer

		B	
		Good performer	Bad performer
A	Good performer	【probability of occurring: $d_a \cdot d_b$】 $(\frac{\Lambda}{2} - k{d_a}^2, \frac{\Lambda}{2} - k{d_b}^2)$	【$d_a \cdot (1-d_b)$】 $(\Lambda - k{d_a}^2, 0 - k{d_b}^2)$
	Bad performer	【$(1-d_a) \cdot d_b$】 $(0 - k{d_a}^2, \Lambda - k{d_b}^2)$	【$(1-d_a)(1-d_b)$】 $(\frac{\Lambda}{2} - k{d_a}^2, \frac{\Lambda}{2} - k{d_b}^2)$

Here"α" denotes the ratio of reward to the cost of effort(Λ/k).

A's expected gain is

$E_a = (d_a \cdot d_b)\left(\frac{\alpha}{2} - {d_a}^2\right) + (d_a \cdot (1-d_b))\,(\alpha - k{d_a}^2) + ((1-d_a) \cdot d_b)$

$(0 - {d_a}^2) + (1-d_a)(1-d_b)\,(\frac{\alpha}{2} - {d_a}^2)$

$= -{d_a}^2 + \frac{\alpha}{2}\, d_a + \frac{\alpha}{2}(1-d_b).$

Similarly, B's expected gain is $E_b = -{d_b}^2 + \frac{\alpha}{2}\, d_b + \frac{\alpha}{2}(1-d_a)$. From the first-order condition, the equilibrium levels of efforts are $d_a = \frac{\alpha}{4}$ and $d_b = \frac{\alpha}{4}$.

As $0 \le d \le 1$, α should be 4 or less.

3. Biased relative performance appraisal

In the performance appraisal, when the performance is clear like sales results, it is easy to evaluate, but many operations are difficult to quantify. Subjective emotions or favoritism may enter into the evaluations for these operations.

If there is an obvious difference in performance and the better performer is treated lower, the employees' motivation must deteriorate. However, when there is no palpable difference, managers may show favoritism.

The model, similar to Chapter 1, analyzes the effect of the manager's bias.

Proposition 1. In the case of $\alpha < 2$, the effort is maximized when bias for the opposite side is working. In the case of $\alpha > 2$, the effort is maximized when a slight bias for one's own self is working. From a managerial point of view, it is reasonable to show favoritism when the reward is small, and assume a neutral posture when the reward is large.

Proof. The probability that A is evaluated to be better denotes $p(0 \le p \le 1)$ if the performances are almost identical. Table 2 shows the payoff.

Table 2: Payoff matrix with manager's bias

		B	
		Good performer	Bad performer
A	Good performer	【d_a ・d_b】 $(\alpha p - {d_a}^2, \alpha(1-p) - {d_b}^2)$	【d_a ・$(1-d_b)$】 $(\alpha - {d_a}^2, 0 - {d_b}^2)$
	Bad performer	【$(1-d_a)$ ・d_b】 $(0 - {d_a}^2, \alpha - {d_b}^2)$	【$(1-d_a)(1-d_b)$】 $(\alpha p - {d_a}^2, \alpha(1-p) - {d_b}^2)$

A's expected gain is as follows:

$E_a = -{d_a}^2 + (\alpha - \alpha p - \alpha d_b + 2\alpha p d_b)\, d_a + \alpha p - \alpha p d_b.$

From the first-order condition, A's expected gain takes the maximum at

$d_a = \frac{\alpha - \alpha p - \alpha d_b + 2\alpha p d_b}{2}$ (1).

Similarly B's expected gain is $E_b = -{d_b}^2 + (\alpha p + \alpha d_a - 2\alpha p d_a) d_b + \alpha - \alpha p - \alpha d_a + \alpha p d_a$. B's expected gain takes the maximum at

$d_b = \frac{\alpha p + \alpha d_a - 2\alpha p d_a}{2}$ (2).

By solving the simultaneous equations of (1) and (2), we can get values of the effort at the equilibrium,

$d_a = \frac{2\alpha - 2\alpha p - \alpha^2 p + 2\alpha^2 p^2}{4 + \alpha^2 - 4\alpha^2 p + 4\alpha^2 p^2}$ (3).

$d_b = \frac{\alpha(1-2p)(2\alpha - 2\alpha p - \alpha^2 p + 2\alpha^2 p^2) + \alpha p(4 + \alpha^2 - 4\alpha^2 p + 4\alpha^2 p^2)}{2(4 + \alpha^2 - 4\alpha^2 p + 4\alpha^2 p^2)} = \frac{\alpha^2 + 2\alpha p - 3\alpha^2 p + 2\alpha^2 p^2}{4 + \alpha^2 - 4\alpha^2 p + 4\alpha^2 p^2}$ (4).

First, the relationship between manager's bias and employee's effort is examined. As d_a and d_b are symmetrical, how the manager's bias affects employee's effort is analyzed using d_a. The level of effort is differentiated with respect to bias

$\frac{dd_a}{dp} = \frac{4\alpha^3(2-\alpha)p^2 + 4\alpha^2(\alpha-2)^2 p - \alpha(\alpha-2)(\alpha^2 - 4\alpha - 4)}{(4 + \alpha^2 - 4\alpha^2 p + 4\alpha^2 p^2)^2} = 0.$

Then, the bias maximizes the effort, when $p = \frac{-(2-\alpha) \pm 2\sqrt{2}}{2\alpha}$.

In the case of $\alpha < 2$, d_a takes the maximum at $p = \frac{-(2-\alpha) - 2\sqrt{2}}{2\alpha}$ and takes the minimum at $p = \frac{-(2-\alpha) + 2\sqrt{2}}{2\alpha}$. When d_a takes the maximum, p is always negative and when $p = 0$, the effort will be maximized. In the case of $\alpha > 2$, d_a takes the minimum at $p = \frac{-(2-\alpha) - 2\sqrt{2}}{2\alpha}$ and takes the global maximum at $p = \frac{-(2-\alpha) + 2\sqrt{2}}{2\alpha}$. Then, the organizational profit is analyzed.

Supposing one good performer produces sales of "S", the expectation of organizational sales is $S(d_a + d_b) = \frac{S\alpha(2 - 4\alpha p + 4\alpha p^2 + \alpha)}{(4 + \alpha^2 - 4\alpha^2 p + 4\alpha^2 p^2)}$ (5).

