



A PROOF OF BEAL'S CONJECTURE

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ABSTRACT. It is proved in this paper that the equation $z^\xi = x^\mu + y^\nu$ has no solution in relatively prime positive integers x, y, z , with ξ, μ, ν odd primes at least 3. This is equivalent to Fermat's Last Theorem which is stated as follows: If x, y, z are positive integers, and π is an odd prime satisfying $z^\pi = x^\pi + y^\pi$, then x, y, z are not relatively prime.

- (1) (Beal's conjecture) The equation $z^\xi = x^\mu + y^\nu$ has no solution in relatively prime positive integers x, y, z , with ξ, μ, ν primes at least 3.
- (2) (Fermat's Last Theorem) If x, y, z are relatively prime positive integers and π is an odd prime then $z^\pi = x^\pi + y^\pi$ implies x, y, z are not relatively prime.

The proof of (2) is given in [1], [2], in terms of cyclic groups. It is proved that if x, y, z are positive integers satisfying $z^\pi = x^\pi + y^\pi$ for an odd prime π , then x, y, z are not relatively prime. Thus, it is enough to prove that if $z^\xi = x^\mu + y^\nu$, with positive integers x, y, z , and ξ, μ, ν primes at least 3, then z^ξ, x^μ, y^ν and x, y, z are not relatively prime.

Proof. $2 \implies 1$. $\xi \geq \mu \geq \nu \geq 3$.

(Beal's conjecture) The equation $z^\xi = x^\mu + y^\nu$ has no solution in relatively prime positive integers x, y, z , with ξ, μ, ν primes at least 3.

Proof.

$$(z^\xi)^\xi = (x^\xi)^\mu + (y^\xi)^\nu = (x^\mu)^\xi + (y^\nu)^\xi,$$

and by Fermat's Last Theorem., z^ξ, x^μ, y^ν and x, y, z are not relatively prime. □

$1 \implies 2$. Obvious. □

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REFERENCES

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