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Volume 10, Issue 2
Published online: December 28, 2016|

Journal of Progressive Research in Mathematics
www.scitecresearch.com/journals

Indexed Absolute Riesz Summability Factor Using Delta Quasi Monotone Sequence

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Abstract.

A result concerning absolute indexed Riesz Summability factor of an infinite series using δ-quasi monotone sequence has been established.

Keywords: Quasi-increasing; Quasi - f - power increasing; δ - Quasi monotone; index absolute Summability; summability factor.

AMS Classification No: 40A05, 40D15, 40F05

1. Introduction

We ask that authors follow simple guidelines. In essence, we ask you to make your paper look exactly like this document. A sequence (a_n) of positive numbers is said to be almost increasing if there exists a positive sequence (b_n) and two positive constants A and B such that

(1.1) $$Ab_n \le a_n \le Bb_n \text{ , for all } n \in N .$$

It is said to be quasi-β-power increasing, if there exists a constant K depending upon β with $K \ge 1$ such that

(1.2) $$K n^{\beta} a_n \ge m^{\beta} a_m \ ,$$

for all $n \ge m$ .Particularly, if $\beta = 0$, then (a_n) is a quasi-increasing sequence. It is clear that for any non-negative β, every almost increasing sequence is a quasi-β-power increasing sequence. But the converse is not true in general, as $(n^{-\beta})$ is quasi-β-power increasing but not almost increasing.

Let $f = (f_n)$ be a positive sequence of numbers. Then the positive sequence (a_n) is said to be quasi-f-power increasing, if there exists a constant K depending upon f with $K \ge 1$ such that

(1.3) $$K\, f_n\, a_n \geq f_m\, a_m$$

for $n \geq m \geq 1.$ Clearly, if (a_n) is a quasi- f -power increasing sequence, then the $(a_n\, f_n)$ is a quasi- increasing sequence.

Let $\delta = (\delta_n)$ be a positive sequence of numbers. Then the positive sequence (a_n) is said to be δ -quasi monotone , $a_n \to 0,\ a_n > 0$ ultimately and $\Delta d_n \geq -\delta_n$, where $\Delta d_n = d_n - d_{n-1}$.

Let $\sum a_n$ be an infinite series with sequence of partial sums $\{s_n\}$. Let (p_n) be a sequence of positive numbers such that

$$P_n = \sum_{\nu=0}^{n} p_\nu \to \infty, \text{ as } n \to \infty \ .$$

Then the sequence-to-sequence transformation

(1.4) $$T_n = \frac{1}{P_n} \sum_{\nu=0}^{n} p_\nu\, s_\nu \ ,\ P_n \neq 0,$$

defines the $(\overline{N}, p_n)$- mean of the sequence (s_n) generated by the sequence of coefficients (p_n). The series $\sum a_n$ is said to be summable $\left|\overline{N}, p_n\right|_k$, $k \geq 1$, if

(1.5) $$\sum_{n=1}^{\infty} \left(\frac{P_n}{p_n}\right)^{k-1} \left|T_n - T_{n-1}\right|^k < \infty \ .$$

The series $\sum a_n$ is said to be summable $\left|\overline{N}, p_n; \delta\right|_k, k \geq 1, \delta \geq 0,$ if

(1.6) $$\sum_{n=1}^{\infty} \left(\frac{P_n}{p_n}\right)^{\delta k + k - 1} \left|T_n - T_{n-1}\right|^k < \infty \ .$$

The series $\sum a_n$ is said to be summable $\left|\overline{N}, p_n, \alpha_n(\delta)\right|_k, k \geq 1, \delta \geq 0,$ if

(1.7) $$\sum_{n=1}^{\infty} (\alpha_n)^k \left(\frac{P_n}{p_n}\right)^{k-1} \left|T_n - T_{n-1}\right|^k < \infty \ .$$

For any real number μ, the series Σa_n is said to be summable by the summabilty method $\left|\overline{N}, p_n, \alpha_n(\delta), \mu\right|_k, k \geq 1, \delta \geq 0,$ if

(1.8) $$\sum_{n=1}^{\infty}(\alpha_n)^{k\mu}\left(\frac{P_n}{p_n}\right)^{\mu(k-1)}\left|T_n-T_{n-1}\right|^k<\infty.$$

For $\mu=1$, the summability method $\left|\bar{N},p_n,\alpha_n(\delta),\mu\right|_k, k\geq 1,\delta\geq 0,$ reduces to the method $\left|\bar{N},p_n,\alpha_n(\delta)\right|_k, k\geq 1,\delta\geq 0,$

2. Known Theorems

Dealing with quasi- f - power increasing sequence, Palo et al[4] prove the following theorem.

