



The Solving of Fermat's Theorem

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Abstract

This article is dedicated to the proof of Fermat's theorem in general form. It is shown that besides the second degree equation, Fermat's equation does not contain any other integer solutions. It is suggested to review 4 methods to proof the Theorem for integers x, y . The proof for Fermat's theorem should be considered closed.

Keywords: Fermat function; acute triangle.

Introduction:

For more than 350 years professional mathematicians and amateurs try to prove the Fermat's theorem, but till now there is no generally accepted evidence of it. However, an interest for this mysterious theorem does not fade and remains debatable nowadays. In present article we propose to consider a simple method.

$$y^n + x^n = z^n \quad (1)$$

The method can be seen into two subsets. The first of which contains only those x and y all exponents n , which may contain the solution of equation (1) in integers x, y, z . The second subset includes only non-integer solutions. Separated by the subset of feasible by expanding equation (1) on components for biome Newton and preparation on their basis of the equation, taking into account the limitations adopted for finding integer solutions. For this we represent the equation (1) in a form suitable for decomposition:

$$(x - a)^n + x^n - (x + b)^n = 0 \quad (2)$$

Here: $x - a$ variable number, $a < x$ - integer; n - integer, exponent; b - integer or non-integer number depending on the relation x, a and n .

The essence of the proof is to determine the appropriate values x, y, z to satisfy equations (1) and (2) method of successive approximations. The problem is solved in relation to 45⁰ sector I quadrant plane coordinates (x, y) , because due to lack of information, coordinate z is equal to 0. The results

can be extended to the rest 7 sectors of the plane (x, y), thereby determining the scope of the conditions of Fermat's theorem.

So using the binomial formula to the expression (2). We get

$$\begin{aligned} (x-a)^n + x^n &= 2x^n - nx^{n-1}a + c_n^2 x^{n-2} a^2 - c_n^3 x^{n-3} a^3 \dots \pm a^n \\ (x+b)^n &= x^n + nx^{n-1}b + c_n^2 x^{n-2} b^2 + c_n^3 x^{n-3} b^3 \dots + b^n \\ \Delta &= x^n - nx^{n-1}(a+b) + c_n^2 x^{n-2}(a^2-b^2) - c_n^3 x^{n-3}(a^3+b^3) \dots + (a^n \pm b^n) = 0 \end{aligned} \tag{3}$$

We call the expression (3) the basic equation in the search for integer solutions of the equation (2). Suitable values $x, y=(x-a), z=(x+b)$, satisfying the equations (1) and (2), We will look at the condition $a=b=1$. Rationale for assumptions (restrictions) follows. Believing $a = b$, the equation (3) into the form:

$$x^n - 2nx^{n-1}a - 2c_n^3 x^{n-3} a^3 - 2c_n^5 x^{n-5} a^5 - \dots (a^n \pm a^n) = 0 \tag{4}$$

We denote $P(a,n) = 2c_n^3 x^{n-3} a^3 + 2c_n^5 x^{n-5} a^5 + \dots (a^n \pm a^n)$ Additive after the first two terms in equation (4). Then equation (4) takes the form:

$$x^n - 2nx^{n-1}a - P(a,n) = 0$$

Dividing the equation by all members x^n we obtain an expression for the desired x

$$x = 2na + P(a,n)/x^{n-1}, \text{ Where } P(a,n)/x^{n-1} \geq 0 \tag{5}$$

With $a = b = 1$ expression (5) takes the form:

$$x = 2n + P(1,n)/x^{n-1} \tag{6}$$

Suitable values $y = x-1$ and $z = x+1$ are determined through a well-known x. From (5) and (6) it is clear, that the $n > 2$ coordination of the left and right sides of the equations (1) and (2) possibly taking into account only the additive $P(1,n)/x^{n-1}$.

Based on the foregoing, integers x and y of Fermat's theorem should be unambiguously attributed to the second subset $y^n + x^n = z^n$.

The table below shows the results of calculations for the harmonization $n=2,3,4$ and 5.

n	X	y=x-1	z=x+1	x ⁿ	y ⁿ	x ⁿ + y ⁿ	z ⁿ	Δ%
2	4	3	5	16	9	25	25	-
3	6,055	5,055	7,055	221	129	350	350	-
4	8,125	7,125	9,125	4350	2540	6890	6890	-
5	10,200	9,200	11,200	107000	66000	173000	175000	1,25

Based on the above we can draw the following preliminary conclusions:

1-Approval of the left and right sides of the equations (1) and (2) impossible without additives $P(a,n)/x^{n-1}$.

2-If the equation $y^n + x^n = z^n$ based additive $P(a,n)$ expressed in number segments and projected on a plane (x,y) , then it has at $n>2$ forms an acute-angled triangle whose sides are at all $a = b = 1$ expressed in whole numbers: $x = 2n+P(1, n)/x^{n-1}$; $y = 2n-1+ P(1, n)/x^{n-1}$; $z = 2n+1+ P(1,n)/x^{n-1}$, that is confirmed at the next examination of additives $P(1,n)/x^{n-1}$.