The profit at this time equals (5)$-\alpha$k.

$\frac{8\alpha^2 S(2p-1)(2-\alpha)}{(4 + \alpha^2 - 4\alpha^2 p + 4\alpha^2 p^2)^2}$ is obtained by differentiating the profit (5) by p. When p$=\frac{1}{2}$, if $\alpha < 2$, the expectation of sales is minimized and if $\alpha > 2$ it is maximized.

When $\alpha > 2$, comparing the sums of employees' efforts in the case of p$=\frac{1}{2}$ with $p = \frac{1}{2} + \frac{(\sqrt{2}-1)}{\alpha}$ which is the personal largest effort in the previous case. The sum of the former is larger than that of the latter by $\frac{(3-2\sqrt{2})(\alpha-2)}{4(2-\sqrt{2})}$. Although a little bias maximizes effort when looking at the employee personally, it is confirmed that the sum of employees' efforts becomes the largest when there is no bias. Therefore, having no bias is desirable for the whole organization.

Next, a comparison is made between the profit with bias and that without bias.

If there is no bias, the sales of the organization are $Sd_a + Sd_b = \frac{S\alpha}{2}$ (6).

As the costs in both cases are the same,αk, the difference in profits is the difference in sales that equals (5)–(6) $= \frac{4(2-\alpha)\alpha^2Sp^2-4(2-\alpha)\alpha^2Sp+(2-\alpha)\alpha^2S}{2(4+\alpha^2-4\alpha^2p+4\alpha^2p^2)}$.

The denominator of this fraction is always positive as it takes the minimum value of 8 when $p = \frac{1}{2}$. Considering the sign of the numerator, in the case $\alpha < 2$ it takes the minimum value of zero when p$=\frac{1}{2}$ and the other is positive. In other words, the profit is larger if there is a bias. On the contrary, in the case $\alpha > 2$, it takes the maximum value of zero when p$=\frac{1}{2}$ and the other is negative, and the profit is larger if there is no bias. When $\alpha = 2$, bias does not affect profit.

4. Two-period relative performance appraisal

Now, imagine two-period games played by two employees (t = 1,2).

Each employee chooses the level of effort. The level of A's effort is denoted by $d_{at}(0 \le d_{at} \le 1)$ and that of B is denoted by $d_{bt}(0 \le d_{bt} \le 1)$. Table 3 shows the payoff of the first and second periods.

Table 3: Payoff matrix for two-period appraisals

		B Good performer	B Bad performer
A	Good performer	【d_{at} ・d_{bt}】 $(\frac{\alpha}{2} - {d_{at}}^2, \frac{\alpha}{2} - {d_{bt}}^2)$	【d_{at} ・$(1 - d_{bt})$】 $(\alpha - {d_{at}}^2, 0 - {d_{bt}}^2)$
	Bad performer	【$(1 - d_{at})$ ・d_{bt}】 $(0 - {d_{at}}^2, \alpha - {d_{bt}}^2)$	【$(1 - d_{at})(1 - d_{bt})$】 $(\frac{\alpha}{2} - {d_{at}}^2, \frac{\alpha}{2} - {d_{bt}}^2)$

As with the basic model, to take the maximum value, the degrees of both efforts are $d_{at} = \frac{\alpha}{4}$.

4.1 Bias toward the first-period inferior

Based on the earlier seniority system in Japan, as long as employees were working, there was no need for different treatments.

Then, an additional premise is as follows. A manager hesitates to make the first-period inferior the inferior again in the second period. If both employees are good or bad performers in the second period, the evaluation of the first-period inferior will be higher. Table 4 shows the payoff of the second period, where A is the first-period superior and B is the first-period inferior.

Table 4: Payoff matrix with bias toward the first-period inferior

		B (first-period inferior) Good performer	B (first-period inferior) Bad performer
A (first-period superior)	Good performer	【d_{a2} ・d_{b2}】 $(0 - {d_{a2}}^2, \alpha - {d_{b2}}^2)$	【d_{a2} ・$(1 - d_{b2})$】 $(\alpha - {d_{a2}}^2, 0 - {d_{b2}}^2)$

Bad performer	【$(1-d_{a2})\cdot d_{b2}$】 $(0-{d_{a2}}^2, \alpha-{d_{b2}}^2)$	【$(1-d_{a2})(1-d_{b2})$】 $(0-{d_{a2}}^2, \alpha-{d_{b2}}^2)$

The expected gain of A (the first-period superior) is $E_{a2} = \alpha d_{a2} - \alpha d_{a2} d_{b2} - {d_{a2}}^2$.

The expected gain of B (the first-period inferior) is $E_{b2} = \alpha - \alpha d_{a2} + \alpha d_{a2} d_{b2} - {d_{b2}}^2$.

From the first-order condition, we get the following:

$\frac{dE_{a2}}{dd_{a2}} = -2d_{a2} + \alpha - \alpha d_{b2} = 0$ (7).

$\frac{dE_{b2}}{dd_{b2}} = -2d_{b2} + \alpha d_{a2} = 0$ (8).

By solving the equations of (7) and (8) simultaneously, we obtain values for the effort at the equilibrium, $d_{a2} = \frac{2\alpha}{4+\alpha^2}$, $d_{b2} = \frac{\alpha^2}{4+\alpha^2}$.

These values are compared with the level of effort without bias, $\frac{\alpha}{4}$.

Regarding d_{a2}, if $\alpha < 2$, the effort level is higher when there is a bias, if $\alpha > 2$,the effort level is higher with no bias.

Regarding d_{b2}, the level of effort with bias is always higher.

Next, the difference between the total gains is considered when A wins and loses at the first period.

The total expected gain (r is a discount factor) for the first-period superior is

$(\alpha - {d_{a1}}^2) + (\alpha d_{a2} - \alpha d_{a2} d_{b2} - {d_{a2}}^2)r$ (9).

Total expected gain for the first-period inferior is

$(0 - {d_{a1}}^2) + (\alpha - \alpha d_{b2} + \alpha d_{a2} d_{b2} - {d_{a2}}^2)r$ (10).

Supposing r = 1, the difference is

(9)–(10)$=\alpha(d_{a2} + d_{b2} - 2d_{a2}d_{b2}) \geq 0$ (11).

