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Conformal Change of Finsler Special (α, β)-Metric is of Douglas Type

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Abstract. In this present article, we are devoted to study the necessary and sufficient conditions for a Finsler space with a special (α, β)-Metric i.e., $F = c_1\alpha + C_2\beta + \frac{\beta^2}{\alpha} : C_2 \neq 0$; to be a Douglas space and also to be Berwald space, where α is Riemannian metric and $\beta$is differential 1-form. In the second part of this article we are discussing about conformal change of Douglas space with special (α, β)-Metric metric.

Key words: Finsler space; (α, β)-metrics; Conformal change; Douglas space; Berwald space.

AMS Subject Classification (2010): 53C40; 53C15; 53C25; 53C60.

1. INTRODUCTION

The theory of Finsler space with (α, β)-metric has been developed into faithful branch of Finsler geometry. The study of Finsler space with (α, β)-metric was studied by many authors and it is quite old concept, but it is a very important aspect of Finsler geometry and its application to physics.

An n-dimensional Finsler space F^n is Douglas space or Douglas type if and only if the Douglas tensor vanishes identically. In 1997, S. Bacso and M. Matsumoto [2], introduced the notation of Douglas space as a generalization of Berwlad space from the view point of geodesic equations. The condition for some Finsler space with an (α, β)-metric to be Douglas space obtained by M. Matsumoto [3]. Gauree Shankar and Ravindra Yadav [5], studied the Finsler space with third Approximate Matsumoto metric. H. S. park and E. S. Choi [6], worked on Finsler space with the 2nd Approximate Matsumoto metric. H. S. park and E. S. Choi [7], worked on Finsler space with an Approximate Matsumoto metric of Douglas type.

Let (M, L) be a Finsler space, where M is an n-dimensional c^∞ manifold and $L(x, y)$ is a Finsler metric function. If $\sigma(x)$ is a function in each cordinate neighborhood of M, the change $L(x, y) \to e^{\sigma(x)}\bar{L}(x, y)$ is called a conformal change. This change was introduced by M. S. Kneblman [11], and deeply investigated by many authors. The conformal theory of Finsler metrics based on the theory of Finsler space was developed by M. Matsumoto, M. Hasiguchi ([10], [12]) in 1976 and studied the conformal change of a Finsler metric. B. N. Prasad [16], studied conformal change of Douglas space with (α, β)-metric. S. K. Narasimhamurthy, Vasantha D. M and Ajith [14], worked on conformal change of Douglas space with the special (α, β)-metric.

In this present article, we are devoted to study the necessary and sufficient condition for a special Finsler space with the metric $F = c_1\alpha + c_2\beta + \frac{\beta^2}{\alpha}; c_2 \neq 0$, to be a Douglas space and also to be a Berwald space. Further we discuss about the conformal change of Douglas space.

2. PRELIMINARIES

In Finsler geometry, so called (α, β)-metrics are those Finsler metrics are defined by Riemannian metric $\alpha = \sqrt{a_{ij}(x)y^i y^j}$ and a 1-form $\beta = b_i(x)y^i$ on an n-dimensional manifold$M$. They are expressed in the form

$$F = \alpha\phi(s), s = \frac{\beta}{\alpha}$$

where $\phi(s)$ is c^∞ positive function on $(-b_0, b_0)$. It is known that $F = \alpha\phi\left(\frac{\beta}{\alpha}\right)$ is a positive definite Finsler metric for any α and $\beta$with $\|\beta\|_\alpha < b_0$ if and only if ϕ satisfies the following:

$$\phi(s) - s\phi'(s) + (b^2 - s^2)\phi''(s) > 0, \qquad |s| \leq b < b_0.$$

In the local coordinates, the geodesics of a Finsler metric $F = F(x, y)$ are characterized by

$$\frac{d^2x^i}{dt^2} + 2G^i\left(x, \frac{dx}{dt}\right) = 0,$$

where $2G^i = \gamma^i_{ijk}(x, y)y^j y^k$ and $\gamma^i_{ijk}(x, y)$ are Christoffel symbols constructed from $g_{ij}(x, y)$ with respect to x^i.

