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An Improved Group Acceptance Sampling Plan for Weighted Binomial on Time Truncated Testing Strategy Using Multiple Testers: Exponential Distributed Lifetime

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Abstract

In this paper an improved group acceptance sampling plan using weighted binomial is developed when the lifetime of the test item follows exponential distribution. The optimal numbers of group are obtained for pre-specified parameters, acceptance number, quality levels, and number of tester per group at different level of consumer risk. The results are discussed with the help of example and figure. The proposed plan is compare with Anburajan and Ramaswamy (2015) and observed more efficient than the existing plan for Exponential Distribution.

Keywords: Improved group acceptance sampling plan; Weighted Binomial; Exponential distribution; Producer and consumer's risks.

1. Introduction

One of the major functions of acceptance sampling is the lot sentencing, means to accept or reject the particular lot under consideration. In lot sentencing the risk of two types of errors are involved known as producer and consumer risks denoted by α, β respectively. The probability of rejecting a good lot is known as producer's risk (α) whereas the probability of accepting a bad lot is known as consumer's risk (β) although these risks can be minimized through different statistical techniques as well. A sampling plan is considered to be a good if it minimizes these risks. A simple sampling plan is one in which a single item is put on a tester however more than one item can be put up on a tester at a same time which form a group and number of items on a tester is known as group size(r). In group acceptance sampling plans the key concern is to minimize the number of groups (g) which is as similar as to minimize the sample size (n) when a single item is put on test (n = rg). The quality of the product is usually evaluated through its average life. Suppose that average true life of the item is μ whereas its specified average life is μ_0 then a suitable statistical null hypothesis can be $\mu \geq \mu_0$ against the alternative of lesser type. Ample of work can be cited in this regard. Amburajan P. and Ramaswamy S. (2015) developed group acceptance sampling plan using weighted binomial when the life time variate follows exponential and weibull distributions. Amburajan P. and Ramaswamy S. (2015) reported that the minimum number of groups required decreases as the test termination time multiplier is increases and such plans are efficient in terms of test time and test cost. Aslam M. and Shahbaz M.Q. (2007) developed an acceptance sampling plan when the test items follows generalized exponential distribution and compare it with log-logistic provided by Kantam at el. (2006). Aslam M. and Shahbaz M.Q. (2007) reported that proposed plan is more efficient in terms of cost and time to reach the final decision as compare to Kantam at el. (2006).Epstein B.(1954) has discussed truncated life tests for exponential distribution. Gupta R.D. and Kundu D. (2003) studied for Weibull and generalized exponential distributions. Mir and Ahmed (2009) discussed about sized-biased distributions and their applications with respect to non-experimental, non-replicated and nonrandom categories. Mir and Ahmed (2009) concluded that proposed sized–biased models considered more efficient than simple classical and simple generalized models. Radhakrishnan R. and Alagirisamy K. (2011) discuss the construction of group acceptance sampling plan using weighted binomial distribution. Ramaswamy S. and Amburajan P. (2012) developed group acceptance sampling plan using weighted binomial when the life time variate follows inverse Rayleigh and log-logistic distributions.

In this paper first time, the improved group acceptance sampling plan (IGASP) proposed by Aslam et al. (2011) is reconsidered using weighted binomial distribution when the lifetime variate of the test item follows exponential distribution.

2. Design of proposed plan/Methodology

Aslam et al. (2011) proposed the following improved group acceptance sampling plan (IGASP).

1) Select the number of groups g and allocate predefined r items to each group so that the sample size for a lot will be $n = rg$.
2) Select the acceptance number c for a group and the experiment time t_0.
3) Carry out the experiment for the g groups simultaneously and record the number of failures for each group.
4) Accept the lot if at most c failures occurs in each of all groups.
5) Truncate the experiment, if more than c failures occur and reject the lot or, at time t_0.

The stated plan is based on two known plan parameters $g$and c. The proposed plan reduces to the ordinary acceptance sampling plan when $r = 1$.

The lot acceptance probability with weighted binomial for the stated improved group acceptance sampling plan is as follows:

$$\text{L(P)} = \sum_{i=1}^{c} \binom{rg-1}{i-1} p^{i-1}(1-p)^{rg-i} \quad (1)$$

Here p is the probability of failure of an item before the termination time t_0. As we know that p is the function of cumulative distribution of the underlying distribution , which is exponential in this case hence

$$\text{P} = 1 - exp^{-\frac{1.2779\,a}{\frac{\mu}{\mu_0}}} \quad (2)$$

In acceptance sampling plans the scale parameter is often known as quality parameter and the quality of an item is accessed by the ratio of the true average life of an item to its specified average life $(\frac{\mu}{\mu_0})$. It is convenient to determine the termination time t_0 as a multiple of the specified average life μ_0. So we will consider $t_0 = a\mu_0$ for a constant a e.g a=0.5 means that the experiment time is just half of the specified average life (Aslam and Jun, (2009)