As $d_{a2} + d_{b2} - 2d_{a2}d_{b2} = \left(\sqrt{d_{a2}} - \sqrt{d_{b2}}\right)^2 + 2\left(\sqrt{d_{a2}}\sqrt{d_{b2}} - d_{a2}d_{b2}\right) \geq 0$, equation (11) is positive.

Therefore, the first-period superior is advantageous.

In this case, the employees regard the first period gain as $\alpha(d_{a2} + d_{b2} - 2d_{a2}d_{b2})$.

That is, the degree of the equilibrium effort of the first period is not $\frac{\alpha}{4}$ but $\frac{\alpha(d_{a2}+d_{b2}-2d_{a2}d_{b2})}{4}$. When $d_{a2} = \frac{2\alpha}{4+\alpha^2}$ and $d_{b2} = \frac{\alpha^2}{4+\alpha^2}$ is substituted for this, the result is $\frac{8\alpha^3-2\alpha^4+4\alpha^3+\alpha^5}{4(4+\alpha^2)^2}$.

Then, comparing it to $\frac{\alpha}{4}$,$\frac{8\alpha^3-2\alpha^4+4\alpha^3+\alpha^5}{4(4+\alpha^2)^2} - \frac{\alpha}{4} = 2\alpha(-\alpha^3 - 2\alpha^2 - 8) < 0$ shows that the level of efforts in the equilibrium becomes lower if there is bias.

Again supposing one good performer produces sales of "S," the total profit with bias through two periods becomes $\frac{S(8\alpha^3+16\alpha+8\alpha^2+\alpha^5)}{2(4+\alpha^2)^2} - 2\alpha k$. The total profit without bias through two periods is $S\alpha - 2\alpha k$. Comparing these profits,

$\left[\frac{S(8\alpha^3+16\alpha+8\alpha^2+\alpha^5)}{2(4+\alpha^2)^2} - 2\alpha k\right] - [S\alpha - 2\alpha k] = S\alpha(-\alpha^4 - 16\alpha^2 + 8\alpha - 8) < 0.$

The total profit without bias is always larger than with bias.

The bias toward the first-period inferior will negatively impact the degree of efforts of the employees and the organization's profit.

4.2 The effect of bias based on the result of the first period

In this section, a more generalized case is analyzed.

Proposition 2. In the case of a tie for the second period, when the manager is inferred to evaluate the first-period inferior as the superior, the total profit becomes smaller. On the contrary in the case of a tie for the second period, when the manager is inferred to evaluate the superior in the first period as the superior, the total profit with bias is greater in most cases.

Proof. A case is analyzed where employees assume that A, the superior from the first period, is evaluated as the superior when performances are almost equal in the second period. Then, the probability above is denoted by $q(0 \le q \le 1)$. Table 5 shows the payoff for the second period.

Table 5: Payoff matrix with bias for the second period

		B(first-period inferior)	
		Good Performer	Bad Performer
A (first-period superior)	Good Performer	【$d_{a2} \cdot d_{b2}$】 $(\alpha q - {d_{a2}}^2, \alpha(1-q) - {d_{b2}}^2)$	【$d_{a2} \cdot (1-d_{b2})$】 $(\alpha - {d_{a2}}^2, 0 - {d_{b2}}^2)$
	Bad Performer	【$(1-d_{a2}) \cdot d_{b2}$】 $(0-{d_{a2}}^2, \alpha - {d_{b2}}^2)$	【$(1-d_{a2})(1-d_{b2})$】 $(\alpha q - {d_{a2}}^2, \alpha(1-q) - {d_{b2}}^2)$

A's expected gain is $E_{a2} = -{d_{a2}}^2 + (\alpha - \alpha q - \alpha d_{b2} + 2\alpha q d_{b2})\, d_{a2} + \alpha q - \alpha q d_{b2}$. From the first-order condition, A's expected gain takes the maximum value at $d_{a2} = \frac{\alpha - \alpha q - \alpha d_{b2} + 2\alpha q d_{b2}}{2}$. Similarly B's expected gain takes the maximum value at $d_{b2} = \frac{\alpha q + \alpha d_{a2} - 2\alpha q d_{a2}}{2}$. From these, we get equilibrium levels of efforts,

$$d_{a2} = \frac{2\alpha - 2\alpha q - \alpha^2 q + 2\alpha^2 q^2}{4 + \alpha^2 - 4\alpha^2 q + 4\alpha^2 q^2}$$

$$d_{b2} = \frac{\alpha(1-2q)(2\alpha - 2\alpha q - \alpha^2 q + 2\alpha^2 q^2) + \alpha q(4 + \alpha^2 - 4\alpha^2 q + 4\alpha^2 q^2)}{2(4 + \alpha^2 - 4\alpha^2 q + 4\alpha^2 q^2)} = \frac{\alpha(2q - 3\alpha q + 2\alpha q^2 + \alpha)}{4 + \alpha^2 - 4\alpha^2 q + 4\alpha^2 q^2}.$$

Then, the sum of equilibrium efforts is as follows:

$$d_{a2} + d_{b2} = \frac{2(2\alpha - 2\alpha q - \alpha^2 q + 2\alpha^2 q^2) + \alpha(1-2q)(2\alpha - 2\alpha q - \alpha^2 q + 2\alpha^2 q^2) + \alpha q(4 + \alpha^2 - 4\alpha^2 q + 4\alpha^2 q^2)}{2(4 + \alpha^2 - 4\alpha^2 q + 4\alpha^2 q^2)}$$

$$= \frac{\alpha(2 - 4\alpha q + 4\alpha q^2 + \alpha)}{(4 + \alpha^2 - 4\alpha^2 q + 4\alpha^2 q^2)} \quad (12).$$

In the same way, as described above, if the discount factor is one, we obtain the first-period equilibrium effort as,

$$\frac{1}{4}\left[2\alpha q + \alpha(1-2q)\frac{\alpha(2 - 4\alpha q + 4\alpha q^2 + \alpha)(4 + \alpha^2 - 4\alpha^2 q + 4\alpha^2 q^2) - 2\alpha(2\alpha - 2\alpha q - \alpha^2 q + 2\alpha^2 q^2)(2q - 3\alpha q + 2\alpha q^2 + \alpha)}{(4 + \alpha^2 - 4\alpha^2 q + 4\alpha^2 q^2)^2}\right] \quad (13).$$