2.1. Theorem

Let $f=(f_n)=\left(n^{\beta}(\log n)^{\gamma}\right)$ be a sequence and (X_n) be a quasi- f -power sequence.

Let (λ_n) a sequence of constants such that

(2.1.1) $$\lambda_n\to 0,\ as\ \ n\to\infty,$$

(2.1.2) $$\sum_{n=1}^{\infty}nX_n|\Delta||\Delta\lambda_n|<\infty$$

(2.1.3) $$|\lambda_n|X_n=O(1),$$

(2.1.4) $$\sum_{n=v+1}^{m}(\alpha_n)^{k\mu}\left(\frac{P_n}{p_n}\right)^{(\mu-1)(k-1)-1}\frac{1}{P_{n-1}^{k}}=O\left((\alpha_m)^{k\mu}\left(\frac{P_n}{p_n}\right)^{(\mu-1)(k-1)-1}\right),$$

(2.1.5) $$\sum_{n=1}^{m}(\alpha_n)^{k\mu}\left(\frac{P_n}{p_n}\right)^{(\mu-1)(k-1)-1}\frac{|t_n|^k}{X_n^{k-1}}=O(X_m),$$

(2.1.6) $$\sum_{n=1}^{m}(\alpha_n)^{k\mu}\left(\frac{p_n}{P_n}\right)^{(\mu-1)(k-1)}\frac{|t_n|^k}{nX_n^{k-1}}=O(X_m),$$

where (t_n) is the nth $(C,1)$mean of the sequence (na_n).

Then the series $\sum a_n\lambda_n$ is summable $\left|\bar{N},p_n,\alpha_n(\delta)\right|_k, k\geq 1,\delta\geq 0.$

Dealing with quasi- δ - quasi monotone sequence, Sarangi et al. [4] Proved the following theorem:

2.2. Theorem

Let $f=(f_n)=\left(n^{\beta}(\log n)^{\gamma}\right)$ be a sequence and (X_n) be a quasi- f -power sequence. Suppose also that there exists a sequence of numbers (A_n) such that it is $\delta-$quasi – monotone with

(2.2.1) $$\sum n\delta_n X_n<\infty$$

(2.2.2) $$\Delta A_n\leq\delta_n \quad \text{for all } n.$$

Let (λ_n) a sequence of constants such that

(2.2.3) $$\lambda_n \to 0, \; as \;\; n \to \infty,$$

(2.2.4) $$|\lambda_n| X_n = O(1),$$

and

(2.2.5) $$|\Delta\lambda_n| \le |A_n| \;\; \text{for all} \;\; n.$$

Then the series $\sum a_n \lambda_n$ is summable $\left|\bar{N}, p_n; \sigma, \mu\right|_k, k \ge 1, \sigma \ge 0.$, if

(2.2.6) $$\sum_{n=\nu+1}^{m} \left(\frac{P_n}{p_n}\right)^{\sigma k-1} \frac{1}{P_{n-1}} = O\left(\frac{P_\nu}{p_\nu}\right)^{\sigma k-1},$$

(2.2.7) $$\sum_{n=1}^{m} \left(\frac{P_n}{p_n}\right)^{\mu(\sigma k+k-1)-k} \frac{|t_n|^k}{X_n^{k-1}} = O(X_m),$$

(2.2.8) $$\sum_{n=1}^{m} \left(\frac{P_n}{p_n}\right)^{\mu(\sigma k+k-1)-k+1} \frac{|t_n|^k}{nX_n^{k-1}} = O(X_m),$$

where (t_n) is the n th $(C,1)$ mean of the sequence $(n\,a_n)$.

However, extending to summability method $\left|\bar{N}, p_n, \alpha_n(\delta), \mu\right|_k, k \ge 1, \delta \ge 0,$ in this paper, we prove the following theorem.