A Finsler space F^n is said to be Douglas space [2], if

$$D^{ij} \equiv G^i(x, y)y^j - G^j(x, y)y^i, \tag{2.1}$$

are homogeneous polynomial in (y^i) of degree 3. In [2], proved that the Finsler space F^n is of Douglas type if and only if the Douglas tensor

$$D^h_{ijk} = C^h_{ijk} - \frac{1}{n+1}\left(G_{ijk}y^h + G_{ij}\delta^h_k + G_{jk}\delta^h_i + G_{ki}\delta^h_j\right)$$

Vanishes identically, where $G^h_{ijk} = \dot{\partial}_k G^h_{ij}$ is the hv-curvature tensor of the Berwald connection $B\Gamma$.

Finsler space with an (α, β)-metric is a Douglas space if and only if $B^{ij} - B^i y^j - B^j y^i$ are homogeneous polynomials in (y^i) of degree three. The space $R^n = (M, \alpha)$ is called the associated Riemannian space with F^n ([1], [12]). The Covariant differentiation with respect to Levi-Civita connection $\{\gamma^i_{jk}\}$ of R^n is denoted by$(|)$. From the differential 1-form, $\beta(x, y) = b_i(x)y^i$, we define

$$2r_{ij} = b_{i|j} + b_{j|i},\ 2s_{ij} = b_{i|j} - b_{j|i},\ s^i_j = a^{ih}s_{hj},\ s_j = b_i s^i_j.$$

The Berwald connection $B\Gamma = \left(G^i_{jk}, G^i_j\right)$ of F^n plays one of the leading roles in the present paper. Denote by B^i_{jk} the difference tensor [13] of G^i_{jk} fromγ^I_{jk}:

$$G^i_{jk}(x, y) = \gamma^i_{jk}(x) + B^i_{jk}(x, y).$$

With the subscripts 0, transvection by y^i,

$$G^i_j = \gamma^i_{0j} + B^i_j \text{ and } 2G^i = \gamma^i_{00} + 2B^i.$$

We have the function $G^i(x, y)$ of F^n with the (α, β)-metric are written in the form [13],

$$2G^i = \left\{\gamma^i_{00}\right\} + 2B^i,$$

$$B^i = \frac{\alpha L_\beta}{L_\alpha} + C^*\left[\frac{\beta L_\beta}{\alpha L_\alpha}y^i - \frac{\alpha L_{\alpha\alpha}}{L_\alpha}\left(\frac{y^i}{\alpha} - \frac{\alpha}{\beta}b^i\right)\right], \tag{2.2}$$

where $L_\alpha = \frac{\partial L}{\partial \alpha}$, $L_\beta = \frac{\partial L}{\partial \beta}$, $L_{\alpha\alpha} = \frac{\partial^2 L}{\partial\alpha\partial\alpha}$, the subscript 0 means contraction by y^i and we put

$$C^* = \frac{\alpha\beta\,(r_{00}L_\alpha - 2\alpha s_0 L_\beta)}{2(\beta^2 L_\alpha + \alpha\gamma^2 L_{\alpha\alpha})},$$

Where $\gamma^2 = b^2\alpha^2 - \beta^2$, $b^i = a^{ij}\, b_j$ and $b^2 = a^{ij}\, b_i b_j$.

Since$\gamma^i_{00} = \gamma^i_{jk}(x,y)y^i y^j$ are homogeneous polynomials in (y^i) of degree two.

From (2.1) and (2.2), we have

$$B^{ij} = \frac{\alpha L_\beta}{L_\alpha}\left(s^i_0 y^j - s^j_0 y^i\right) + \frac{\alpha^2 L_{\alpha\alpha}}{\beta L_\alpha} C^*(b^i y^j - b^j y^i). \tag{2.3}$$

Thus, a Finsler space Fn with an (α,β)-metric is Douglas space if and only if $B^{ij} = B^i y^j - B^j y^i$

are hp (3).

We use the following lemma later [8].

Lemma-2.1: *If $\alpha^2 \equiv 0(mod\beta)$, i.e., $a_{ij}(x)y^i y^j$ contains $b_i y^i$ as a factor, then the dimension is equal to 2 and b^2 vanishes. In this case, we have 1-form $\delta = d_i(x)y^i$ satisfying $\alpha^2 = \beta\delta$ and $d_i b^i = 2$.*

3. FINSLER SPACE WITH (α,β)-METRIC OF BERWALD TYPE.

In this section, we find the condition for a Finsler space F^n with the metric $F = c_1\alpha + c_2\beta + \frac{\beta^2}{\alpha}; c_2 \neq 0$ to be a Berwald space.