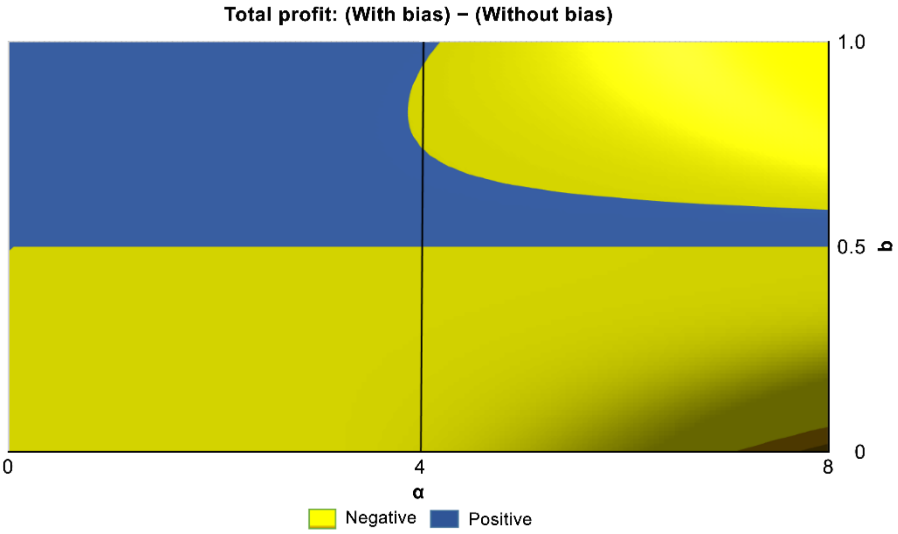
If the company gets "S" when one employee is a good performer, the total expected value of profit in the first and second periods is $S(2 \times (13) + (12)) - 2\alpha k$, and the difference from $S\alpha - 2\alpha k$, which is the total expected value without bias, is

$$\{16\alpha^5 q^5 - (56\alpha^5 - 32\alpha^4)q^4 + (68\alpha^5 - 48\alpha^4 + 16\alpha^3)q^3(38\alpha^5 - 24\alpha^4 + 40\alpha^3 - 32\alpha^2)q^2 + (10\alpha^5 - 4\alpha^4 + 32\alpha^3 - 48\alpha^2 + 32\alpha)q - \alpha^5 - 8\alpha^3 + 16\alpha^2 - 16\alpha\}S \,/\, 4(4 + \alpha^2 - 4\alpha^2 q + 4\alpha^2 q^2)^2$$

$$= (2q-1)\{8\alpha^5 q^4 - (24\alpha^5 - 16\alpha^4)q^3 + (22\alpha^5 - 16\alpha^4 + 8\alpha^3)q^2 - (8\alpha^5 - 4\alpha^4 + 16\alpha^3 - 16\alpha^2)q + \alpha^5 + 8\alpha^3 - 16\alpha^2 + 16\alpha\}S/2(4 + \alpha^2 - 4\alpha^2 q + 4\alpha^2 q^2)^2.$$

Figure 1 shows the difference; the vertical axis shows q between 0 and 1 at intervals of 0.01 and the horizontal axis shows α between 0 and 8 at intervals of 0.05.

(Figure 1) Difference of profit between with bias for the first-period superior and without bias



In the range of $\alpha \leq 4$, the total profit is greater with bias, except in the following cases. The total profit without bias exceeds only in the case of $3.9 \leq \alpha \leq 4$ *and* $0.76 \leq q \leq 0.92$ when $q > 1/2$. In other words, unless α is near 4 and q is intermediate between 1/2 and 1, a bias toward the first period superior is likely to have greater total profit than with no bias.

Moreover, the difference takes the maximum value at $q = 1$ in the range of $0 < \alpha \leq 4$. Though it is difficult for a manager to indicate the value of bias to employees, it is easier to imply that a first-period superior will win when the performances are the same in the second period. Then, the company can obtain more profit than it would if the

manager shows fairness or slight favoritism. When evaluating work, in the second period, if the employees' performances are identical, a manager should make the first-period superior win.

Next, we compare cases where the manager has a bias with cases without bias in each period.

The difference of the second period is $\frac{4(2-\alpha)\alpha^2Sq^2-4(2-\alpha)\alpha^2Sq+(2-\alpha)\alpha^2S}{2(4+\alpha^2-4\alpha^2q+4\alpha^2q^2)}$ as with (5)–(6) of Section 3.

When $\alpha < 2$, if $q = 1/2$, the minimum value is zero, and the remainder is positive.

In other words, bias is more profitable in the second period.

Conversely, in the case $\alpha > 2$ the maximum value zero is taken at $q = 1/2$, otherwise it is negative. Hence without bias the company obtains greater profit for the second period.

The difference in the first period is $\frac{2[\{\alpha-2\alpha q+(4\alpha q-2\alpha)d_{b2}\}d_{a2}+2\alpha q-2\alpha q d_{b2}+\alpha d_{b2}]}{4}-\frac{\alpha}{2}$

$=\frac{\alpha}{2}(2q-1)\{1+2d_{a2}d_{b2}-(d_{a2}+d_{b2})\}$.

If we substitute the value with d_{a2} and d_{b2}, we get

$\alpha(2q-1)\{8\alpha^4q^4-16\alpha^4q^3+(10\alpha^4+8\alpha^3+8\alpha^2)q^2-(2\alpha^4+8\alpha^3+8\alpha^2)q+$

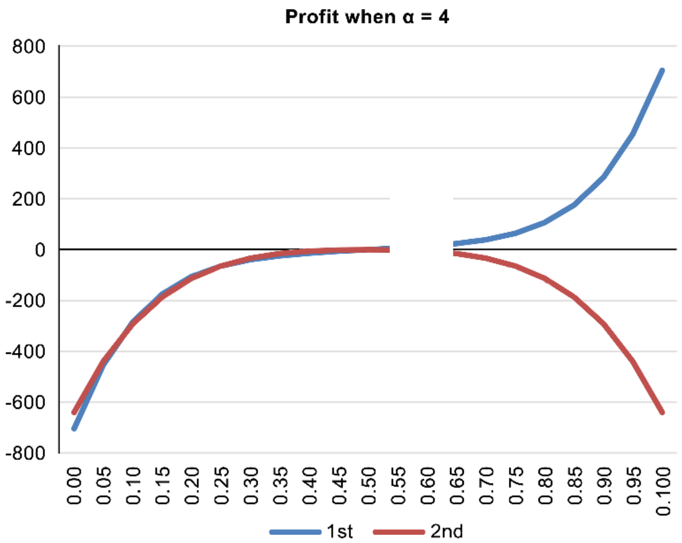
$2\alpha^3+4\alpha^2-8\alpha+16\}S/2(4+\alpha^2-4\alpha^2q+4\alpha^2q^2)^2$ (14).

