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Optimal managerial decision model in a closed-loop supply chain for a retailer to collect used products with different transfer policy

T. C. Weng[1], C. T. Hsu[2]
[1]Department of Human Resource Development, National Kaohsiung University of Science and Technology, 82444, Taiwan & tzuchun@nkust.edu.tw
[2]Department of Recreation & Sport Management, Shu-Te University of Science and Technology, 82445, Taiwan & ajexhsu@stu.edu.tw

Abstract

In recent years, the concerns of environmental and resource issues are growing. Due to a great deal of attention for closed-loop supply chains, collection behaviour of used products from public consumers becomes more and more important. This paper proposes an inventory model for retailer with new products and used products and the retailer is the decision maker who follows the rule of Economic Ordering Quantity (EOQ) model. We assumed that the retailer not only sells products but also collects those sold used products. The proposed problem faced by the retailer is formulated as a cost-minimization problem where the replenishment quantity of new products and the return rate of used products are the decision variables and an efficient iterative algorithm is provided to search for the solution. Furthermore, we extend our model to consider the "not lot for lot policy" inventory model for forward channel and reverse channel and determine the optimal management policy. Finally, we discuss the economical decision modelling for a retailer to analyse the willingness of collecting used products.

Keywords: Closed-loop supply chains; used products; inventory; replenishment quantity; return rate.

1. Introduction

For the past few years, many people are so deeply concerned about the problem of environmental change, especially global resources waste. Resources waste is not only an environmental problem, but also an economic and capital loss. Due to the fact, there is a great deal of attention for closed-loop supply chains in order to minimize economy-wide use of factors of production in satisfying final product demand (Duchin and Levine 2019). Recovery behaviour of used products from consumers becomes more and more significant and important. Used-products recovery process includes recycle, reuse, refurbish, repair, remanufacture, and cannibalize. Recycling materials and products multiple times before the end of their useful lifetimes is one of used-products recovery ways to protect our environments (Geissdoerfer et al. 2017). In recent years, there are more researches concerning about the recycling of resources and used-product. Shumon et al. (2011) stated that environmental concern for managing waste and product disposals has encouraged some enterprises to adjust their business operations and processes to make them more efficient in order to minimize waste. Recycle of used-product becomes more and more significant and important to the industry. Industrial applications include photo copiers, tires, personal computers and et cetera.

Over the last few decades, there has been an increase in the number of publications on reverse logistics. Prahinski and Kocabasoglu (2006) indicated that reverse logistics is the process which retrieving and collecting the used product from the end consumers in order to pursue the purposes of creating and capturing value or more proper disposal. Ilgin and Gupta (2010) stated that reverse logistics activities are involved and associated with the collection, reuse, recovery or disposal of used products from public consumers. Shulman et al. (2010) proposed a bilateral monopoly model and investigated the influence of the reverse channel structure. Govindan et al. (2015) analysed and categorized recently published papers which are included total of 382 papers in the fields of closed-loop supply chain from 2007 to 2013. Abbey and Guide (2018) detailed many remanufacturing industries in closed-loop supply chains. We note that the importance of reverse logistics can be explicitly found.

Wei et al. (2015) indicated a unique opportunity may be offered by reverse logistics process to improve their profits and to serve social responsibility for companies. The front behaviour in all the reverse logistic is the collection of the used products and reuse of products or resources. How to increase the amount of the collection is an important issue that may offer manufacturers to decrease production cost with remanufacturing. Savaskan et al. (2004) found that the retailer, who is closer to the customer, is the most effective undertaker of used-product collection behaviour in the reverse logistic. In this paper, we proposed that the retailer is contracted to the collection of used products. Another critical decision relevant to the economic activities of a retailer is the replenishment quantity of new products which are going to sell to public consumers. From an economic perspective, the appropriate decision on the replenishment quantity is dependent on cost components incurred to the retailer such as the fixed replenishment cost and the inventory holding cost.

In this paper, the retailer is assumed to be an EOQ-based decision maker. The retailer can sell new products to public consumers and collect those sold used products from public consumers simultaneously. The contributions of this paper are about the collection behaviour and replenishment decisions, as well as the batch policy of used-products procedure in different cost consideration and economical decision modelling for a retailer to analyse the willingness of collecting used products. Under these model assumptions, the following research questions will be focused to answer: (1) How can the retailer optimally determine the decisions for the replenishment quantity and the return rate of used-products simultaneously? (2) Is it economically more beneficial for a retailer with the not lot for lot transport policy than with lot for lot transport policy? (3) How does the retailer which is involved in collecting the used sold products enhance incentives to increase return rate?

The rest of this paper is organized as follows. In section 2, we review and discuss the related literature of this paper. In section 3, we present assumptions and notation of closed-loop supply chain model and mathematically formulate the model. And then we propose an algorithm to solve the model and analyse the result in section 4. In section 5, we improve the model which considers the not lot for lot policy. In section 6, we discuss the economical decision modelling for a retailer to analyse the willingness of collecting used products. In section 7, we give some conclusions and possible future research at the end of this research.

2. Literature Review

In this section, the literatures concerned with the reverse logistics and EOQ Model with remanufactured product in the supply chain are reviewed. With the following literature review, we can propose some different contributions to several streams of literature.

2.1 Reverse logistics

Closed-loop supply chain is a kind of supply chain network including return processes from customers. Channel members have the intention of integrating all supply chain activities which include forward logistics and reverse logistics to create additional value and benefits. Closed-loop supply chains, covering recycling and reverse logistics, are expected to enable businesses to meet the growing demands with regard to corporate social responsibility and wider social goals in order to reduce the resource-intensity of contemporary economic life (Wells and Seitz 2005).

Savaskan and Van Wassenhove (2006) investigated the relationship between reverse logistics design and product pricing decision in two competing retailers. Jayant et al. (2012) described that reverse logistics is a process through collecting, recycling, reusing, and reducing the amount of materials in order to become more environmentally efficient for the world. In the past, reverse logistics has often been viewed as the unavoidable part of supply chain management and seen as a necessary business cost, a regulatory commitment issue or a green initiative. Now, however, more companies are regarding it as a strategic activity that can enhance supply chain competitiveness over the long time. Mahapatra et al. (2013) proposed a modified reverse supply chain model with the stationary demand and analysed the relationship between different parameters of the model.