3. Theorem

Let $f = (f_n) = \left(n^\beta (\log n)^\gamma\right)$ be a sequence and (X_n) be a quasi- f -power sequence. Suppose also that there exists a sequence of numbers (A_n) such that it is $\delta -$ quasi – monotone with

(3.1) $$\sum n \delta_n X_n < \infty$$

(3.2) $$\Delta A_n \le \delta_n \;\; \text{for all} \;\; n.$$

Let (λ_n) a sequence of constants such that

(3.3) $$\lambda_n \to 0, \; as \;\; n \to \infty,$$

(3.4) $$|\lambda_n| X_n = O(1),$$

and

(3.5) $$|\Delta\lambda_n| \le |A_n| \;\; \text{for all} \;\; n.$$

(3.6) $$\sum_{n=\nu+1}^{m} (\alpha_n)^{k\mu} \left(\frac{P_n}{p_n}\right)^{(\mu-1)(k-1)-1} \frac{1}{P_{n-1}^k} = O\left((\alpha_m)^{k\mu}\left(\frac{P_n}{p_n}\right)^{(\mu-1)(k-1)-1}\right),$$

(3.7) $$\sum_{n=1}^{m} (\alpha_n)^{k\mu} \left(\frac{P_n}{p_n}\right)^{(\mu-1)(k-1)-1} \frac{|t_n|^k}{X_n^{k-1}} = O(X_m),$$

(3.8) $$\sum_{n=1}^{m} (\alpha_n)^{k\mu} \left(\frac{p_n}{P_n}\right)^{(\mu-1)(k-1)} \frac{|t_n|^k}{nX_n^{k-1}} = O(X_m),$$

where (t_n) is the n th $(C,1)$ mean of the sequence $(n\,a_n)$.

Then the series $\sum a_n \lambda_n$ is summable $\left|\overline{N}, p_n, \alpha_n(\delta)\right|_k, k \geq 1, \delta \geq 0.$

In order to prove the theorem we require the following lemma.

4. Lemma

Let $f = (f_n) = \left(n^{\beta} (\log n)^{\gamma}\right), 0 \leq \beta < 1, \gamma \geq 0$ be a sequence and (X_n) be a quasi - f - power increasing sequence. Let (A_n) be a sequence of numbers such that it is $\delta-$quasi – monotone satisfying (3.1) and (3.2). then

(4.1) $$n X_n |A_n| = O(1)$$

and

(4.2) $$\sum_{n=1}^{m} X_n |A_n| < \infty, \text{as } m \to \infty.$$

4.1. Proof Of The Lemma

As $A_n \to 0$ and $n^{\beta} (\log n)^{\gamma} X_n$ is non-decreasing, we have

$$n X_n |A_n| = n^{1-\beta} (\log n)^{-\gamma} \left(n^{\beta} (\log n)^{\gamma} X_n\right) \sum_{\upsilon=n}^{\infty} \Delta |A_\upsilon|$$

$$= O(1) n^{1-\beta} (\log n)^{\gamma} n \sum_{\upsilon=n} \nu^{\beta} (\log \nu)^{\gamma} X_\upsilon |\Delta A_\upsilon|$$

$$= O(1) \sum_{\upsilon=n}^{\infty} \nu^{1-\beta} (\log \nu)^{-\gamma} \nu^{\beta} (\log \nu)^{\gamma} X_\upsilon |\Delta A_\upsilon|$$

$$= O(1) \sum_{\upsilon=n}^{\infty} \nu X_\upsilon |\Delta A_\upsilon|.$$

$$= O(1) \sum_{\upsilon=n}^{\infty} \nu X_\upsilon |\delta_\upsilon|$$

$$= O(1)$$

This establishes (4.1).

Next

$$\sum_{n=1}^{m} X_n |A_n| = \sum_{n=1}^{m-1} \left(\sum_{r=1}^{n} X_r\right) \Delta |A_n| + |A_m| \left(\sum_{r=1}^{m} X_r\right)$$

$$= O(1)\sum_{n=1}^{m-1}\left(\sum_{r=1}^{n} r^{-\beta}(\log r)^{-\gamma}\ r^{\beta}(\log r)^{\gamma}\ X_r\right)\Delta|\ A_n\ |$$

$$+ O(1)\left(\sum_{r=1}^{m} r^{-\beta}\ (\log r)^{-\gamma}\ r^{\beta}(\log r)^{\gamma}\ \ X_r\right)|\ A_m\ |$$