As equation (14) is always positive in case $\alpha \leq 7.65$ and α in not exceeding 4 from Section 2, positive bias toward the superior employee in the first period increases the overall effort.

Equation (14), which is the difference between the case with bias toward the first-period superior and the case with no bias, takes the minimum value of zero, and then the difference increases in the case $\alpha \leq 2+2\sqrt{2}$ if $q = 1/2$.

Where α is close to 4, the difference graph curve for the second period is gentler than that of the first period. The inflection point appears faster than the difference of the first period (see Fig. 2), such that it is considered that the total profit without bias exceeds only for some intervals when $q > 1/2$ in Figure 1.

(Figure 2) The difference of each-period's profit between with bias and without bias



When considering the first and second periods separately, the profit is greater in the second period if there is bias when the reward is small ($\alpha < 2$); the profit is greater if there is no bias when the reward is large ($\alpha > 2$). For the first period, if the bias to the first-period superior is assumed for the second period, the profit will be greater than without bias.

4.3 The case that two consecutive inferior faces major disadvantages

Next, a case is considered where X is added as a negative gain if an employee becomes inferior for two successive periods. For example, if an employee is deemed inferior for two consecutive periods, they will be disposed of or dismissed. Table 6 shows the payoff.

Table 6: Payoff matrix with additional disadvantage for two consecutive inferior

		B first-period inferior	
		Good Performer	Bad Performer
A First-period superior	Good Performer	Probability of occurring 【$d_{a2}\cdot d_{b2}$】 $(\frac{\alpha}{2} - {d_{a2}}^2, \frac{\alpha-X}{2} - {d_{b2}}^2)$	【$d_{a2}\cdot(1-d_{b2})$】 $(\alpha - {d_{a2}}^2, 0 - {d_{b2}}^2 - X)$
	Bad Performer	【$(1-d_{a2})\cdot d_{b2}$】 $(0 - {d_{a2}}^2, \alpha - {d_{b2}}^2)$	【$(1-d_{a2})(1-d_{b2})$】 $(\frac{\alpha}{2} - {d_{a2}}^2, \frac{\alpha-X}{2} - {d_{b2}}^2)$

The expected gain of A (superior of the first period) is $E_{a2} - {d_{a2}}^2 + \frac{\alpha}{2}\, d_{a2} + \frac{\alpha}{2}(1 - d_{b2})$.

The expected gain of B (inferior of the first period) is $E_{b2} - {d_{b2}}^2 + (\frac{\alpha+X}{2})d_{b2} + \frac{\alpha}{2}(1 - d_{a2}) - \frac{X}{2}(1 + d_{a2})$.

From the first-order condition, we obtain $d_{a2} = \frac{\alpha}{4}\ d_{b2} = \frac{\alpha+X}{4}$.

By the addition of X, d_{b2} is increased by $\frac{X}{4}$.

Then, the total gain difference between the superior and the inferior from the first period for A is considered.

The total expected gain for the first-period superior is

$(\alpha - {d_{a1}}^2) + -(-{d_{a2}}^2 + \frac{\alpha}{2}\, d_{a2} + \frac{\alpha}{2}(1 - d_{b2}))r$ (15).

Here, "r" is a discount factor.

The total expected gain for the first-period inferior is

$(0 - {d_{a1}}^2) + (-{d_{a2}}^2 + (\frac{\alpha+X}{2})d_{a2} + \frac{\alpha}{2}(1 - d_{b2}) - \frac{X}{2}(1 + d_{b2}))r$ (16).

(15)–(16) $= \alpha + \left\{\frac{X}{2}(1 + d_{b2}\right) - \frac{X}{2} d_{a2}\}r \geq 0$ (17).

As equation (17) is positive, the superior from the first period is more advantageous.

In this case, the gain of the first-period superior is not the only α; the employees devise a strategy for the first period regarding $[\alpha + \{\frac{X}{2}(1 + d_{b2}) - \frac{X}{2} d_{a2}\}r]$ as the gain. Here, it is assumed that r = 1. In other words, the value of the equilibrium effort in the first period is not $\frac{\alpha}{4}$ but $\frac{\alpha + \left\{\frac{X}{2}(1+d_{b2}\right) - \frac{X}{2}d_{a2}\}}{4}$.

When $d_{a2} = \frac{\alpha}{4}\ d_{b2} = \frac{\alpha+X}{4}$ are substituted in this expression, the value of the equilibrium effort in the first period is $\frac{\alpha}{4} + \frac{X^2+4X}{32}$.

The total profit without bias is now $S\alpha - 2\alpha k$. Then, the total profit with bias and the disadvantage X becomes $\frac{S(16\alpha+X^2+8X)}{16} - 2\alpha k$, and $\frac{S(X^2+8X)}{16}$ is increased.

If the inferior for two consecutive periods leads to major disadvantages, there is an increase in equilibrium effort level and total profit when compared with a scenario in which the disadvantages are absent.

5. The effect of bias (cases where the performance difference can be grasped precisely)

So far, employees who reach the level of manager's satisfaction are defined as "good performers" and employees who do not reach the level of satisfaction are termed "bad performers"; however, consider a case where the rigorous performance differences are not reflected, such as cases where quantitative performance is difficult to grasp, as in internal affairs, human resources, and accounting. In contrast, the evaluation of sales representatives, for example, when the manager's satisfaction level is one million JPY, an employee who earns 1.1 million JPY and an employee who earns 1.5 million JPY should not be evaluated equally. In this section, the bias's influence is analyzed in cases where performance differences can be strictly described.

5.1 General cases

The basic settings for the analysis are as follows:

An employee's performance is given by $z_i = hd_i + \varepsilon_i, i$ = A, B. Let the random shock that affects the performance of i be ε_i. $\Delta\varepsilon$ denotes $\varepsilon_i - \varepsilon_j$.