2.2 EOQ model with remanufactured product

Early researchers at collection of the used products focused on manufacturer. Inderfurth (2004) considered an EPQ model including set-up cost, inventory holding cost and set-up times and investigated lot-sizing decisions in a hybrid production/rework system characterized by defective products with (P, 1) policy. Chen et al. (2016) considered an EOQ model with selling the products to public consumers and collecting the used sold products from public consumers. The major purpose of this study is to investigate and focus on the role of the retailer. The retailer can sell new products to public consumers as well as collect those sold used products from public consumers simultaneously, who is contracted to the collection of used products by manufacturer.

There are numerous studies in the literature dealing with the sources of collecting used products. Hong and Yeh (2012) proposed a retailer collection and a non-retailer collection model in a close-loop supply chain and compare the resulting performance measurement for both models. Hong et al. (2013) stated a topical issue of choosing the appropriate reverse channel under hybrid dual-channel collection from customers. The secondary purpose of this study is to discuss the different collection behaviours. In the past research, the collection agent deals with the used products sold in the last replenishment cycle. In contrast, we extend the assumption that the collection agents at current replenishment cycle can collect used products sold in the all past cycle.

A key feature differentiating this paper from the extant literature is that an inventory model for retailer with new products and used products and the retailer is the decision maker who follows the rule of EOQ model. Furthermore, this paper extends the model to consider the "not lot for lot policy" inventory model for forward channel and reverse channel and determine the optimal management policy. Next, we present our modelling notation and assumptions.

3. Retailer's Replenishment Model without allowable Shortages

Retailer is the first level of product recovery in reverse logistics, so this paper focus on the sale and collection behaviour for the retailer. The decision-making retailer is assumed to sell the products to public consumers, as well as collect the used sold products from public consumers. Fig 1 depicts the product flow in the closed-loop supply chain which includes the retailer replenishment with new products from the manufacturer, the retailer selling new products to the public consumers, the retailer collecting used products from the public consumers and the retailer transferring used products to the manufacturer or the remanufacturer.

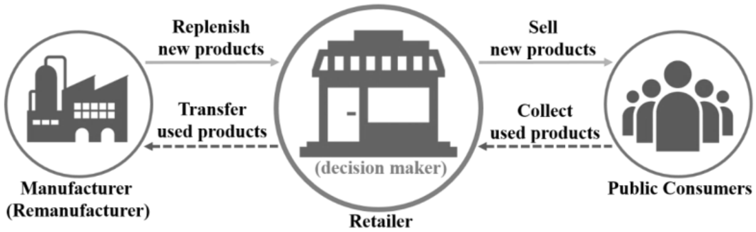

Fig 1: The product flows of the retailer.

3.1. The Forward Channel

The retailer replenishes products from the manufacturer and sells new products to the public consumers, determining the each replenishment quantity Q. We assume that there is a single product and the replenishment of the product is instantaneous without delaying. The demand rate D of the product for the public consumers is deterministic and constant over time. The shortage or delivery lag of new products is not allowed for public consumers. The replenishment cycle time for retailer to replenish products is $\frac{Q}{D}$. The planning horizon in the forward channel is unlimited and infinite.

3.2. The Reverse Channel

The retailer manages and handles the actual collection operation, determining the collection quantity U per transfer to the manufacturer. The retailer incurs the acquisition cost $AC(\tau)$ and transfer profit $TP(\tau)$; thus, τ is the collection rate of products from the previous generation's sales volume (under the implicit assumption that all returns are transferable to the manufacturer).

It is assumed that those products all sold in the past replenishment cycles can be collected at the current replenishment cycle (De Giovanni and Zaccour, 2014). The return rate of the products sold in the past i-th replenishment cycle follows a geometric series with initial value τ and common ratio r, $0 \le r < 1$. The return rate of the products sold in the past i-th replenishment cycle is given by τr^{i-1}. Therefore, the cumulative return rate of used products R at the current replenishment cycle can be expressed by

$$R = \sum_{i=1}^{\infty} \tau r^{i-1} = \tau + \tau r + \tau r^2 + \tau r^3 + \cdots\cdots + \tau r^{\infty} = \frac{\tau}{1-r} \tag{1}$$

It is noted that the total number of collected used products should be non-negative and cannot be exceed the number of products sold at each replenishment cycle, $0 \le \frac{\tau}{1-r} \le 1$. Hence, the constraint $0 \le \tau \le 1 - r$ holds.

The collection quantity U per transfer can be shown as follows.

$$U = R \times Q = \frac{\tau}{1-r} \times Q = \frac{\tau Q}{1-r} \tag{2}$$

Fig 2 shows the inventory level of forward channel and reverse channel.

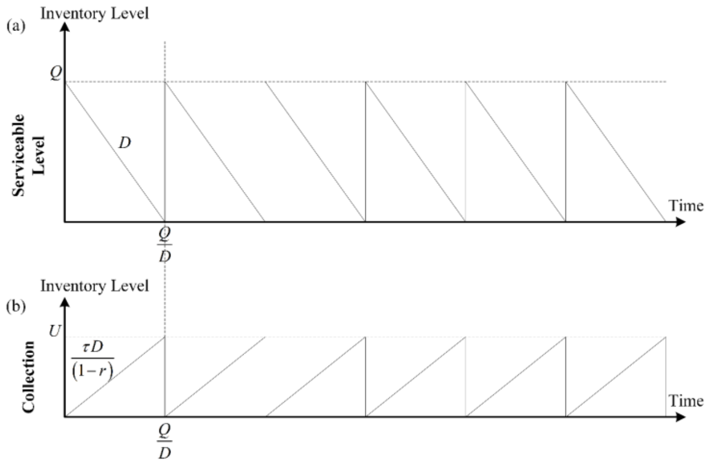

Fig 2: Inventory levels of forward channel and reverse channel.

The retailer collects and handles a returned product which includes the collecting fee A paid by the retailer to the public consumers. And then, the retailer transfers all collected used-product to the manufacturer and receives a transfer price b per unit. The salvage value $b-A$ of a collected used product is assumed to be positive, i.e., $b-A>0$ (Jena and Sarmah, 2014). For the condition, the retailer is willing to collect and handle a returned product. The acquisition cost $AC(\tau)$ and the transfer profit $TP(\tau)$ can be shown as follows.

$$AC(\tau) = A \times U = A \times \frac{\tau Q}{1-r} = \frac{A\tau Q}{1-r} \tag{3}$$

$$TP(\tau) = b \times U = b \times \frac{\tau Q}{1-r} = \frac{b\tau Q}{1-r} \tag{4}$$

3.3. The Collection Cost

We model the investment cost to achieve a collection rate τ of used product for the retailer as $I(\tau)$. The investment cost function in collecting used products is modelled by $C_L\tau^2$ (Rubio and Jiménez-Parra, 2014), where C_L is a scaling parameter.