$$= O(1)\sum_{n=1}^{m-1}\left(n^{\beta}(\log n)^{\gamma}\ X_n\right)\Delta|\ A_n\ |\sum_{r=1}^{n} r^{-\beta-\epsilon}(\log r)^{-\gamma}\ \ r^{\epsilon}$$

$$+ O(1)\, m^{\beta}\ X_m|\ A_m\ |(\log m)^{\gamma}\sum_{r=1}^{m} r^{-\beta-\epsilon}\ (\log r)^{-\gamma}\ r^{\epsilon}, \epsilon < 1-\beta.$$

$$= O(1)\sum_{n=1}^{m-1}\left(n^{\beta}(\log n)^{\gamma}\ X_n\right)\Delta|\ A_n\ |n^{\epsilon}\ \ (\log n)^{-\gamma}\sum_{r=1}^{n} r^{-\beta-\epsilon}$$

$$+ O(1)\, m^{\beta}\ X_m|\ A_m\ |(\log m)^{\gamma}\ m^{\epsilon}(\log m)^{-\gamma}\sum_{r=1}^{m} r^{-\beta-\epsilon}$$

$$= O(1)\sum_{n=1}^{m} n^{\beta+\epsilon} X_n \Delta|\ A_n\ |\left(\int_1^n u^{-\beta-\epsilon} du\right) + O(1)\, m^{\beta+\epsilon}\ X_m|\ A_m\ |\left(\int_1^m u^{-\beta-\epsilon} du\right)$$

$$= O(1)\sum_{n=1}^{m} nX_n\Delta|A_n| + O(1)\, m\, X_m|\ A_m\ |$$

$$= O(1)\ .$$

This establishes (4.2).

5. Proof Of The Theorem

Let (T_n) be the sequence of $(\overline{N}, p_n)$ mean of the series $\sum_{n=1}^{\infty} a_n \lambda_n$, then

$$T_n = \frac{1}{P_n}\sum_{\upsilon=0}^{n} p_{\upsilon}\sum_{r=0}^{\upsilon} a_r\ \lambda_r$$

$$= \frac{1}{P_n}\sum_{\upsilon=0}^{n}\left(P_n - P_{\upsilon-1}\right) a_{\nu}\ \lambda_{\nu}$$

Hence for $n \geq 1$

$$T_n - T_{n-1} = \frac{p_n}{P_n P_{n-1}}\sum_{\upsilon=1}^{n} P_{\upsilon-1} a_{\nu}\ \lambda_{\nu}$$

$$= \frac{p_n}{P_n P_{n-1}}\sum_{\upsilon=1}^{n} \nu\, a_{\nu}\left(\frac{1}{\nu} P_{\nu-1}\ \lambda_{\nu}\right)$$

$$= \frac{(n+1)}{n}\frac{p_n}{P_n} t_n \lambda_n + \frac{p_n}{P_n P_{n-1}} \sum_{\nu=1}^{n-1} p_{\nu-1} t_\nu \lambda_\nu \frac{\nu+1}{\nu} + \frac{p_n}{P_n P_{n-1}} \sum_{\nu=1}^{n-1} P_\nu t_\nu \frac{\nu+1}{\nu} \Delta\lambda_\nu \quad + \frac{p_n}{P_n P_{n-1}} \sum_{\nu=1}^{n-1} P_\nu t_\nu \frac{\lambda_{\nu+1}}{\nu}$$

$$= T_{n1} + T_{n2} + T_{n3} + T_{n4} \text{ (say)}.$$

In order to prove the theorem ,using Minkowski's inequality it is enough to show that

$$\sum_{n=1}^{m} (\alpha_n)^{k\mu} \left(\frac{P_n}{p_n}\right)^{\mu(k-1)} \left|T_{nj}\right| < \infty \text{ , } j = 1,2,3,4.$$