$G(x)$ is the cumulative distribution function of $\Delta\varepsilon$, and the density function of $G(x)$ is $g(x)$. As for $g(x)$, assume $g(x) > 0, g'(0) = 0, g''(0) < 0, g'(x) < 0\ if\ x > 0, g'(x) > 0\ if\ x < 0$. The cost of effort is $V(d) = kd^2$.

The Nash equilibrium is considered when bias "c" is implemented for A, supposing biases such as receiving valuable training and assigning jobs that lead to a good evaluation.

A chooses d_a to maximize the following equation assuming the effort level of B, $p(\ z_a + c >\ z_b)\Lambda + \Big(1 -$ p*za+c> zb*0 –V(*da).*

From the first-order condition, we get the following:

$\Lambda g(hd_b - hd_a - c)h = 2k\ d_a$ (18).

B also chooses d_b to maximize the following equation,

$p(\ z_a + c <\ z_b)\Lambda + \Big(1 - p(z_a + c <\ z_b)\Big)\, 0 - V(\ d_b)$.

From the first-order condition, we get the following:

$\Lambda g(hd_b - hd_a - c)h = 2k\ d_b$ (19).

The equilibrium solution is $d_a = d_b = d$.

As $d_a = d_b$, we get $\Lambda g(-c)h = 2kd$.

When we assume $\alpha = \frac{\Lambda}{k}$, $d = \frac{\alpha h}{2} g(-c)$ (20).

As $\frac{\partial d}{\partial c} = -\frac{\alpha h}{2} g'(-c) < 0$, d is a decreasing function of c. Then, d is an increasing function of α.

5.2 The cases where special conditions are set on one side

5.2.1 Bias for those who do not have harsh conditions

Consider a case where a big disadvantage is set only to B if the employee becomes the inferior. For example, assume that B will be disposed of if the employee becomes inferior due to past delinquency.

When defining the loss of disposition as 'X' ($\frac{X}{k} = x$), B chooses d_b to maximize the following formula: p($z_a + c <$ zbΛ−1 −pza+c<zbX−V(db).

From the first-order condition, we get $(\alpha + x)g(hd_b - hd_a - c)h = 2\ d_b$.

Similarly, $\alpha g(hd_b - hd_a - c)h = 2\ d_a$ is derived from the first-order condition for A. Then, we get d_b=$\frac{(\alpha+x)}{\alpha} d_a$. So, $\frac{\alpha h}{2} g\left(\frac{hx}{\alpha} d_a - c\right) = d_a$ (21).

When (20) and (21) are compared, if $\frac{hx}{\alpha} d_a < 2c$, (20) < (21). That is, the equilibrium efforts will increase through the existence of 'X'; however even when $\frac{hx}{\alpha} d_a > 2c$, the total amount of efforts may increase depending on the size of 'X' for d_b=$\frac{(\alpha+x)}{\alpha} d_a$.

Next, the paper considers the relationship between effort and bias.

Proposition 3. The maximum value of d_a exists when $\frac{hx}{\alpha} d_a = c$.

When $d_a > \frac{\alpha c}{hx}$ (c is small), d_a is an increasing function of c.

When $d_a < \frac{\alpha c}{hx}$ (c is large) and $\frac{h^2 x}{2} g'\left(\frac{hx}{\alpha} d_a - c\right) < 1$, d_a becomes a decreasing function of c.

Proof. Supposing $f\left(c, d_a(c)\right) \equiv \frac{\alpha h}{2} g\left(\frac{hx}{\alpha} d_a - c\right) - d_a = 0$, we get $\frac{\partial f}{\partial c} + \frac{\partial f}{\partial d_a} d'_a(c) = 0$.

As $\frac{\partial f}{\partial c} = -\frac{\alpha h}{2} g'\left(\frac{hx}{\alpha} d_a - c\right)$ and $\frac{\partial f}{\partial d_a} = \frac{h^2 x}{2} g'\left(\frac{hx}{\alpha} d_a - c\right) - 1$, we get $d'_a(c) = \frac{\frac{\alpha h}{2} g'\left(\frac{hx}{\alpha} d_a - c\right)}{\frac{h^2 x}{2} g'\left(\frac{hx}{\alpha} d_a - c\right) - 1}$. The extreme value of d_a exists at [$\frac{hx}{\alpha} d_a = c$].

As $d''_a(c) = \frac{\frac{\alpha h}{2} g''\left(\frac{hx}{\alpha} d_a - c\right)\left(\frac{hx}{\alpha} d_a' - c\right)\{\frac{h^2 x}{2} g'\left(\frac{hx}{\alpha} d_a - c\right) - 1\} - \frac{\alpha h}{2} g'\left(\frac{hx}{\alpha} d_a - c\right)\frac{h^2 x}{2} g''\left(\frac{hx}{\alpha} d_a - c\right)\left(\frac{hx}{\alpha} d_a' - c\right)}{\{\frac{h^2 x}{2} g'\left(\frac{hx}{\alpha} d_a - c\right) - 1\}^2} < 0$, the maximum value of d_a exists at [$\frac{hx}{\alpha} d_a = c$].

When $d_a > \frac{\alpha c}{hx}$, $d_a(c) > 0$. When $d_a < \frac{\alpha c}{hx}$ and the denominator of (22) is positive, $d_a(c) < 0$, but it cannot be maximum. If the denominator is negative, $d_a(c) > 0$. Then, regarding the manager's utility function, it is assumed that the manager places more importance on employees' utilities. The manager's utility function is written as

$$U_s = l(d_a^* + d_b^*)$$
$$+ m\left\{\mathrm{p}(z_a + c > z_b)\alpha - {d_a^*}^2 + \mathrm{p}(z_a + c < z_b)\alpha - \left(1 - \mathrm{p}(z_a + c < z_b)\right)x - {d_b^*}^2\right\} - nc^2.$$

The first term shows the evaluation of the manager. The better the performance of the two employees is, the greater the manager's utility becomes. The second term shows the sum of the employees' gain. The total happiness of the employees leads to that of the manager. The third term indicates the cost of favoritism; a manager's reputation for being unfair will spread.

Differentiating the utility of the manager for bias using the envelope theorem, we get

$$\frac{\partial U_s}{\partial c} = \frac{l(2\alpha + x)}{2} d_a'(c) - mg(\frac{hx}{\alpha} d_a - c)h(2\alpha + x)\, d_a'(c) - mx(1 + hd_a'(c))g\left(\frac{hx}{\alpha} d_a - c\right) - 2nc = 0$$

When $d_a > \frac{\alpha c}{hx}$ (c is small), the first term is positive, the second, third, and fourth terms are negative.