From (2.2), the Berwald connection $B\Gamma = (G^i_{jk}, G^i_j, 0)$ of F^n with (α,β)-metric [13], is given by

$$G^i_j = \dot{\partial}_j G^i = \gamma^i_{0j} + B^i_j,$$

$$G^i_{jk} = \dot{\partial} G^i_j = \gamma^i_{jk} + B^i_{jk},$$

where $B^i_j = \dot{\partial}_j B^i$ and $B^i_{jk} = \dot{\partial}_k B^i_j$. According to [13], B^i_{jk} are uniquely determined by

$$L_\alpha B^k_{ji} y^j y^k + \alpha L_\beta\left(B^k_{ji} b_k - b_{j|i}\right)y^i = 0, \tag{3.1}$$

where $B_{jki} = a_{kr} B^r_{ji}$.

For the special (α,β)-metric $F = c_1\alpha + c_2\beta + \frac{\beta^2}{\alpha}; c_2 \neq 0$,

$$L_\alpha = c_1 - \frac{\beta^2}{\alpha^2},\ L_\beta = c_2 + \frac{2\beta}{\alpha},\ L_{\alpha\alpha} = \frac{2\beta^2}{\alpha^3}. \tag{3.2}$$

Substituting (3.2) in (3.1), we have

$$(c_1\alpha^2 - \beta^2)B_{jki}\, y^j y^k + \alpha^2(c_2\alpha + 2\beta)\left(B_{jki}\, b^k - b_{j|i}\right)y^j = 0, \tag{3.3}$$

where $B_{jki} = a_{kr} B^r_{ji}$.

According to [15], we suppose that F^n is a Berwald space, then B^i_{jk} and $b_{i|j}$ are functions of position alone. Then (3.3) is separated as rational and irrational terms in (y^i) as follows:

$$(c_1\alpha^2 - \beta^2)B_{jki}\, y^j y^k + 2\alpha^2\beta\left(B_{jki}\, b^k - b_{j|i}\right)y^j + \alpha\{c_2\alpha^2\left(B_{jki}\, b^k - b_{j|i}\right)y^j\} = 0, \tag{3.4}$$

which yields two equations,

$$(c_1\alpha^2 - \beta^2)B_{jki}\, y^j y^k + 2\alpha^2\beta\left(B_{jki}\, b^k - b_{j|i}\right)y^j = 0, \tag{3.5}$$

$$c_2\alpha^2\left(B_{jki}\, b^k - b_{j|i}\right)y^j = 0. \tag{3.6}$$

Substituting (3.6) in (3.5), we have $B_{jki}\, y^j y^k = 0$, and hence $B_{jki} + B_{kji} = 0$. Since B_{jki} is symmetric in (j,i), we get $B_{jki} = 0$ easily, and from (3.5) or (3.6), we have

$$b_{j|i} = 0. \tag{3.7}$$

Conversely, if $b_{j|i} = 0$, then $B_{jki} = 0$ are uniquely determined from (3.3).

According to ([8], [18]), Thus we state that

Theorem-3.1: *A Finsler space with a special (α,β)-metric $F = c_1\alpha + c_2\beta + \frac{\beta^2}{\alpha}; c_2 \neq 0$, is Berwlad space if and only if* $b_{j|i} = 0$.

4. FINSLER SPACE WITH (α,β)-METRIC OF DOUGLAS TYPE

In this section, we characterize that the condition for Finsler space F^n with a special (α,β)-metric

$$F = c_1\alpha + c_2\beta + \frac{\beta^2}{\alpha}; c_2 \neq 0\,, \tag{4.1}$$

to be a Douglas type.

In F^n with the metric (4.1), then the equation (2.3), becomes

$$(c_1\alpha^2 - \beta^2)\{\alpha^2(c_1 + 2b^2) - 3\beta^2\}B^{ij} - \alpha^2(\alpha c_2 + 2\beta)\{\alpha^2(c_1 + 2b^2) - 3\beta^2\}\left(s_0^i y^j - s_0^j y^i\right)$$
$$-\alpha^2\{(\alpha^2 c_1 - \beta^2)r_{00} - 2\alpha^2(c_2\alpha + 2\beta)s_0\}(b^i y^j - b^j y^i) = 0. \tag{4.2}$$