To answer this question we represent it after the reductions in the following form

$$P(1,n)/x^{n-1} = 2c_n^3/x^2 + 2c_n^5/x^4 + 2c_n^7/x^6 \dots (1 \pm 1)/x^{n-1}$$

The numerator of each term of the expansion represented a combination of c_n^k , distribution is symmetric, and Gaussian-like, relative to the center $(n+1)/2$. The denominator function x^2 increases with each member of the square law.

The first term of the expansion, because of the small x^2 has the greatest value, and can be expressed by a whole number with decimal places (for $n=15 - 1, 1 \dots$; for $n=25 - 1, 8 \dots$). The last term is the smallest value of the large denominator

$$x^{n-1} (\text{for } n = 3 - 2/6^2; \text{ for } n=15 - \text{order } 2/30^{14}; \text{ for } n = 25 - 2/50^{24})$$

The first half of the expansion in the amount significantly higher than the second due to a sharp increase in the numerator. All terms of the expansion of the second half is less than 1 by reducing the numerators and denominators of the further increasing and intensively decreasing as the distance from the center. As a result, the total amount expansions $n>14$

(for $n \leq 14$ additive < 1) will always be determined by the integers from decimal places, All these numbers are non-integer, which indicates the validity and provability of Fermat's theorem.

3- It is known that the equation of the second degree $y^2 + x^2 = z^2$ is solved in whole numbers, and its projection on the plane (x,y) is a right-angled triangle. We can assume that for higher degrees of n there is a rectangular projection, in which a solution of Fermat's equation will occur at integer x, y, z . Such an assumption is justified for the degree $n = 3$ Such an assumption is justified for the degree... x, y, z , in which the equation

$$(x-2a)^3 + (x-a)^3 + x^3 = (x+b)^3, \text{ there are integers } 3,4,5,6 \text{ and they are multiples which satisfy } 3^3 + 4^3 + 5^3 = 6^3.$$

.Physically, these numbers express the sum of the cubes in integers by analogy with the $n = 2$, where the sum of the squares is the sum of squares. In fact, we have a new version of Fermat's theorem...

4-.Distortion projections (triangles), as n , increases due to reflection on the (x, y) unusual structures of a higher order. From this we can conclude that the solution of Fermat's theorem in integers is associated with the presence of rectangular projections, and for non-integer solutions – distorted

projections in the form of acute triangles.

This is confirmed by the following mathematical calculations. preliminarily solve triangle ABC Theorem cosines relative $\cos C$, where C is the angle between the parties a and b $\cos C = (a^2 + b^2 - c^2)/2ab$. Substitute for the parties a, b and c analogs of the triangular projections at $a = b = 1$:

$$a \rightarrow x; b \rightarrow y = x - 1; c \rightarrow z = x + 1, \text{ where } x = 2n + P(1, n)/x^{n-1}$$

After the transformation operations obtain:

$$\cos C_n = 0,5 - 1,5/x_n - 1 \quad (7)$$

According to the calculations of the resulting formula

N	2	3	4	5	10	∞
$x-1$	3	5.054	7.125	9.200	19.0..	∞
$\cos C$	0	0.202	0.289	0.337	0.421	0.5
C°	90	78	73	70	65	60

From which it follows:

-distortion of the triangles in the $n > 2$ due to the change of the angle C from 90° for $n=2$ to 60° for $n \rightarrow \infty$ while the triangles are transformed from rectangular in acute and in the limit - in equilateral.

-In acute triangle, no integer solutions of the form as their party formed non-integer.

-The solution of Fermat's theorem in integers belongs only to a rectangular projection on the plane (x,y) numerical equations segments $y^2 + x^2 = z^2$

5-The second sector is a quadrant analogue pierogi- mirror image of the first in the $y > x$ with all the ensuing results.

6-During the analysis of the proof of Fermat's theorem in general, 4 compact methods are received to proof the theorem for integer x, y, and when you want to prove that the $n > 2$ number z is an integer.

The first method: should be evidence of reviewing of an acute triangle, for which

$Z_0^2 = x^2 + y^2 - 2xy \cos C$. We need to prove that Z_0 is an integer. There are known x and y - integers, a $\cos C$ determined taking into account the constraints $a=b=1$. It varies $0 < \cos C < 0.5$ (look at formula 7), and is a function of non-integer, irrational x. Hence, $\cos C$ is also a non-integer number with many digits after point. This eliminates the whole expression, becomes $2xy \cos C$, which in turn makes non integral Z_0^2 and extraction of the square root Z_0 .

The basis of the second method: also is incorporated consideration of the acute-angled triangle. Thus, $Z_0^2 = x^2 + y^2 - 2xy \cos C$ is always smaller than the corresponding $Z_n^2 = x^2 + y^2$ angled triangle and a segment number Z_0^2 is inside a numeric segment $Z_n^2 = x^2 + y^2$.

Given that, the assumed limitations $y = x-1$ differs by one; the root is extracted from Z_0^2 , and will have a non-integer value, since between numbers $x-1$ and x No other integers.