Table 1. Notation

Symbol	Description
c	Purchasing cost of product (per unit).
K_p	Replenishment cost (per replenishment).
K_c	Setup cost (per collection activity).
h_p	Inventory holding cost of new product (per unit per unit time).
h_c	Inventory holding cost of used product (per unit per unit time), the inventory holding cost of new product must be more than the inventory holding cost of used product, i.e., $h_p - h_c > 0$ (Ketzenberg, 2009).
D	Demand rate of product for customers.
Q	Replenishment quantity (per replenishment).
U	The inventory level for used product which is collected.
A	Unit cost of collecting and handling a returned product which includes the collecting fee paid by the retailer to consumers.
b	Unit price of a collected used product sold by the retailer to the manufacturer.
τ	Return rate of collecting used products.
$I(\tau)$	Investment cost of the retailer in collecting used-product activities, which is assumed to be a function of return rate τ. $I(\tau)=C_L\tau^2$.

The total cost per replenishment cycle (*TCPRC*), which is the sum of the costs minus the benefit for the retailer. Namely, $K_p + cQ + \frac{h_p Q^2}{2D} + K_c + \frac{h_c \tau Q^2}{2(1-r)D} + C_L \tau^2 - \frac{(b-A)\tau Q}{1-r}$. The corresponding total cost per unit time (*TCPUT*) can be obtained by dividing the total cost per replenishment cycle by the cycle length. The objective of the model is to minimize total cost per unit time subject to the return rate constraint. Namely,

$$\min TCPUT(Q,\tau) = \frac{K_p D}{Q} + cD + \frac{h_p Q}{2} + \frac{K_c D}{Q} + \frac{h_c \tau Q}{2(1-r)} + C_L \tau^2 - \frac{(b-A)\tau D}{(1-r)} \tag{5}$$

$$\text{subject to: } 0 \le \tau \le 1-r \, . \tag{6}$$

In order to solve the proposed nonlinear programming problem shown in Equation (5) and (6), the constraint $0 \le \tau \le 1-r$ is ignored and the first partial derivatives of $TCPUT(Q,\tau)$ with respect to Q and τ can be obtained, respectively. The corresponding first-order necessary condition (FONC) are given by Equations (7) and (8).

$$\frac{\partial TCPUT(Q,\tau)}{\partial Q} = -\frac{K_p D}{Q^2} - \frac{K_c D}{Q^2} + \frac{h_p}{2} + \frac{h_c \tau}{2(1-r)} = 0 \tag{7}$$

$$\frac{\partial TCPUT(Q,\tau)}{\partial \tau} = \frac{h_c Q}{2(1-r)} + 2C_L \tau - \frac{(b-A)D}{(1-r)} = 0 \tag{8}$$

By examining Equations (7) and (8), the closed-form solutions of optimal (Q^*, τ^*) is very difficult to obtain. However, for fixed τ, $TCPUT(Q,\tau)$ is convex with respect to Q because

$$\frac{\partial^2 TCPUT(Q,\tau)}{\partial Q^2} = \frac{2K_p D}{Q^3} + \frac{2K_c D}{Q^3} > 0 \tag{9}$$

Given fixed τ, by rearranging Equation (7), $\hat{Q}$ can be obtained as Equation (10).

$$\hat{Q} = \sqrt{\frac{2(1-r)(K_p + K_c)D}{h_p(1-r) + h_c \tau}} \tag{10}$$

For fixed Q, $TCPUT(Q,\tau)$ is convex with respect to τ because

$$\frac{\partial^2 TCPUT(Q,\tau)}{\partial \tau^2} = 2C_L > 0 \tag{11}$$

Similarly, for fixed Q, by rearranging Equation (8), $\hat{\tau}$ can be obtained as Equation (12).

$$\hat{\tau} = \frac{2(b-A)D - h_c Q}{4C_L(1-r)} \tag{12}$$

Proposition 1. With the constraint $0 \le \tau \le 1-r$ and fixed τ, then there exists a unique $\hat{Q} > 0$ regarding to the constraint of return rate $0 \le \tau \le 1-r$ to minimize *TCPUT*(*Q*). (**Appendix A**)

Proposition 1 shows that an optimal replenishment quantity per replenishment exists and is unique, $\hat{Q}$ can only be obtained by a nonlinear search algorithm since there is no closed form expression for $\hat{Q}$. Next, we will introduce the nonlinear search algorithm to minimize *TCPUT*(*Q*).

3.4. Analysis and Results

According to the result of Proposition 1, an efficient algorithm can be developed to search for the optimal Q^* and τ^* which minimizes *TCPUT*.

While performing the search algorithm, if the return rate $\hat{\tau}$ obtained from Equation (12) does not fall into the interval (0, 1−*r*], then the return rate $\hat{\tau}$ is set to be equal to (1−*r*). In such a case, the initial value for the replenishment quantity is set to be EOQ. The proposed iterative procedure to search for the optimal replenishment quantity, and return rate is outlined as follows.

Algorithm

Initial Parameter Step:

Input the values of relevant model parameters c, b, A, C_L, K_p, K_c, h_p, h_c, D, and r.

Check the conditions $b>A$, $h_p>h_c$ and $0 \le r<1$. If false, input the values of parameters again.

Set $Q_0 = \sqrt{\frac{2K_pD}{h_p}}$,

$$f(Q) = -K_pD - K_cD + \frac{h_pQ^2}{2} + \frac{h_c(b-A)DQ^2}{4C_L(1-r)^2} - \frac{h_c^2Q^3}{8C_L(1-r)^2} \text{ and}$$

$$f'(Q) = h_pQ + \frac{h_c(b-A)DQ}{2C_L(1-r)^2} - \frac{3h_c^2Q^2}{8C_L(1-r)^2}.$$

Iteration Algorithm Step:

Set $n=1$ and calculate $Q_n = Q_{n-1} - \frac{f(Q_{n-1})}{f'(Q_{n-1})}$.

Do {

Set $n = n+1$ and $Q_n = Q_{n-1} - \frac{f(Q_{n-1})}{f'(Q_{n-1})}$.