The minimum number of groups (g) can be obtained by satisfying the following inequality

$$\sum_{i=1}^{c} \binom{rg-1}{i-1} \left(1 - exp^{-\frac{1.2779\,a}{\frac{\mu}{\mu_0}}}\right)^{i-1} \left(exp^{-\frac{1.2779\,a}{\frac{\mu}{\mu_0}}}\right)^{rg-i} \leq \beta \quad (3)$$

Where β is the consumer risk

3. Notations

a	-	Test termination time multiplier
c	-	Acceptance number
g	-	Number of groups
n	-	Sample size
p	-	Probability of failure
L(p)	-	Probability of acceptance
r	-	Number of items in a group
t_0	-	Termination time
α	-	Producer's risk
β	-	Consumer's risk
μ	-	True average life
μ_0	-	Specified average life

4. Results and Discussion

The minimum number of groups (g) are obtained through simulations by solving nonlinear equation (3) with the pre specified consumer risk(β), acceptance numbers(c),group size (r) and

quality ratio at ($\mu = \mu_0$) its worst case, because otherwise we need to reject the null hypothesis ($\frac{\mu}{\mu_0}$)≥1. The optimal number of groups placed in following table 1

Table1: Minimum number of groups for specified plan parameters

β	r	c	a					
			0.7	0.8	1.0	1.2	1.5	2.0
0.25	2	0	2	2	2	1	1	1
	3	1	2	2	2	2	2	1
	4	2	2	2	2	2	2	1
	5	3	2	2	2	2	2	2
	6	4	2	2	2	2	2	2
	7	5	2	2	2	2	2	2
0.10	4	0	1	1	1	1	1	1
	5	1	2	2	1	1	1	1
	6	2	2	2	2	1	1	1
	7	3	2	2	2	2	1	1
	8	4	2	2	2	2	1	1
	9	5	2	2	2	2	2	1
0.05	5	0	1	2	1	1	1	1
	6	1	2	2	1	1	1	1
	7	2	2	2	2	1	1	1
	8	3	2	2	2	2	1	1
	9	4	2	2	2	2	1	1
	10	5	2	1	2	2	1	1
0.01	7	0	1	1	1	1	1	1
	8	1	2	2	1	1	1	1
	9	2	2	2	1	1	1	1
	10	3	2	2	2	1	1	1
	11	4	2	2	2	2	1	1
	12	5	2	2	2	2	1	1

Table 1 shows the optimal number of groups required for the pre- defined consumer risks, acceptance numbers, test items and constant a. The groups are calculated for the number of settings of the plan parameters and it can be observed that maximum of 2 group size is required which seems economical. May be more efficient results obtain with some other distributions.

Table2: Probability of acceptance for the weighted improved group acceptance sampling plan with c=2 when the lifetime of items follows the exponential distribution

β	G	r	a	μ/μ_0					
				2	4	6	8	10	12
0.25	2	4	0.7	0.5338035	0.8631924	0.9451539	0.9730010	0.9848207	0.9906466
	2	4	0.8	0.4460934	0.8201303	0.9249710	0.9622869	0.9785232	0.9866507
	2	4	1.0	0.2995515	0.7261157	0.8766687	0.9354268	0.9622869	0.9761534
	2	4	1.2	0.1930821	0.6285373	0.8201303	0.9020302	0.9413528	0.9622869
	2	4	1.5	0.0944774	0.4888880	0.7261157	0.8421123	0.9020302	0.9354268
	1	4	2.0	0.6464673	0.9034294	0.9621059	0.9815310	0.9896755	0.9936614
0.10	2	6	0.7	0.2014107	0.6394361	0.8271678	0.9064108	0.9441849	0.9642014
	2	6	0.8	0.1355681	0.5586272	0.7757094	0.8745632	0.9236589	0.9503517
	2	6	1.0	0.0581662	0.4117188	0.6668873	0.8018418	0.8745632	0.9161791
	1	6	1.2	0.4600582	0.8259868	0.9275399	0.9636017	0.9792783	0.9871223
	1	6	1.5	0.3142492	0.7346026	0.8807960	0.9376487	0.9636017	0.9769908
	1	6	2.0	0.1538237	0.5769174	0.7863287	0.8807960	0.9275399	0.9529078
0.05	2	7	0.7	0.1141311	0.5262991	0.7536917	0.8604746	0.9143960	0.9440184
	2	7	0.8	0.0685039	0.4378855	0.6880835	0.8166356	0.8848106	0.9234336
	2	7	1.0	0.0232658	0.2910058	0.5575782	0.7210663	0.8166356	0.8742019
	1	7	1.2	0.3046596	0.7290929	0.8781193	0.9362082	0.9627494	0.9764480
	1	7	1.5	0.1766532	0.6083677	0.8072614	0.8940551	0.9362082	0.9588134
	1	7	2.0	0.0649816	0.4238299	0.6753342	0.8072614	0.8781193	0.9186029
0.01	2	9	0.7	0.0331250	0.3340280	0.5993875	0.7530315	0.8400226	0.8913739
	2	9	0.8	0.0157210	0.2501928	0.5149525	0.6872792	0.7913739	0.8553832
	1	9	1.0	0.2053866	0.6425408	0.8288258	0.9073444	0.9447520	0.9645689
	1	9	1.2	0.1183096	0.5311386	0.7566236	0.8622300	0.9155017	0.9447520
	1	9	1.5	0.0486717	0.3833410	0.6425408	0.7843473	0.8622300	0.9073444
	1	9	2.0	0.0099413	0.2053866	0.4619610	0.6425408	0.7566236	0.8288258