The impact on improving performance is equal to the sum of the employees' utility decrease because of the lower probability of winning (increase in the opponent's effort), the increasing likelihood that the disadvantage X is given, and the cost increase that extending favoritism entails.

Then, the manager's utility has the maximum value, if the following equation holds: $\frac{l(2\alpha+x)}{2} d_a''(c) < m\alpha g'(\frac{hx}{\alpha} d_a -$ *c)(hxαdα′−1)h2α+xαdα″c+mxg′hxαdα−c(hxαdα′−1)+2n.*

If the manager is only interested in performance, the utility function is only the first term, and positive bias $(c = \frac{hx}{\alpha} d_a)$ will be fostered toward employees without harsh conditions

When $d_a < \frac{\alpha c}{hx}$ (c is large) and the denominator of $d_a'(c)$ is negative, the first term is negative, the second term is positive, the third term is negative or positive, and the fourth term is negative.

5.2.2 Bias toward those with harsh conditions

Here, considers a case where bias is implemented to B under the condition that B will be disposed of if B becomes the inferior.

Proposition 4. Bias toward the employee with harsh conditions reduces both sides' efforts.

Proof. B chooses d_b to maximize the following formula,

$$\mathrm{p}(z_a < z_b + c)\Lambda - \left(1 - \mathrm{p}(z_a < z_b + c)\right) X - \mathrm{V}(d_b).$$

We obtain $(\alpha + x)g(hd_b - hd_a + c)h = 2\, d_b$ from the first-order condition.

Similarly, $\alpha g(hd_b - hd_a - c)h = 2\, d_a$ is derived from the first-order condition for A.

Then, we get $d_b = \frac{(\alpha+x)}{\alpha} d_a$. Hence, $\frac{\alpha h}{2} g\left(\frac{hx}{\alpha} d_a + c\right) = d_a$.

The value of d_a is smaller compared to d_a without X $(d = \frac{\alpha h}{2} g(c))$. As for $d_b (= \frac{(\alpha+x)h}{2} g\left(\frac{hx}{\alpha} d_a + c\right))$, it depends on the size of X. The value of d_a becomes smaller compared to the equilibrium effort when "c" is implemented to A and X (against B) exists.

Now, consider the relationship between effort and bias.

Supposing $f(c, d_a(c)) \equiv \frac{\alpha h}{2} g\left(\frac{hx}{\alpha} d_a + c\right) - d_a = 0$, we get $\frac{\partial f}{\partial c} + \frac{\partial f}{\partial d_a} d_a'(c) = 0$.

As $\frac{\partial f}{\partial c} = \frac{\alpha h}{2} g'\left(\frac{hx}{\alpha} d_a + c\right)$ and $\frac{\partial f}{\partial d_a} = \frac{h^2 x}{2} g'\left(\frac{hx}{\alpha} d_a + c\right) - 1$, we get

$$d_a'(c) = \frac{-\frac{\alpha h}{2} g'\left(\frac{hx}{\alpha} d_a + c\right)}{\frac{h^2 x}{2} g'\left(\frac{hx}{\alpha} d_a + c\right) - 1}.$$

When c is positive (bias toward the employee with harsh conditions), d_a is a decreasing function because it has no extremes and because the numerator is positive and the denominator is negative. Bias toward those with harsh conditions reduces both sides' efforts. Therefore, sympathy for the employee with harsh conditions is bad for company's performance.

Again, consider the manager's utility function (the employee's utility is put into the manager's utility).

Differentiating the manager's utility concerning bias using the envelope theorem, we get

$$\frac{\partial U_s}{\partial c} = \frac{l(2\alpha+x)}{2} d_a'(c) - (2\alpha + x)hmg\left(\frac{hx}{\alpha} d_a + c\right) d_a'(c) + mx(1 - hd_a'(c))g\left(\frac{hx}{\alpha} d_a + c\right) - 2nc = 0.$$

The first term is negative, the second and third terms are positive, and the fourth term is negative. Hence, the manager's utility has an extreme value if the equation holds. The sum of the utility increases because of the increasing probability of winning and the decreasing likelihood that the disadvantage, X, is given is equal to the sum of the decrease in utility because of decreasing performance and the increase in cost.

Next, consider a relationship between effort and a disadvantage to one side (X).

Proposition 5. In the case that c is positive (bias toward those where harsh conditions are not set), when $d_a > \frac{\alpha c}{hx}\left(x > \frac{\alpha c}{hd_a}\right)$, d_a is the increasing function of x. When $d_a < \frac{\alpha c}{hx}$, if the denominator is negative, d_a is the decreasing function of x.

Proof. Supposing $f\left(x, d_a(x)\right) = \frac{\alpha h}{2} g\left(\frac{hx}{\alpha} d_a - c\right) - d_a = 0$, we get $\frac{\partial f}{\partial x} + \frac{\partial f}{\partial d_a} d_a'(x) = 0$.

As $\frac{\partial f}{\partial x} = \frac{\alpha h}{2} g'\left(\frac{hx}{\alpha} d_a - c\right)\left(\frac{h}{\alpha} d_a\right)$ and $\frac{\partial f}{\partial d_a} = \frac{h^2 x}{2} g'\left(\frac{hx}{\alpha} d_a - c\right) - 1$, we get

$$d_a'(x) = \frac{\frac{h^2}{2} g'\left(\frac{hx}{\alpha} d_a - c\right)}{\frac{h^2 x}{2} g'\left(\frac{hx}{\alpha} d_a - c\right) - 1}.$$

The external value of d_a exists at $\frac{hx}{\alpha} d_a = c$.

As $d_a''(x) = \frac{\frac{h^2}{2} g''\left(\frac{hx}{\alpha} d_a - c\right)\left(\frac{h}{\alpha} d_a + \frac{hx}{\alpha} d_a'\right)\{\frac{h^2 x}{2} g'\left(\frac{hx}{\alpha} d_a - c\right) - 1\} - \frac{h^2}{2} g'\left(\frac{hx}{\alpha} d_a - c\right)\frac{h^2 x}{2} g''\left(\frac{hx}{\alpha} d_a - c\right)\left(\frac{h}{\alpha} d_a + \frac{hx}{\alpha} d_a'\right)}{\{\frac{h^2 x}{2} g'\left(\frac{hx}{\alpha} d_a - c\right) - 1\}^2} > 0$,

d_a takes the minimum value at $\frac{hx}{\alpha} d_a = c$.