Suppose that F^n is a Douglas space, then B^{ij} are $hp(3)$. Separating the rational and irrational terms of y^i in (4.2), which yields

$$\{\alpha^2(c_1 + 2b^2) - 3\beta^2\}\left[(c_1\alpha^2 - \beta^2)B^{ij} - 2\alpha^2\beta\left(s_0^i y^j - s_0^j y^i\right)\right] - \alpha^2\{(\alpha^2 c_1 - \beta^2)r_{00} - 4\alpha^2\beta s_0\} \times (b^i y^j - b^j y^i) -$$
$$\alpha\left[c_2\alpha^2\{\alpha^2(c_1 + 2b^2) - 3\beta^2\}\left(s_0^i y^j - s_0^j y^i\right) - 2\alpha^4 c_2 s_0 (b^i y^j - b^j y^i)\right] = 0, \tag{4.3}$$

Then, we get the following two equations:

$$\{\alpha^2(c_1 + 2b^2) - 3\beta^2\}\left[(c_1\alpha^2 - \beta^2)B^{ij} - 2\alpha^2\beta\left(s_0^i y^j - s_0^j y^i\right)\right] - \alpha^2\{(\alpha^2 c_1 - \beta^2)r_{00} - 4\alpha^2\beta s_0\}$$
$$\times (b^i y^j - b^j y^i) = 0, \tag{4.4}$$

$$\alpha^2\{\alpha^2(c_1 + 2b^2) - 3\beta^2\}\left(s_0^i y^j - s_0^j y^i\right) - 2\alpha^4 s_0 (b^i y^j - b^j y^i) = 0. \tag{4.5}$$

Substituting (4.5) in (4.4), we get

$$(c_1\alpha^2 - \beta^2)\{\alpha^2(c_1 + 2b^2) - 3\beta^2\}B^{ij} - \alpha^2(\alpha^2 c_1 - \beta^2)(b^i y^j - b^j y^i)r_{00} = 0. \tag{4.6}$$

Only the term $3\beta^4 Bij$ of (4.6) does not contain α^2. Hence, we must have $hp(5)$, v_5^{ij} satisfying

$$3\beta^4 Bij = \alpha^{2v_5^{ij}}. \tag{4.5}$$

Case-(i): $\alpha^2 \not\equiv 0(mod\ \beta)$.

In this case, (4.7) is reduced to $B^{ij} = \alpha^2 v^{ij}$, where v^{ij} are $hp(1)$. Thus (4.6) gives

$$\{\alpha^2(c_1 + 2b^2) - 3\beta^2\}v^{ij} - (b^i y^j - b^j y^i)r_{00} = 0. \tag{4.8}$$

Contracting (4.8) by $b_i y_j$, where $y_j = a_{jk} y^k$, we have

$$\alpha^2\left\{(c_1 + 2b^2)v^{ij} b_i y_j - b^2 r_{00}\right\} = \beta^2(3v^{ij} b_i y_j - r_{00}). \tag{4.9}$$

Since $\alpha^2 \not\equiv 0(mod\ \beta)$, there exist a function $h(x)$ satisfying

$$(c_1 + 2b^2)v^{ij} b_i y_j - b^2 r_{00} = h(x)\beta^2 \quad \text{and} \quad 3v^{ij} b_i y_j - r_{00} = h(x)\alpha^2.$$

Eliminating $v^{ij} b_i y_j$ from the above two equations, we obtain

$$(b^2 - c_1)r_{00} = h(x)\{(c_1 + 2b^2)\alpha^2 - 3\beta^2\}. \tag{4.10}$$

From (4.10), we get

$$b_{i|j} = k\{(c_1 + 2b^2)a_{ij} - 3b_i b_j\}, \tag{4.11}$$

Where $k = \frac{h(x)}{b^2 - c_1}$. Here, $h(x)$ is a scalar function, i.e., b^i is a gradient vector.

Conversely, if (4.11) holds, then $s_{ij} = 0$ and we get (4.10). Therefore, (4.2) is written as follows:

$$B^{ij} = k\{\alpha^2(b^i y^j - b^j y^i)\}, \tag{4.12}$$

which are $hp(3)$, that is, F^n is a Douglas space.

Case-(ii): $\alpha^2 \equiv 0(mod\ \beta)$.

In this case, there exists 1-form δ such that $\alpha^2 = \delta\beta$, $b^2 = 0$ and the dimension is two by the lemma (2.1). Therefore (4.7) is reduced to $B^{ij} = \delta\omega_2^{ij}$, where ω_2^{ij} are $hp(2)$. Hence, the equation (4.5) leads to

$$2\delta s_0(b^i y^j - b^j y^i) - (c_1\delta - 3\beta)\left(s_0^i y^j - s_0^j y^i\right) = 0. \tag{4.13}$$

Transvecting the above equation by $y_i b_j$, we have $s_0 = 0$.