} Until $Q_n - Q_{n-1} < \varepsilon$

Set $Q = Q_n$ and calculate $\tau = \frac{(b-A)D}{2C_L(1-r)} - \frac{h_cQ}{4C_L(1-r)}$.

If $\tau > 1-r$, then $\tau = 1-r$ and $Q = \sqrt{\frac{2(K_p+K_c)D}{h_p+h_c}}$

Calculate $TCPUT(Q,\tau)$.

Final Output Step:

Output the results $(Q^*,\tau^*) = (Q,\tau)$ and $TCPUT^* = TCPUT(Q,\tau)$.

In order to validate the model that we consider an example with the following parameters. The values of parameters are c = 10, b = 2, A = 1, C_L = 20000, K_p = 12000, K_c = 2000, h_p = 0.3, h_c = 0.1, D = 10000 and r = 0.1. By employing the algorithm, this numerical example can be solved very efficiently. Assuming $\varepsilon = 10^{-8}$, the optimal solution Q^* = 29291.4763 and τ^* = 0.2371, and the corresponding total cost per unit time (TCPUT) is 108048.9865.

4. Sections Retailer's Replenishment Decisions in the EOQ Model with Allowable Shortages under Used-Product Collection Policy

In this section, shortage or delivery lag is allowed, determining the shortage level per replenishment S. Under these definitions and assumptions, the total net cost per replenishment cycle includes the costs per replenishment cycle and the benefit from collecting used products per replenishment cycle. Therefore, the total net cost per replenishment cycle, which is the costs minus the benefit is given by

$$K + cQ + \frac{h(Q-S)^2}{2D} + \frac{pS^2}{2D} + C_L\tau^2 - (b-A)Q\left(\frac{\tau}{1-r}\right) \tag{13}$$

The corresponding total net cost per unit time can be obtained by dividing the total net cost per replenishment cycle by the cycle length. The objective of the basic model is to minimize total cost per unit time (*TCPUT*) subject to the return rate constraint. Namely,

$$\text{Minimize } TCPUT(Q,S,\tau) = \frac{KD}{Q} + cD + \frac{h(Q-S)^2}{2Q} + \frac{pS^2}{2Q} + \frac{C_L D\tau^2}{Q} - (b-A)D\left(\frac{\tau}{1-r}\right) \quad (14)$$

$$\text{subject to: } 0 \le \frac{\tau}{1-r} \le 1 \quad (15)$$

In order to solve the proposed nonlinear programming problem shown in expression (14), we first ignore the constraint $0 \le \frac{\tau}{1-r} \le 1$ and take the first partial derivatives of $TCPUT(Q, S, \tau)$ with respect to τ, S and Q, respectively. The corresponding first-order necessary condition (FONC) are separately given by equations (16), (17) and (18)

$$\frac{\partial TCPUT}{\partial \tau} = \frac{2C_L D\tau}{Q} - \frac{(b-A)D}{1-r} = 0 \quad (16)$$

$$\frac{\partial TCPUT}{\partial S} = -\frac{h(Q-S)}{Q} + \frac{pS}{Q} = 0 \quad (17)$$

$$\frac{\partial TCPUT}{\partial Q} = -\frac{KD}{Q^2} + \frac{h(Q-S)}{Q} - \frac{h(Q-S)^2}{2Q^2} - \frac{pS^2}{2Q^2} - \frac{C_L D\tau^2}{Q^2} = 0 \quad (18)$$

By solving the simultaneous equations (16), (17) and (18), we can have the optimal τ^*, S^* and Q^*, which are expressed in equations (19), (20) and (21)

$$\tau^* = \frac{(b-A)}{2C_L(1-r)}\sqrt{\frac{2KD}{h\left(\frac{p}{h+p}\right) - \frac{D(b-A)^2}{2C_L(1-r)^2}}} \quad (19)$$

$$S^* = \frac{h}{h+p}\sqrt{\frac{2KD}{h\left(\frac{p}{h+p}\right) - \frac{D(b-A)^2}{2C_L(1-r)^2}}} \quad (20)$$

$$Q^* = \sqrt{\frac{2KD}{h\left(\frac{p}{h+p}\right) - \frac{D(b-A)^2}{2C_L(1-r)^2}}} \quad (21)$$

From the observations of equations (19), (20) and (21), we found out that if the salvage value ($b-A$) approaches zero or the scaling parameter C_L is relatively large, then the optimal replenishment quantity Q^* approaches $\sqrt{\frac{2KD}{h}\left(\frac{h+p}{p}\right)}$ and the optimal allowable shortage level per replenishment S^* approaches $\frac{h}{h+p}\sqrt{\frac{2KD}{h}\left(\frac{h+p}{p}\right)}$, which are the traditional EOQ formula with shortages, and the optimal return rate τ^* approaches zero.

Proposition 2 shows that, when the constraint $0 \le \frac{\tau}{1-r} \le 1$ is ignored, the point (Q^*, S^*, τ^*) is the optimal solution such that total cost per unit time has a minimum.

Proposition 2. The Hessian Matrix for $TCPUT(Q, S, \tau)$ (as expressed in equation (14)) is positive definite. (**Appendix B**)

We note that Proposition 2 also shows the convexity of the objective function $TCPUT$ with respect to τ, S and Q. We now consider the constraint $0 \le \frac{\tau}{1-r} \le 1$. Because of the convexity of the objective function shown in

Proposition 1, if the optimal return rate τ^* of used-products obtained from equation (19) does not fall into the constraint interval [0, 1−r], then the maximum value of the equation (19) and the minimum value of the objective function occur at the boundary point 1−r.

$$\left(\tau^*, S^*, Q^*\right) = \left(1-r, \frac{h}{h+p}\sqrt{\frac{2\left(KD + C_L D(1-r)^2\right)(h+p)}{hp}}, \sqrt{\frac{2\left(KD + C_L D(1-r)^2\right)(h+p)}{hp}}\right) \tag{22}$$

Proposition 3. The policy with collecting used products consideration is more beneficial than the policy without collecting used products consideration. (**Appendix C**)

The cost ratio is defined as the ratio between the cost of optimal policies with and without collecting used products consideration, which can be shown as equation (23).

$$\text{Cost Ratio (CR)} = \frac{TCPUT^*(Q,S,\tau)}{TCPUT_{\text{EOQ}}{}^*(Q,S)} \tag{23}$$

The graph of cost ratio (CR) with respect to scaling parameter C_L can be depicted as Figure 3.