Applying H$\ddot{o}$lder's inequality, we have

$$\sum_{n=1}^{m} (\alpha_n)^{k\mu} \left(\frac{P_n}{p_n}\right)^{\mu(k-1)} \left|T_{n,1}\right|^k$$

$$= \sum_{n=1}^{m} (\alpha_n)^{k\mu} \left(\frac{P_n}{p_n}\right)^{\mu(k-1)} \left|\frac{(n+1)}{n}\frac{p_n}{P_n} t_n \lambda_n\right|^k$$

$$= O(1) \sum_{n=1}^{m} (\alpha_n)^{k\mu} \left(\frac{P_n}{p_n}\right)^{(\mu-1)(k-1)-1} \frac{\left|t_n\right|^k}{X_n^{k-1}} \left(X_n \left|\lambda_n\right|\right)^{k-1} \left|\lambda_n\right|$$

$$= O(1) \sum_{n=1}^{m} (\alpha_n)^{k\mu} \left(\frac{P_n}{p_n}\right)^{(\mu-1)(k-1)-1} \frac{\left|t_n\right|^k}{X_n^{k-1}} \left|\lambda_n\right|$$

$$= O(1) \sum_{n=1}^{m-1} \left(\sum_{\nu=1}^{n} (\alpha_\nu)^{k\mu} \left(\frac{P_\nu}{p_\nu}\right)^{(\mu-1)(k-1)-1} \frac{\left|t_\nu\right|^k}{X_\nu^{k-1}} \right) \Delta\left|\lambda_n\right|$$

$$+ O(1) \left(\sum_{\nu=1}^{m} (\alpha_\nu)^{k\mu} \left(\frac{P_\nu}{p_\nu}\right)^{(\mu-1)(k-1)-1} \frac{\left|t_\nu\right|^k}{X_\nu^{k-1}} \right) \left|\lambda_m\right|$$

$$= O(1) \sum_{n=1}^{m-1} X_n \left|A_n\right| + O(1) X_m \left|\lambda_m\right|$$

$$= O(1)$$

Next,

$$\sum_{n=1}^{m}(\alpha_n)^{k\mu}\left(\frac{P_n}{p_n}\right)^{\mu(k-1)}\left|T_{n,2}\right|^k$$

$$=\sum_{n=1}^{m}(\alpha_n)^{k\mu}\left(\frac{P_n}{p_n}\right)^{\mu(k-1)}\left|\frac{p_n}{P_nP_{n-1}}\sum_{\nu=1}^{n-1}p_{\nu-1}t_\nu\lambda_\nu\frac{(\nu+1)}{\nu}\right|^k$$

$$=O(1)\sum_{n=1}^{m}(\alpha_n)^{k\mu}\left(\frac{P_n}{p_n}\right)^{(\mu-1)(k-1)-1}\frac{1}{P_{n-1}}\sum_{\nu=1}^{n-1}p_{\nu-1}\left|t_\nu\right|^k\left|\lambda_\nu\right|^k\left(\frac{1}{P_{n-1}}\sum_{\nu=1}^{n-1}p_\nu\right)^{k-1}$$

$$=O(1)\sum_{\nu=1}^{m}p_{\nu-1}\left|t_\nu\right|^k\left|\lambda_\nu\right|^k\sum_{n=\nu+1}^{m}(\alpha_n)^{k\mu}\left(\frac{P_n}{p_n}\right)^{(\mu-1)(k-1)-1}\frac{1}{P_{n-1}}$$

$$=O(1)\sum_{\nu=1}^{m}(\alpha_\nu)^{k\mu}\left(\frac{P_\nu}{p_\nu}\right)^{(\mu-1)(k-1)-1}\left|t_\nu\right|^k\left|\lambda_\nu\right|^k$$

$$=O(1)$$

Again,

$$\sum_{n=1}^{m}(\alpha_n)^{k\mu}\left(\frac{P_n}{p_n}\right)^{\mu(k-1)}\left|T_{n,3}\right|^k$$

$$=\sum_{n=1}^{m}(\alpha_n)^{k\mu}\left(\frac{P_n}{p_n}\right)^{\mu(k-1)}\left|\frac{p_n}{P_nP_{n-1}}\sum_{\nu=1}^{n-1}p_\nu t_\nu\frac{(\nu+1)}{\nu}\Delta\lambda_\nu\right|^k$$

$$=O(1)\sum_{n=1}^{m}(\alpha_n)^{k\mu}\left(\frac{P_n}{p_n}\right)^{(\mu-1)(k-1)-1}\frac{1}{P_{n-1}^k}\sum_{\nu=1}^{n-1}p_\nu^k\frac{\left|t_\nu\right|^k}{X_\nu^{k-1}}\left|\Delta\lambda_\nu\right|\left(\sum_{\nu=1}^{n-1}X_\nu\left|\Delta\lambda_\nu\right|\right)^{k-1}$$