To sum up, as for cases where harsh conditions are set on one side, when biasing employees without harsh conditions, the equilibrium effort level increases while the bias is small. As the bias increases, it goes down, but the total amount of effort may increase depending on the size of the harsh conditions, X. With a bias toward employees with harsh conditions, the equilibrium effort decreases, becoming a decreasing function of bias.

5.3 The case that the manager implements the bias after recognizing the performances of two employees

Under the condition that the inferior employee is disposed of, the manager retrospectively evaluates the employees' performances. The manager's utility function is

$U_s = l$ × (sum of employees' performance)$+m$×(sum of the utility of employees)
$-$n × (arbitrary adjustment=difference of performances)^2.

The manager checks the performance of the two people and evaluates them by the following process. In the case of $z_a < z_b$, the manager's utility when B wins is larger than when A wins. Therefore, the manager rewards B.

In the case of $z_a > z_b$, the difference in the manager's utilities between A winning and B winning is $-mX + n \cdot (z_a - z_b)^2$. The utility of A winning and B winning is indiscriminate when $z_a - z_b = \sqrt{\frac{mX}{n}}$.

Then, two employees assume to be biased $\sqrt{\frac{mX}{n}}$ to B and choose the level of effort.

B chooses d_b to maximize the following formula: $\mathrm{p}(z_a < z_b + \sqrt{\frac{mX}{n}})\Lambda - (1 - \mathrm{p})(z_a < z_b + \sqrt{\frac{mX}{n}})X - kd_b^2$. The first-order condition is $(\alpha + x)g(hd_b - hd_a + \sqrt{\frac{mX}{n}})h = 2\,d_b$. The first-order condition for A is $\alpha g(hd_b - hd_a + \sqrt{\frac{mX}{n}})h = 2\,d_a$.

Then, we get $d_b = \frac{(\alpha + x)}{\alpha} d_a$.

As stated earlier, the bias toward B reduces equilibrium effort.

Next, consider the relationship between effort and profit.

First, $\left(X, d_a(X)\right) \equiv \frac{\alpha h}{2} g\left(\frac{hX}{\alpha k} d_a + \sqrt{\frac{mX}{n}}\right) - d_a = 0$ are defined, transforming the first-order condition for A.

We substitute [$\frac{\partial f}{\partial X} = \frac{\alpha h}{2} g'\left(\frac{hX}{\alpha k} d_a + \sqrt{\frac{mX}{n}}\right)(\frac{h}{\alpha k} d_a + \frac{1}{2}\sqrt{\frac{m}{n}} X^{-\frac{1}{2}})$] and [$\frac{\partial f}{\partial d_a} = \frac{h^2 X}{2k} g'\left(\frac{hX}{\alpha k} d_a + \sqrt{\frac{mX}{n}}\right) - 1$] into [$\frac{\partial f}{\partial X} + \frac{\partial f}{\partial d_a} d_a'(X) = 0$].

Then, we get $d_a'(X) = \frac{-\frac{\alpha h}{2} g'\left(\frac{hX}{\alpha k} d_a + \sqrt{\frac{mX}{n}}\right)(\frac{h}{\alpha k} d_a + \frac{1}{2}\sqrt{\frac{m}{n}} X^{-\frac{1}{2}})}{\frac{h^2 X}{2k} g'\left(\frac{hX}{\alpha k} d_a + \sqrt{\frac{mX}{n}}\right) - 1} < 0$.

The larger X is, the smaller d_a becomes.

The manager checks the achievements of the two and rewards B with a win if $z_a < z_b$; however, B wins even when $z_a > z_b$, if $-mX + n \cdot (z_a - z_b)^2 < 0$

As we saw earlier, the assumptions implementing the bias to the employee with harsh conditions will reduce both sides' efforts. However, if the manager's utility decreases with the decrease in the employee's utility, the manager's evaluation conflicts with the performance improvements that are desirable for the organization.

Organizations should create workplaces where the manager is not afraid to be hated by employees or does not become attached to them.

6. Discussion and conclusions

Companies have limited funds to pay for their employees. Regarding allocating funds to employees, few disagree with making differences based on the degree of contribution. However, it is difficult to grasp the varying levels of contribution. If managers do not differentiate, the employees' motivation may be lowered. Therefore, performance

appraisal systems are conducted with an objective evaluation, which leaves some ambiguity, and the manager's bias can play a mediating role.

Apart from idealism, according to this analysis, in cases where performances are difficult to quantify, a small bias for a single period and bias for the first-period superior from the two periods can raise the organization's overall profit. Similar to this study, Meyer (1992) analyzed a two-period principal-agent model, showing that the principal has zero bias in the first period and a strictly positive bias toward the first-period winner in the second period. Meyer's model allows players to recognize bias levels and measure performances quantitatively. This paper, however, examines situations in which performance differences are unclear and employees act on the basis of assumptions about a manager's evaluation tendencies. The results confirm that the bias to the first-period superior increases the total profit; however, in some cases, the total profit will increase without bias. Effort in Meyer's model is a decreasing function concerning bias in the second period, but bias increases effort in this study when the reward is small. This is because the rewards for the first and second periods are configured to minimize costs in Meyer's model although those are exogenous in the current model. In reality, labor costs are not so flexible.

In cases where performances can be quantified, setting a condition that the losers of two consecutive periods will be dismissed increases employees' efforts and the organization's profit. The bias for employees without preset severe conditions may increase the organization's profit. These results justify policies that purge incompetent employees and implement bias to capable employees; incompetent employees should not be treated generously and capable employees should be treated well. If we regard the severe conditions as handicap in the second period for the inferior of the first period, bias for the first-period superior can raise the organization's overall profit such as in cases where performances are difficult to quantify.

However, when the manager evaluates after checking the employees' performances in cases where performances can be quantified, the manager may extend favoritism to those with harsh conditions, thus lowering the equilibrium effort level. Further analysis is required to discern whether the difference between the winner's reward and the damage of dismissal is valid, compared to the difference between employees' performances and whether it is good to continue the jungle system based on the utility of each employee and manager.

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