Substituting $s_0 = 0$ in (4.13), we have

$$\left(s_0^i y^j - s_0^j y^i\right) = 0. \tag{4.14}$$

Transvecting the (4.14) by y_j, we have $s_0^i = 0$, implies $s_{ij} = 0$.

Therefore, (4.6) reduces to

$$(c_1\delta - 3\beta)\omega_2^{ij} - r_{00}(b^i y^j - b^j y^i) = 0.$$

Contracting the above equation by$b_i y_j$, we get

$$(c_1\delta - 3\beta)\omega_2^{ij} b_i y_j + r_{00}\beta^2 = 0,$$

which is written as

$$c_1\delta\omega_2^{ij}b_iy_j = \beta\left(3\omega_2^{ij}b_iy_j - \beta r_{00}\right).$$

Therefore, there exists an $hp(2), \lambda = \lambda_{ij}(x)y^iy^j$ such that

$$\omega_2^{ij}b_iy_j = \beta\lambda, \qquad 3\omega_2^{ij}b_iy_j - \beta r_{00} = c_1\delta\lambda.$$

Eliminating $\omega_2^{ij}b_iy_j$ from the above equations, we get

$$\beta r_{00} = \lambda(3\beta - c_1\delta), \tag{4.15}$$

Which implies there exists an $hp(1), v_0 = v_i(x)y^i$, such that

$$r_{00} = v_0(3\beta - c_1\delta), \ \ \lambda = v_0\beta. \tag{4.16}$$

From r_{00} is given by (4.16) and $s_{ij} = 0$, we get

$$b_{i|j} = \frac{1}{2}\{v_i\left(3b_j - c_1d_j\right) + v_j\left(3b_i - c_1d_i\right)\}. \tag{4.17}$$

Where, b_i is gradient vector.

Conversely, if (4.17) holds, then $s_{ij} = 0$, and we get $r_{00} = v_0(3\beta - c_1\delta)$. Therefore, (4.2) is written as follows:

$$B^{ij} = -v_0\delta(b^iy^j - b^jy^i), \tag{4.18}$$

Which are $hp(3)$.

Therefore, F^n is a Douglas space. Thus, we have

Theorem-4.2: A Finsler space with a special (α,β)-metric $F = c_1\alpha + c_2\beta + \frac{\beta^2}{\alpha}; c_2 \neq 0$ is a Douglas space if and only if

$(i)\ \alpha^2 \not\equiv 0(mod\ \beta), b^2 \neq 1; b_{i|j}$ is written in the form (4.11),

$(ii)\alpha^2 \equiv 0(mod\ \beta): n = 2$ and $b_{i|j}$ is written in the form (4.17),

where $\alpha^2 = \beta\delta,\ \delta = d_i(x)y^i, v_0 = v_i(x)y^i$.

5. CONFORMAL CHANGE OF DOUGLAS SPACE WITH THE SPECIAL (α,β) METRIC.

Let $F^n = (M^n, L)$ and $\bar{F}^n = (Mn, \bar{L})$ be two Finsler spaces on the same underlying manifold M^n. If the angle in F^n is equal to that in $\bar{F}^n$for any tangent vectors, then F^n is called conformal to $\bar{F}^n$ and the change $L \to \bar{L}$ of the metric is called a conformal change. In other words, if there exists a scalar function $\sigma = \sigma(x)$ such that $\bar{L} = e^\sigma L$, then the change is called conformal change.

For an (α,β)-metrics, $\bar{L} = e^\sigma L(\alpha,\beta)$ is equivalent to $\bar{L} = (e^\sigma\alpha, e^\sigma\beta)$ by homogeneity. Therefore, a conformal change of (α,β)-metric is expressed as $(\alpha,\beta) \to (\bar{\alpha},\bar{\beta})$, where $\bar{\alpha} = e^\sigma\alpha,\ \bar{\beta} = e^\sigma\beta$. Therefore, According to [10]:

$$\bar{a}_{ij} = e^{2\sigma}a_{ij},\ \bar{b}_i = e^\sigma b_i,\ \bar{a}^{ij} = e^{-2\sigma}a^{ij},\ \bar{b}^i = e^{-\sigma}b^i,\ b^2 = a^{ij}b_ib_j = \bar{a}^{ij}\bar{b}_i\bar{b}_j. \tag{5.1}$$

From (5.1), it follows that, the conformal change of Chrostoffel symbols is given by

$$\bar{\gamma}_{jk}^i = \gamma_{jk}^i + \delta_j^i\sigma_k + \delta_k^i\sigma_j - \sigma^ia_{jk}, \tag{5.2}$$

Where $\sigma_j = \partial_j\sigma$ and $\sigma^i = a^{ij}\sigma_j$.