$$=O(1)\sum_{\nu=1}^{m}p_\nu^k\frac{\left|t_\nu\right|^k}{X_\nu^{k-1}}\left|\Delta\lambda_\nu\right|\sum_{n=\nu+1}^{m+1}(\alpha_n)^{k\mu}\left(\frac{P_n}{p_n}\right)^{(\mu-1)(k-1)-1}\frac{1}{P_{n-1}^k}$$

$$=O(1)\sum_{\nu=1}^{m}(\alpha_\nu)^{k\mu}\left(\frac{P_\nu}{p_\nu}\right)^{(\mu-1)(k-1)}\frac{\left|t_\nu\right|^k}{\nu X_\nu^{k-1}}\left(\nu\left|\Delta\lambda_\nu\right|\right)$$

$$=O(1)\left(\sum_{\nu=1}^{m-1}\left(\sum_{r=1}^{\nu}(\alpha_r)^{k\mu}\left(\frac{P_r}{p_r}\right)^{(\mu-1)(k-1)}\right)\frac{\left|t_r\right|^k}{rX_r^{k-1}}\right)\left(\nu\left|\Delta\lambda_\nu\right|\right)$$

$$+O(1)\left(m\left|\Delta\lambda_m\right|\right)\sum_{r=1}^{m}(\alpha_r)^{k\mu}\left(\frac{P_r}{p_r}\right)^{(\mu-1)(k-1)}$$

$$=O(1)\sum_{\nu=1}^{m-1}X_\nu\left(-\left|\Delta\lambda_\nu\right|+(\nu+1)\left|\Delta\right|\left|\Delta\lambda_\nu\right|\right)+O(1)mX_m\left|\Delta\lambda_m\right|$$

$$=O(1)$$

Finally,

$$\sum_{n=1}^{m}(\alpha_n)^{k\mu}\left(\frac{P_n}{p_n}\right)^{\mu(k-1)}\left|T_{n,4}\right|^k$$

$$=\sum_{n=1}^{m}(\alpha_n)^{k\mu}\left(\frac{P_n}{p_n}\right)^{\mu(k-1)}\left|\frac{p_n}{P_nP_{n-1}}\sum_{\nu=1}^{n-1}p_\nu t_\nu\frac{\lambda_{\nu+1}}{\nu}\right|^k$$

$$=O(1)\sum_{n=1}^{m}(\alpha_n)^{k\mu}\left(\frac{P_n}{p_n}\right)^{(\mu-1)(k-1)-1}\frac{1}{P_{n-1}^k}\sum_{\nu=1}^{n-1}\frac{p_\nu}{\nu}\left|t_\nu\right|^k\left|\lambda_\nu\right|^k\left(\sum_{\nu=1}^{n-1}\frac{p_\nu}{\nu}\right)^{k-1}$$

$$=O(1)\sum_{\nu=1}^{m}\frac{p_\nu}{\nu}\left|t_\nu\right|^k\left|\lambda_\nu\right|^k\sum_{n=\nu+1}^{m}(\alpha_n)^{k\mu}\left(\frac{P_n}{p_n}\right)^{(\mu-1)(k-1)-1}\frac{1}{P_{n-1}}$$

$$=O(1)\sum_{\nu=1}^{m}(\alpha_\nu)^{k\mu}\left(\frac{P_\nu}{p_\nu}\right)^{(\mu-1)(k-1)}\frac{\left|t_\nu\right|^k}{\nu X_\nu^{k-1}}\left(X_\nu\left|\lambda_\nu\right|\right)^{k-1}\left|\lambda_\nu\right|$$

$$=O(1)\sum_{\nu=1}^{m-1}\left\{\sum_{r=1}^{\nu}(\alpha_r)^{k\mu}\left(\frac{P_r}{p_r}\right)^{(\mu-1)(k-1)}\frac{\left|t_r\right|^k}{rX_r^{k-1}}\right\}\left|\Delta\lambda_\nu\right|$$

$$+O(1)\sum_{r=1}^{m}(\alpha_r)^{k\mu}\left(\frac{P_r}{p_r}\right)^{(\mu-1)(k-1)}\frac{\left|t_r\right|^k}{rX_r^{k-1}}$$

$$=O(1)\sum_{\nu=1}^{m-1}X_\nu\left|\Delta\lambda_\nu\right|+O(1)mX_m\left|\Delta\lambda_m\right|$$

$$=O(1).$$

This completes the proof of the theorem.

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