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UNIQUE FACTORIZATION FERMAT'S LAST THEOREM BEAL'S CONJECTURE

JAMES E. JOSEPH

Dedicated to all of my teachers and students

Abstract. In this paper, the following statememt of Fermat's Last Theorem is proved. If x, y, z are positive integers, π is an odd prime and $z^\pi = x^\pi + y^\pi, x, y, z,$ are all even. Also, in this paper, is proved (Beal's conjecture): The equation $z^\xi = x^\mu + y^\nu$ has no solution in relatively prime positive integers $x, y, z,$ with $\xi, \mu, \nu,$ primes at least 3.

§1. Fermat's Last Theorem

For other theorems named after Pierre de Fermat, see [1]. The 1670 edition of Diophantus' Arithmetica includes Fermat's commentary, particularly his "Last Theorem" (Observatio Domini Petri de Fermat). In number theory, Fermat's Last Theorem (sometimes called Fermat's conjecture, especially in older texts) states that no three positive integers x, y, and z satisfy the equation $z^\pi = x^\pi + y^\pi$ for any integer value of π greater than two. The case $\pi = 2$ was known to have infinitely many solutions. This theorem was first conjectured by Pierre de Fermat in 1637, in the margin of a copy of Arithmetica where he claimed he had a proof that was too large to fit in the margin. The first proof agreed upon as successful was released in 1994 by Andrew Wiles, using cyclic groups, and formally published in 1995 [2],]3], after 358 years of effort by mathematicians. The unsolved problem stimulated the development of algebraic number theory in the 19th century and the proof of the modularity theorem in the 20th century. It is among the most notable theorems in the history of mathematics It is known that if x, y, z are relatively prime positive integers, $z^4 \neq x^4 + y^4$[1]. In view of this fact, it is only necessary to prove if $x, y, z,$ are relatively prime positive integers, π is an odd prime, $z^\pi = x^\pi + y^\pi$, then $x, y, z,$

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are each divisible by π. Before and since Wiles paper, many papers and books have been written trying to solve this problem in an elegant algebraic way, but none have suceeded. (See [1], and go to a search engine on the computer and search Fermat's Last Theorem). In the remainder of this paper, π will represent an odd prime. The special case $z^4 = x^4 + y^4$ is impossible. In view of this fact, it is only necessary to prove, if x, y, z, are positive integers, π is an odd prime, $z^\pi = x^\pi + y^\pi$, then x, y, z, are all even.

Theorem. If x, v, z are positive integers and $z^\pi = x^\pi + y^\pi$, then x, y, z, are all even.

Proof. Since $z^\pi = x^\pi + y^\pi$, x, y, or z is even, and the other two are odd. It will be shown that all are even, through the following implications.

z even $\implies$ y even $\iff$ x even $\implies$ z even.

z is even $\implies$ y is even

$$z = 2^k c$$

where c, is odd and k is a positive integer.

$$z^\pi = 2^{k\pi} c^\pi = (x + y) \sum_{0}^{\pi-1} x^{\pi-1-k}(-y)^k,$$

where c is odd. If

$$\sum_{0}^{\pi-1} x^{\pi-1-k}(-y)^k \equiv 0 \ (mod\ 2)$$

, then since, $x^{\pi-1-k}(-y)^k$ is odd for $k < \pi - 2$,

$$y^{\pi-2} \equiv 0 \ (mod\ 2),$$

gives y even. In the alternative, the Unique Factorization Theorem gives

$$x + y = 2^{k\pi};$$

$$(x + y)^\pi = \sum_{0}^{\pi} C(\pi, k) x^{\pi-k} y^k \equiv 0 \ (mod\ 2^\pi);$$

$$\sum_{1}^{\pi-1} C(\pi, k) x^{\pi-k} y^k \equiv 0 \ (mod\ 2^\pi);$$

$$\sum_{1}^{\pi-1} C(\pi,k)(x+y-y)^{\pi-k}y^k \equiv 0 \ (mod\ 2^\pi);$$

$$-y^\pi \sum_{1}^{\pi-1} C(\pi,k) \equiv 0 \ (mod\ 2^\pi);$$

$$-2y^\pi(2^{\pi-1}-1) \equiv 0 \ (mod\ 2^\pi);$$

so

$$y \equiv 0 \ (mod\ 2);$$

interchangiing x and y in the above argument leads to x even.

<u>y even $\implies$ x even</u>

$$y^\pi = 2^{k_1\pi}c^\pi = (z-x)\sum_{0}^{\pi-1} z^{\pi-1-k}x^k,$$

where c is odd and k_1 is a positive integer. If

$$\sum_{0}^{\pi-1} z^{\pi-1-k}x^k \equiv 0 \ (mod\ 2)$$

, then $z^{\pi-1-k}(-x)^k$ is odd for $k < \pi - 2$, $x^{\pi-2} \equiv 0 \ (mod\ 2)$, gives x even; in the alternative, the Unique Factorization Theorem gives

$$z - x = 2^{k_1\pi};$$

$$(z-x)^\pi = \sum_{0}^{\pi} C(\pi,k)z^{\pi-k}(-x)^k \equiv 0 \ (mod\ 2^\pi);$$

$$\sum_{1}^{\pi-1} C(\pi,k)z^{\pi-k}(-x)^k \equiv 0 \ (mod\ 2^\pi);$$

$$\sum_{1}^{\pi-1} C(\pi,k)(z-x+x)^{\pi-k}(-x)^k \equiv 0 \ (mod\ 2^\pi);$$

$$-x^\pi \sum_{1}^{\pi-1} C(\pi,k) \equiv 0 \ (mod\ 2^\pi);$$

$$-2x^\pi(2^{\pi-1}-1) \equiv 0 \ (mod\ 2^\pi);$$

so

$$x \equiv 0 \ (mod\ 2).$$

Interchanging x and y in the proof of
<u>y even $\implies$ x even</u>, leads to the proof of
<u>x even $\implies$ y even</u>.

So z even $\implies$ y is even $\iff$ x is even $\iff$ z is even; so x, y, z are all even. $\square$

Fermat's Last Theorem If $x, y, z,$ are relatively prime positive integers, then $z^\pi \neq x^\pi + y^\pi$.

Proof. $x, y, z,$ are each divisible by 2. $\square$

§2. Beal's conjecture

Beal's conjecture. The equation $z^\xi = x^\mu + y^\nu$ has no solution in relatively prime positive integers $x, y, z,$ with $\xi, \mu, \nu,$ primes at least 3.

Proof.

$$(z^\xi)^\xi = (x^\xi)^\mu + (y^\xi)^\nu = (x^\mu)^\xi + (y^\nu)^\xi,$$

and by Fermat's Last Theorem, z^ξ, x^μ, y^ν, and $x, y, z,$ are all divisible bv 2. $\square$

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Retired Professor, Department of Mathematics, Howard University, Washington, DC 20059

E-mail address: jjoseph@Howard.edu

Current address: 35 E Street NW #709, where c is odd, . Washington, DC 20001

E-mail address: j122437@yahoo.com