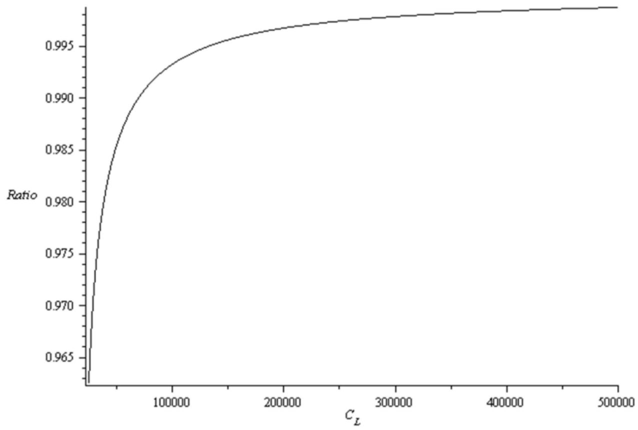

Fig 3: The graph of cost ratio (CR) with respect to scaling parameter C_L.

Fig 3 states the cost ratio (CR) between the optimal total cost per unit time with and without collecting used products consideration which are with respect to scaling parameter C_L changing from 0 to 500000. From equations (19), we know that the return rate changes with scaling parameter C_L. When the scaling parameter is larger, the return rate is smaller. This means that investment cost of the retailer in collecting used-product activities will affect the collecting effect of used products from customers. As the scaling parameter C_L is relatively large, recycling will reduce the willingness for retailer. From Figure 3, the observation points out when the willingness to recycle becomes less and less, the total cost per unit time with collecting used products consideration will be closer to the traditional EOQ formula with backorders allowed and the cost ratio (CR) will approach 1.

5. Collection transfer with not lot for lot

We consider the batch transportation and setup cost, but the lot for lot policy may not be the optimal when the retailer transports the collected used products to the manufacturer. In this section, we propose the not lot for lot policy for retailer. In Fig 4, we depict the "not lot for lot policy" inventory behaviors for forward channel and reverse channel. The retailer transports used products in each batch to the manufacturer is called the cycle length $\frac{nD}{Q}$, and we denote the integer multiple by n.

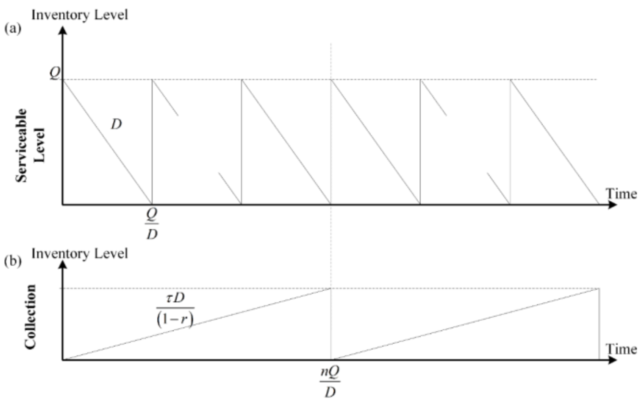

Fig 4: Inventory level of forward channel and reverse channel with not lot for lot policy.

The corresponding total cost per unit time (*TCPUT*) can be obtained by dividing the total cost per replenishment cycle by the cycle length. The objective of the model is to minimize total cost per unit time subject to the return rate and the integer multiple constraint. Namely,

$$TCPUT(Q,\tau,n)=\frac{K_pD}{Q}+pD+\frac{h_pQ}{2}+\frac{K_cD}{nQ}+\frac{nh_c\tau Q}{2(1-r)}+C_L\tau^2-\frac{(b-A)\tau D}{(1-r)} \tag{24}$$

$$\text{subject to: } 0\le\tau\le1-r \tag{25}$$

$$n\in\mathrm{N} \tag{26}$$

In order to solve the proposed mixed-integer nonlinear programming problem shown in Equation (24), the corresponding first-order necessary conditions (FONC) are given by Equations (27) and (28).

$$\frac{\partial TCPUT(Q,\tau,n)}{\partial Q}=-\frac{K_pD}{Q^2}-\frac{K_cD}{nQ^2}+\frac{h_p}{2}+\frac{nh_c\tau}{2(1-r)}=0 \tag{27}$$

$$\frac{\partial TCPUT(Q,\tau,n)}{\partial\tau}=\frac{nh_cQ}{2(1-r)}+2C_L\tau-\frac{(b-A)D}{(1-r)}=0 \tag{28}$$

Given fixed τ, by rearranging Equation (27) and for fixed Q, by rearranging Equation (28), $\hat{Q}$ and $\hat{\tau}$ can be obtained as Equations (29) and (30).

$$\hat{Q}=\sqrt{\frac{2(1-r)(nK_pD+K_cD)}{nh_p(1-r)+n^2h_c\tau}} \tag{29}$$

$$\hat{\tau}=\frac{2(b-A)D-nh_cQ}{4C_L(1-r)} \tag{30}$$

The second order differential equations of Equation (24) for Q and τ are shown as following. However, for fixed τ and n, $TCPUT(Q,\tau,n)$ is convex with respect to Q because

$$\frac{\partial^2TCPUT(Q,\tau,n)}{\partial Q^2}=\frac{2K_pD}{Q^3}+\frac{2K_cD}{nQ^3}>0 \tag{31}$$

Furthermore, for fixed Q and n, $TCPUT(Q,\tau,n)$ is convex with respect to τ because

$$\frac{\partial^2TCPUT(Q,\tau,n)}{\partial\tau^2}=2C_L>0 \tag{32}$$

Proposition 4. We ignore the integer multiple constraint n. $TCPUT(n)$ is a convex function with fixed Q and τ. (**Appendix D**)

Rearranging Equation (56), we can be obtained as Equation (33)

$$n=\sqrt{\frac{2(1-r)K_cD}{h_c\tau Q^2}} \tag{33}$$

It is apparent that searching n^* by starting with *n* is much more efficiently than by starting with $n=1$. Therefore, we can have the following procedure to find the optimal number of replenishments per cycle.

Initial Parameter Procedure:

Input the values of relevant parameters p, b, A, C_L, K_p, K_c, h_p, h_c, D, and r.