Table 2 shows the acceptance probability for the proposed plan when the test item follows exponential distribution. It can be observed in each case when the quality ratio μ/μ_0 increases the probability of acceptance increases in every case. Although there is a decreasing trend in the acceptance probability when the consumer risk is decreasing and also P_a decreases when the number of items to be test increases.

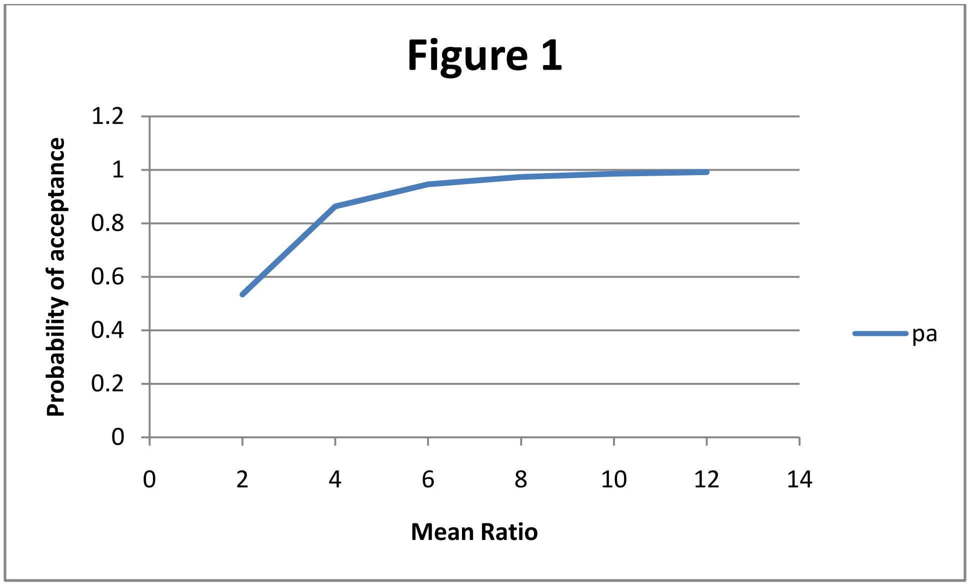

Figure 1: operating characteristic curve for the weighted improved group acceptance sampling plan when the lifetime of an item follows exponential distribution with r= 4, β = 0.25, a = 0.7

Figure 1 shows the increasing trend of the acceptance probability with respect to the mean ratio. As the mean ratio (quality ratio) increases means the quality of the product increase then the lot acceptance probability also increases.

5. Comparison

The forthcoming section elucidates the significance of proposed plan with the help of table3 and corresponding figure 2.

Table 3:Comparison between proposed and *existing plan

Plan	β	r	c	a				
				0.7	0.8	1.0	1.2	1.5
Existing	0.25	2	0	2	2	2	1	1
		3	1	4	3	3	2	2
		4	2	7	5	4	3	2
		5	3	12	9	5	4	2
		6	4	22	14	9	3	2
		7	5	38	26	11	6	2
Proposed	0.25	2	0	2	2	2	1	1
		3	1	2	2	2	2	2
		4	2	2	2	2	2	2
		5	3	2	2	2	2	2
		6	4	2	2	2	2	2
		7	5	2	2	2	2	2

* Extracted the part of table 1 from Amburajan P. and Ramaswamy S. (2015).

