



Proofs of Fermat's Last Theorem and Beal's Conjecture

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ABSTRACT. If π is an odd prime and x, y, z , are relatively prime positive integers, then $z^\pi \neq x^\pi + y^\pi$. In this note, an elegant simple proof of this theorem is given that if π is an odd prime and x, y, z are positive integers satisfying $z^\pi = x^\pi + y^\pi$, then x, y, z , are each divisible by 2 : (Beal's conjecture) The equation $z^\xi = x^\mu + y^\nu$ has no solution in relatively prime positive integers x, y, z , with ξ, μ, ν primes at least 3. is also proved; that is x, y, z are all even.

For other theorems named after Pierre de Fermat, see the book by H. Edwards [1]. The cases $n = 1$ and $\pi = 2$ were known to have infinitely many solutions. This theorem was first conjectured by Pierre de Fermat in 1637 in the margin of a copy of *Arithmetica* where he claimed he had a proof that was too large to fit in the margin. The first proof agreed upon as successful was released in 1994 by Andrew Wiles, using cyclic groups, and formally published in 1995, after 358 years of effort by mathematicians. The unsolved problem stimulated the development of algebraic number theory in the 19th century and the proof of the modularity theorem in the 20th century. It is among the most notable theorems in the history of mathematics. It is known that if x, y, z are relatively prime positive integers, $z^4 \neq x^4 + y^4$ [1]. In view of this fact, it is only necessary to prove if x, y, z , are relatively prime positive integers, π is an odd prime, $z^\pi = x^\pi + y^\pi$, then x, y, z , are each divisible by 2. Before and since Wiles paper, many papers and books have been written trying to solve this problem in an elegant algebraic way, but none have succeeded. (See [1], and go to a search engine on the computer and search Fermat's Last Theorem). In the remainder of this paper, π will represent an odd prime.

§1. Fermat's Last Theorem

Theorem If x, y, z , are positive integers, and $z^\pi = x^\pi + y^\pi$, then

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- (1) $x + y - z$ is even;
- (2) x, y, z are all even.

Proof. (1) x, y, z relatively prime and $z^\pi = x^\pi + y^\pi \implies x + y - z$ is even and x, y , or z is even;

z even $\implies x + y$ even $\implies x - y$ even, since $x - y = x + y - 2y; x > y \implies x + y = 2^k c, x - y = 2^m d, k, m$ positive integers, c, d positive odd integers; solving, gives x, y even.

y even $\implies z + x, z - x$ even; using the same arguments as above, z, x are even □

Fermat's Last Theorem If x, y, z , are relatively prime positive integers, then $z^\pi \neq x^\pi + y^\pi$.

Proof. From the Theorem x, y, z are each divisible by 2. □

§2. **Beal's conjecture** The equation $z^\xi = x^\mu + y^\nu$ has no solution in relatively prime positive integers x, y, z , with ξ, μ, ν primes at least 3.

Proof.

$$(z^\xi)^\xi = (x^\xi)^\mu + (y^\xi)^\nu = (x^\mu)^\xi + (y^\nu)^\xi,$$

and by Fermat's Last Theorem., z^ξ, x^μ, y^ν and x, y, z are each divisible by 2.. □

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