Set $\tau = 0.5$, $Q_0 = \sqrt{\dfrac{2KD}{h_p}}$ and $n = \left\lfloor \sqrt{\dfrac{2(1-r)K_c D}{h_c \tau Q_0^2}} \right\rfloor$

$$f(Q) = -K_p D - \frac{K_c D}{n} + \frac{h_p Q^2}{2} + \frac{n h_c (b-A) D Q^2}{4 C_L (1-r)^2} - \frac{n^2 h_c^2 Q^3}{8 C_L (1-r)^2} \text{ and}$$

$$f'(Q) = h_p Q + \frac{n h_c (b-A) D Q}{2 C_L (1-r)^2} - \frac{3 n^2 h_c^2 Q^2}{8 C_L (1-r)^2}.$$

Search Procedure:

Choose two initial values of n, say n in approximate equation and n-1.

Compute $TCPUT(Q^*, \tau^*, n)$ and $TCPUT(Q^*, \tau^*, n-1)$.

Iteration Algorithm Procedure:

Set $i = 1$ and compute $Q_i = Q_{i-1} - \dfrac{f(Q_{i-1})}{f'(Q_{i-1})}$.

Do {

$i = i + 1$ and $Q_i = Q_{i-1} - \dfrac{f(Q_{i-1})}{f'(Q_{i-1})}$.

} Until $Q_i - Q_{i-1} < \varepsilon$

Set $Q = Q_i$ and compute $\tau = \dfrac{(b-A)D}{2C_L(1-r)} - \dfrac{h_c Q}{4 C_L (1-r)}$.

If $\tau > 1 - r$, then $\tau = 1 - r$ and $Q = \sqrt{\dfrac{2(K_p + K_c)D}{h_p + h_c}}$

Calculate $TCPUT(Q, \tau, n)$.

Final Output Procedure:

If $TCPUT(Q^*, \tau^*, n) > TCPUT(Q^*, \tau^*, n-1)$

Let $n = n - 1$.

Compute $TCPUT(Q^*, \tau^*, n)$ and $TCPUT(Q^*, \tau^*, n-1)$.

Stop procedure until we find $TCPUT(Q^*, \tau^*, n) \le TCPUT(Q^*, \tau^*, n-1)$.

Else if $TCPUT(Q^*, \tau^*, n) \le TCPUT(Q^*, \tau^*, n-1)$

Let $n = n + 1$.

Compute $TCPUT(Q^*, \tau^*, n)$ and $TCPUT(Q^*, \tau^*, n-1)$.

Stop procedure until we find $TCPUT(Q^*, \tau^*, n) > TCPUT(Q^*, \tau^*, n-1)$.

Output $(Q^*, \tau^*, n^*) = (Q_i, \tau_i, n)$ and $TCPUT^* = TCPUT(Q_i, \tau_i, n)$.

We consider an example with the following parameters with the algorithm procedure. The values of parameters are c = 20, b = 3, A = 1, C_L = 60000, K_p = 12000, K_c = 4000, h_p = 0.5, h_c = 0.02, D = 15000 and r = 0.1. Assuming $\varepsilon = 10^{-6}$, the outcome of proposed algorithm procedure to search for the optimal n^* is showed that n^* =5 and the optimal Q^* and τ^* are 26930.3059 and 0.2653. The optimal *TCPUT* with different n are shown in Figure 5.

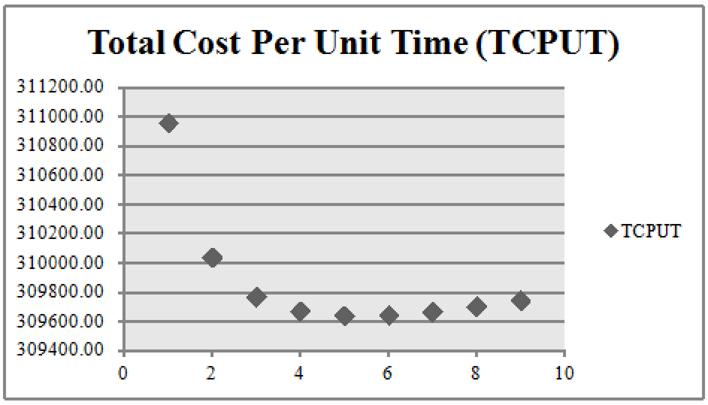

Fig 5: Effect of the different n for the total cost per unit time.

From Fig 5, the result indicates that lot for lot policy is not always the lowest cost decision for retailer. Fixed setup cost per collection activity will have a significant effect on the cycle length which the retailer transports used products in each batch to the manufacturer. The retailer for different cost structures will face different decision-making models and develop the best management decisions.

6. Economical decision modelling for a retailer

There is the growing attention on closed-loop supply chain issues with public environmental awareness. Kleindorfer et al. (2005) identified that closed-loop supply chain management can promote to realize sustainable operations. There are many positive effects which are the protection and saving of primary resources. Souza (2013) indicated that remanufactured engines or parts can sell at a 35% discount relative to the corresponding new part, module or engine. With restorative and regenerative processes, and closed-loop supply chain management can keep materials, components, modules and products at their highest utility, value and benefit.

In this paper, the retailer is involved in collecting the used sold products from public consumers. How to enhance incentives for retailer to increase return rate is a critical issue. The return rate can measure the collection efficiency of the reverse logistics. A higher return rate level is the result of lower consumption of new materials and natural resources. Furthermore, the retailer may need to invest in promoting and advertising activities to enhance public environmental awareness of the programs for collecting used products to achieve a higher return rate. Therefore, it is highly desirable for the retailer to determine the return rate based on benefit and cost analyses for the collection process.

In retailing industries, the traditional retailer only sells new products to public consumers. Now, the retailer should be seriously re-examined from the perspective of environmental consciousness. The retailer engages in the collection of used products in addition to selling new products. Under the assumption, we analyse whether the retailer can obtain lower costs and greater benefits by collecting used products. The formulations of the policies with and without considering the collection of used products can be expressed as Equations (34) and (35).

$$TCPUT(Q,\tau) = \frac{K_p D}{Q} + cD + \frac{h_p Q}{2} + \frac{K_c D}{Q} + \frac{h_c \tau Q}{2(1-r)} + C_L \tau^2 - \frac{(b-A)\tau D}{(1-r)} \quad (34)$$

$$TC_{EOQ}(Q) = \frac{K_p D}{Q} + cD + \frac{h_p Q}{2} \quad (35)$$

The cost saving (CS) when considering the collection of used products from public consumers is defined as the cost difference between the policies without and with considering the collection process of used products, as shown by

$$CS = TC_{EOQ}(Q) - TCPUT(Q,\tau) = \frac{(b-A)\tau D}{(1-r)} - \frac{K_c D}{Q} - \frac{h_c \tau Q}{2(1-r)} - C_L \tau^2 \quad (36)$$

The CS with respect to the return rate τ and replenishment quantity Q can be depicted as shown in Fig 6 and 7.