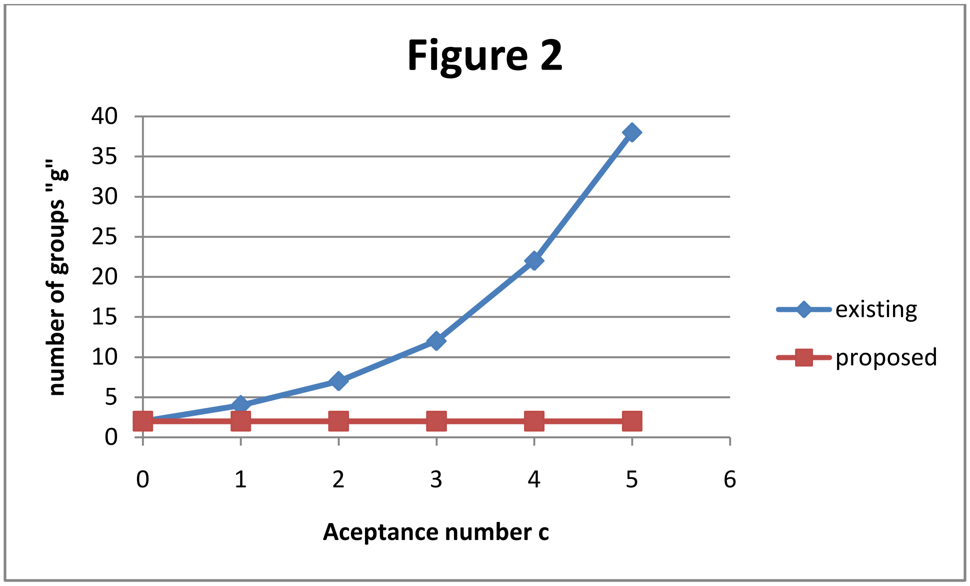

Figure 2: comparison of minimum number of groups between existing and proposed sampling plans when the lifetime of an item follows exponential distribution with r= 4, β = 0.25, a = 0.7

6. Example

With the help of a hypothetical industrial example the practical significance of the proposed plan will be clearer. Let an experimenter wants to test the quality of an electrical item for 1000h.If he has the following sampling plan parameters:

β = 0.25, r=7 c=5 a=0.8 then from Table 1 he required 14 electrical item to test the experiment whereas for the same parameters' setting under existing plan he required 182 such items from Table 3. Hence the proposed plan is more economical as compare to existing plan. The explanation also complies with the Figure 2 graphical comparison of proposed and existing plans.

7. Conclusion

In this paper an improved grouped acceptance sampling plan has been proposed for weighted binomial distribution when the test item follows exponential distribution. The optimal number of groups calculated through simulations, while the rest plan parameters are pre specified. The proposed sampling plan is observed more efficient and economical as compare to existing sampling plan.

Reference

[1] Amburajan, P, Ramaswamy, S. (2015). A Group acceptance sampling plan for weighted binomial on truncatedtests using exponential and weibull distributions. *Journal of Progressive Research in Mathematics*,*2*(1),80-88.

[2] Aslam, M, &Shahbaz, MQ (2007). Economic reliability tests plans using the generalized exponential distribution. *J.Statist.*,*14*,52-59

[3] Aslam, M**,** Jun, C.-H., Lee, H, Ahmad, M, Rasool, M. (2011). Improved group sampling plans based on truncated life tests, *The Chilean Journal of Statistics*,*2*(1),85-97.

[4] Aslam, M**,** Jun, C.-H, Lee, H, Ahmad, M. (2009). Group Acceptance Sampling Plan based on truncated life test for gamma distributed items. *Pakistan Journal of Statistics*,*25*(3) ,333-340.

[5] Kantam, RRL, SrinivasaRao, G,Sriram,,G. (2006). An economic reliability test plan: log-logistic distribution. *J.App.Statist.*, *33*(3), 291-296.

[6] Epstein, B. (1954). Truncated life tests in the exponential case. *Ann. Math. Statist.*, *25*,555-564.

[7] Gupta, RD, &Kundu, D (2003). Generalized exponential distributions.*Australian and New Zealand Journal of Statistics*, *41*(2),173-188.

[8] Mir, KA, & Ahmed, M (2009). Size-Biased distributions and their applications.*Pakistan Journal of Statistic*, *25*(3),283-294.

[9] Radhakrishnan, R, &Alagirisamy, K (2011). Construction of Group Acceptance Sampling Plan usingWeightedbinomial Distribution.*International Journal of Recent Scientific Research*, *2*(7),229-231.

[10] Ramaswamy S, &Amburajan, P (2012). Group acceptance sampling plan using weighted binomial on truncated life tests for Inverse Rayleigh and log-logistic distributions..*Journal of Mathematics, 2*(3), 33-38.