The third method is based on a different principle. Its essence is as following. For a sequence of integers 1,2,3,4 etc. composed of a number of squares:

4 9 16 25 36 49 64 81 100 121 144 169 196

2 4 6 8 10 12 14 16 18 20 22 24 26

Between the numbers of the first row receiving the lower row, represents the number of integer numbers (sequence numbers) disposed between two adjacent squares numbers x and x+1. These whole (or integral) of z^1 . It may not have been withdrawn from their roots of integer values as located between the numbers differ by one, and will matter $x+\Delta$, where $\Delta = z^1/\Delta x^2$

Taking into account that $n > 2$ for acute triangles z_0^2 always less z_n^2 or the corresponding Δx^2 a series of squares, one needs to insert a numeric interval z_0^2 in numerical interval Δx^2 and ensure that the root of the extracted z_0^2 It is an integer.

It is an integer. $n=5$

Assume: $x=2n=10$; $y=2n-1=9$; $\cos C=0,337$ (look at Formula 6 and 7).

$$z_0^2 = 10^2 + 9^2 - 2 \cdot 10 \cdot 9 \cdot 0,337 = 120,34.$$

In a series of squares that number is between numbers 100 и 121, are squares of integers 10 and 11.

Ap. root of 120, 34 is equal to 10.97 – not an integer

Check: $10^5 + 9^5 = 159049$. The fifth root of the number of 159049 is equal to 10, 97. If necessary, z_0^2 it can be changed by (repeated) determination $\cos C$ on three sides of the triangle known.

Note: The numbers refer to the number of squares of varying degrees of acute triangle n . Of the second series are marked in bold and divided by 4, indicate the degree n , to which the pair of numbers chosen from the condition restrictions $a=b=1$, in accordance with the formula (6).

The fourth method: It based on the fact that similar power series can be built to any n . Then for an arbitrary degree $n=k$ it is possible to directly verify that the extracted root of the number of degree $kz^k = x^k + y^k$. It is an integer. *P.S. The question is: under what conditions is not an integer 10,97... , raised to the power $n = 5$, becomes an integer 159049 ?* Begs an answer: the number of 10.97... It should be rational ie after the decimal point have an unlimited number of significant digits. Let us dwell on the basis of the assumptions made in the article (restrictions).

Adoption $a=1$ due to get maximum

$$z_{\max} = \sqrt[n]{(x-a)^n + x^n} = x + b \leq x \sqrt[n]{2}, (*) \text{ under which, for all } a < 1$$

there are no solutions of Fermat's equation in integers, az^n closest to $2x^n$.

Adoption $b=1$ because the 1 It is the only one for all n integer. This is confirmed by the following considerations. From equation

$$x + b \leq x \sqrt[n]{2}$$

From when $ceb \leq x^n (\sqrt[n]{2} - 1)$. Substituting x this closest integer value $2n$, We obtain the formula $b \leq 2n^n (\sqrt[n]{2} - 1)$ for practical calculations, which suggest that near the origin (at a distance x for each degree n) b varies between 1,65 at $n=2$ to 0 at increase n to ∞ . Hence the conclusion: the solution 45° sector everywhere b is an integer excluding obtain whole x, y, z for solving equations (1) and (2), except at one point where $b = 1$, which should be checked for the presence of solutions in integers x, y, z , as it was done above with negative results.

Calculations $a=b=2,3,4$ refer to points at a considerable distance from the origin, multiple factors $a=2,3,4, \dots$

The results of calculations performed in this case are similar to those in the $a=b=1$, except in cases where x It is determined by a number with a finite number of decimal places. Then you can pick up a

proportionality factor a but the multiplication of integers x, y, z It makes them integers, which will be valid

$$(x*a)^n + (y*a)^n = (z*a)^n.$$

In this case, Fermat's last theorem has become unreliable or exceptions $n > 2$. In principle, Fermat's last theorem can be considered accurate if the additive $P(a,n)/x^{n-1}$ is an irrational number. Then it is impossible to use a coefficient of proportionality a .

The irrationality of the additive $P(1,n)/x^{n-1}$ You can be sure, if done repeatedly refinement of x by successive approximations, because the fission entire numerator in addition to the non-integer, repeatedly refines the denominators in the additive exists At least one irrational result of the division, which will transform the whole additive irrational number

Lastly, analyzing the location of the sectors in the plane (x, y) and, given that the odd function x^n and y^n may be positive or negative, can be composed following the layout of these functions on a plane (x, y) , those. Proliferation terms of Fermat's theorem:

the entire plane (x, y) - for even exponents n

- quadrant I - For positive x and y
- quadrant III- for negative x and y
- Quadrants II and IV for odd n will be a difference in the type
- $x^n - y^n$ or $y^n - x^n$, consideration of which Fermat's theorem is not provided.

References

- [1] A method for the proof of Fermat's theorem in general form. Identify the basic equation (3) and the working of the formula (2), (5), (6), (7) for analysis and calculations.
- [2] The solution of Fermat in integer numbers for $n > 2$ due to the formation on the plane (x, y) distorted (acute-angled) projection function $y^n + x^n = z^n$. If the projections in the form of right-angled triangles solutions obtained in whole numbers
- [3] Fermat's theorem applies to the whole plane (x, y) , except the quadrants II and IV, for odd n .