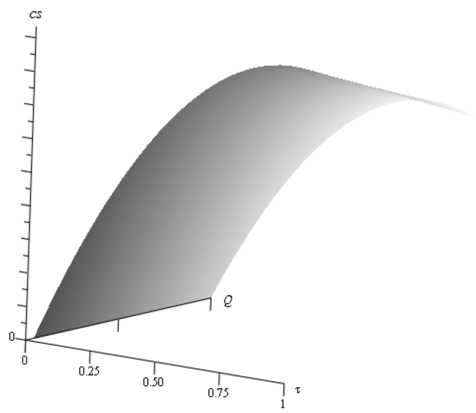

Fig 6: The CS with respect to the return rate and replenishment quantity with higher transfer price for retailer.

Fig 6 indicated that the *CS* with respect to the return rate τ and replenishment quantity Q with higher transfer price for retailer. In this condition, the retailer collects the used products and transfers the used products to the manufacturer, which has a lower cost than the retailer who does not collect the used products. Despite the retailer increases investment cost of the retailer in collecting used-product activities for improving the return rate, the retailer can still decrease cost.

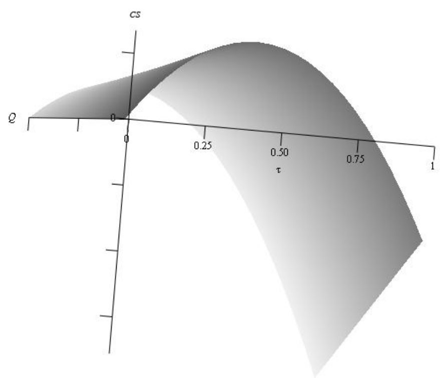

Fig 7: The CS with respect to the return rate and replenishment quantity with lower transfer price for retailer.

Fig 7 indicated that the *CS* with respect to return rate τ and replenishment quantity Q with lower transfer price for retailer. In this condition, the retailer collects the used products and transfers the used products to the manufacturer, which has a higher cost than the retailer who does not collect the used products. Because the revenue for transferring the used products to the manufacturer lower than the cost for investment of the retailer in collecting used-product activities.

Huang et al. (2013) had noted that transfer price and investment cost have an effect on the optimal return rate. The lower transfer price or the higher investment cost will reduce the retailer's willingness to recycle. The similar finding of the research also implies in the Figure 6 and 7.

7. Conclusion

This paper proposes an inventory model for the retailer with selling new products and collecting used products for analyzing the decisions to introduce a reverse-logistics system. The retailer is assumed that sells new products to customers and collects those sold used products from customers at the same time. We identify the decision variables and determine the optimal policies with considering algorithm. First, we formulate mathematical models to minimize the total relevant costs for the retailer. Next, by considering different variable costs, we can analyze the shape of total cost per unit time. Finally, we extend our model to consider the “not lot for lot” policy inventory model for forward channel and reverse channel and determine the optimal management policy.

Rubio and Jiménez-Parra (2014) indicated that reverse logistics is a process of planning, implementing and controlling backward flows of raw materials or products which involves all the activities from manufacturing or distribution to collection, recovery or disposal of used products. For the scope of reverse logistics, there are several possible extensions or interesting issues in the future research. The remanufacturing process is an important issue in the reverse logistics. We can explore the pricing making decision and management implication of different channel members, such as the retailer and the manufacturer, who carry on remanufacturing process with the different investment cost. Furthermore, we can examine the interaction of game theory and collection behaviour among the retailer, the manufacturer and independent third-party collector to analyse the reverse and forward channel decisions.

Appendix A

Because of the constraint of return rate $0 \le \tau \le 1-r$, then we can further substitute equation (12) into the constraint to obtain two interval uniqueness of solution Q which are given by Equation (37)

$$\frac{2(b-A)D - 4C_L(1-r)^2}{h_c} \le Q \le \frac{2(b-A)D}{h_c} \tag{37}$$

And then, we substitute Equation (12) into Equation (7) and arrange to obtain Equation (38).

$$-K_pD - K_cD + \frac{h_pQ^2}{2} + \frac{h_c(b-A)DQ^2}{4C_L(1-r)^2} - \frac{h_c^2Q^3}{8C_L(1-r)^2} = 0 \tag{38}$$

We can know that the optimal solution of replenishment quantity Q is the cubic function. Hence, we can analyze whether the optimal solution could be reasonable. We let following Equation (39)

$$f(Q) = -K_pD - K_cD + \frac{h_pQ^2}{2} + \frac{h_c(b-A)DQ^2}{4C_L(1-r)^2} - \frac{h_c^2Q^3}{8C_L(1-r)^2} \tag{39}$$

By substituting the boundary of replenishment quantity into Equation (39), we can obtain the following four inequalities (40), (41), (42) and (43).

$$f(Q)\Big|_{Q=-\infty} = \infty > 0 \tag{40}$$

$$f(Q)\Big|_{Q=0} = -K_pD - K_cD < 0 \tag{41}$$

$$f(Q)\Big|_{Q=\frac{2D(b-A)}{h_c}} = \frac{\left(2h_pD(b-A)^2 - (K_p+K_c)h_c^2\right)D}{h_c^2} > 0 \tag{42}$$

$$f(Q)\Big|_{Q=\infty} = -\infty < 0 \tag{43}$$

Through Equations (10) and (37), we can prove that the following equation is greater than zero.

$$\frac{\left(2h_pD(b-A)^2 - (K_p+K_c)h_c^2\right)}{4D(b-A)^2} > 0 \tag{44}$$

By the condition of Equation (44), we can prove that Equation (42) is positive. According to determination of roots, we know that we have the solution in the interval between 0 and $\frac{2(b-A)D}{h_c}$. We further analyze whether the solutions in the interval is the only one solution.

Then, we take the first derivative of Equation (7) with respect to Q and get Equation (45).

$$\frac{\partial^2 TCPUT(Q)}{\partial Q^2} = \frac{2K_pD}{Q^3} + \frac{2K_cD}{Q^3} - \frac{h_c^2}{8C_L(1-r)^2} \tag{45}$$

Equation (46) can be illustrated that in this interval, Equation (45) is greater than 0 and strictly increasing.

$$0 < Q < \sqrt[3]{\frac{16C_L D(1-r)^2 (K_p + K_c)}{h_c^2}} \tag{46}$$

By the way, we can prove that there are the existence and uniqueness of a solution $\hat{Q}$ with the constraint $0 \le \tau \le 1-r$.

Appendix B

We first obtain the Hessian Matrix **H** as shown in equation (47).

$$\mathbf{H} = \begin{bmatrix} \frac{\partial^2 TCPUT}{\partial \tau^2} & \frac{\partial^2 TCPUT}{\partial \tau \partial S} & \frac{\partial^2 TCPUT}{\partial \tau \partial Q} \\ \frac{\partial^2 TCPUT}{\partial \tau \partial S} & \frac{\partial^2 TCPUT}{\partial S^2} & \frac{\partial^2 TCPUT}{\partial S \partial Q} \\ \frac{\partial^2 TCPUT}{\partial \tau \partial Q} & \frac{\partial^2 TCPUT}{\partial S \partial Q} & \frac{\partial^2 TCPUT}{\partial Q^2} \end{bmatrix} \tag{47}$$

The equation (48) shows the determinant of the first principal minor H_{11} of the Hessian matrix **H**. The equation (49) and (50) show the determinants of the second principal minor H_{22} and H_{33} of the Hessian matrix **H**.

$$|H_{11}| = \frac{\partial^2 TCPUT}{\partial \tau^2} = \frac{2C_L D}{Q} > 0 . \tag{48}$$

$$|H_{22}| = \frac{\partial^2 TCPUT}{\partial \tau^2}\frac{\partial^2 TCPUT}{\partial S^2} - \left(\frac{\partial^2 TCPUT}{\partial \tau \partial S}\right)^2 = \frac{2C_L D}{Q}\left(\frac{h}{Q} + \frac{p}{Q}\right) > 0 . \tag{49}$$

$$|H_{33}| = \frac{2C_L D}{Q^5}\left(2KD(h+p)\right) > 0 . \tag{50}$$

Since the determinants of the first and the second principal minors are both positive, the Hessian Matrix (47) is positive definite.

Appendix C

For any given $\overline{Q} > 0$, the formulations for the policies with and without collecting used products consideration can be expressed as equations (51) and (52).

$$TCPUT\left(\overline{Q}, S, \tau\right) = \frac{KD}{\overline{Q}} + cD + \frac{h\left(\overline{Q} - S\right)^2}{2\overline{Q}} + \frac{pS^2}{2\overline{Q}} + \frac{C_L D \tau^2}{\overline{Q}} - (b-A)D\left(\frac{\tau}{1-r}\right) \tag{51}$$

$$TCPUT_{EOQ}\left(\overline{Q}, S\right) = \frac{KD}{\overline{Q}} + cD + \frac{h\left(\overline{Q} - S\right)^2}{2\overline{Q}} + \frac{pS^2}{2\overline{Q}} \tag{52}$$

The cost saving from collecting used product consideration is defined as the cost difference between the policies with and without collecting used products consideration, which can be shown as equation (53).

$$CS = TCPUT_{EOQ}\left(\overline{Q}, S\right) - TCPUT\left(\overline{Q}, S, \tau\right) = \left(\frac{(b-A)D}{1-r} - \frac{C_L D \tau}{\overline{Q}}\right)\tau \tag{53}$$

The graph of cost saving (CS) with respect to return rate τ can be depicted as Fig 8.

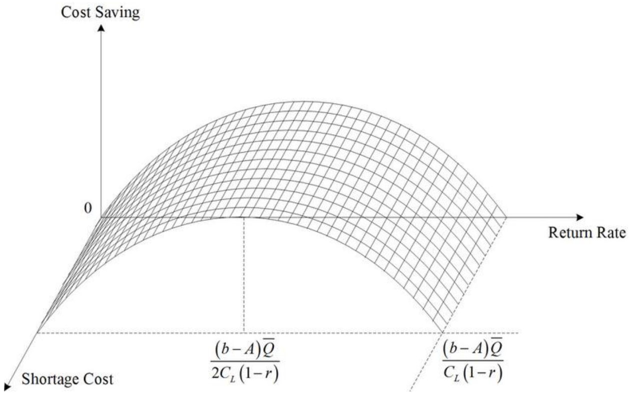

Fig 8: The graph of cost saving (CS) with respect to return rate τ.

In order to find the critical point of equation (53), we take the first derivative of cost saving (CS) with respect to return rate τ and let it to be equal to zero, which is shown as equation.

$$\frac{\partial CS}{\partial \tau} = \frac{(b-A)D}{1-r} - \frac{2C_L D\tau}{\overline{Q}} = 0 \tag{54}$$

By solving equation (54), the critical point is expressed as equations (55).

$$\overline{\tau} = \frac{(b-A)\overline{Q}}{2C_L(1-r)} \tag{55}$$

From equation (53) and Figure 8, we note that the parabola *CS* intersects the *x* axis (return rate) at the two lines, which can be obtained by letting equation (53) equal to zero. The two lines can be expressed as follows

$$\tau = 0 \text{ and } \tau = \frac{(b-A)\overline{Q}}{C_L(1-r)}$$

We note that the critical point lies at the mid-point of interval. The concavity of cost saving (CS) with respect to return rate τ can by further examined by checking the second derivative which can be expressed as follows

$$\frac{\partial^2 CS}{\partial \tau^2} = -\frac{2C_L D}{\overline{Q}} < 0$$

In such a case, the cost saving between the policy with collecting used products consideration (i.e., $0 < \frac{\tau}{1-r} \le 1$) and the policy without collecting used products consideration (i.e., $\tau = 0$) is expected to be positive. Hence, the policy with collecting used products consideration is more beneficial than the policy without collecting used products consideration has been proved.

Appendix D

The first differential equation and second order differential equation of Equation (24) for n are described as following

$$\frac{\partial TCPUT(Q,\tau,n)}{\partial n} = -\frac{K_c D}{n^2 Q} + \frac{h_c \tau Q}{2(1-r)} = 0 \tag{56}$$

$$\frac{\partial^2 TCPUT(Q,\tau,n)}{\partial n^2} = \frac{2K_c D}{n^3 Q} > 0 \quad (57)$$

That function has property of $TCPUT''(n) > 0$.

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