From (5.2) and (2.2), we have the following conformal change:

$$\begin{aligned}
\bar{b}_{i|j} &= e^\sigma\left(b_{i|j} + \rho a_{ij} - \sigma_ib_j\right),\\
\bar{r}_{ij} &= e^\sigma\left[r_{ij} + \rho a_{ij} - \tfrac{1}{2}\left(b_i\sigma_j + b_j\sigma_i\right)\right],\\
\bar{s}_{ij} &= e^\sigma\left[s_{ij} + \tfrac{1}{2}\left(b_i\sigma_j - b_j\sigma_i\right)\right],\\
\bar{s}_j^i &= e^{-\sigma}\left[s_j^i + \tfrac{1}{2}\left(b^i\sigma_j - b_j\sigma^i\right)\right],\\
\bar{s}_j &= s_j + \tfrac{1}{2}\left(b^2\sigma_j - \rho b_j\right),
\end{aligned} \tag{5.3}$$

where $\rho = \sigma_rb^r$.

In [4], a Finsler space with (α,β)-metric $F = c_1\alpha + c_2\beta + \frac{\beta^2}{\alpha}; c_2 \neq 0$ is a Douglas space if and only if

$$b_{i|j} = k\left\{(c_1 + 2b^2)a_{ij} - 3b_ib_j\right\}.$$

By [17], for a conformal change, Finsler space with the special (α,β)-metric $F = c_1\alpha + c_2\beta + \frac{\beta^2}{\alpha}; c_2 \neq 0$ is Douglas space if and only if there exists a function $k(x)$ such that $H_{ij} = 0$, where

$$H_{ij} = b_{i|j} - k\left\{(c_1 + 2b^2)a_{ij} - 3b_ib_j\right\}. \tag{5.4}$$

From (5.1), (5.3) and (5.4), we get

$$\begin{aligned}
\bar{H}_{ij} &= \bar{b}_{i|j} - \bar{k}\{(c_1 + 2\bar{b}^2)\bar{a}_{ij} - 3\bar{b}_i\bar{b}_j\}\\
&= e^\sigma[b_{i|j} - e^\sigma k\left\{(c_1 + 2b^2)a_{ij} - 3b_ib_j\right\} + \rho a_{ij} - \sigma_ib_j],
\end{aligned}$$

Where $\bar{k} = e^{-\sigma}k$.

Therefore

$$\overline{H}_{ij} = e^{\sigma}[H_{ij} + \rho a_{ij} - \sigma_i b_j]. \quad (5.5)$$

Hence, the Douglas space with the metric $F = c_1\alpha + c_2\beta + \frac{\beta^2}{\alpha}; c_2 \neq 0$ is conformally transformed to a Douglas space if and only if $a_{ij} = \sigma_i b_j$, that is,

$$\rho a_{ij} = \frac{1}{2}(\sigma_i b_j + \sigma_j b_i). \quad (5.6)$$

Transvecting (5.6) by b^j , we get

$$\rho b_i = \sigma_i b^2. \quad (5.7)$$

Hence, (5.6) gives $$a_{ij} = \frac{1}{b^2} b_i b_j. \quad (5.8)$$

Contracting (5.8) with $y^i y^j$, we get $b^2\alpha^2 = \beta^2$. If $\alpha^2 \not\equiv 0 (mod\ \beta)$then (5.6) is possibly only when $\rho = 0$ and $\sigma_i = 0$. Thus, the transformation is homothetic. Then we state

Theorem-5.3: If $\alpha^2 \not\equiv 0(mod\ \beta)$, then the Douglas space with a special (α, β)-metric $F = c_1\alpha + c_2\beta + \frac{\beta^2}{\alpha}; c_2 \neq 0$ is conformally transformed to a Douglas space if and only if the transformation is homothetic.

6. CONCLUSION

An n-dimensional Finsler space is a Douglas space or Douglas type if and only if the Douglas tensor vanishes identically. Also it is well known that Douglas space is a generalization of Berwald space from the view point of geodesic equation. In Finsler geometry, we generalized the various types of changes; conformal change, c-conformal change, Randers change, β-conformal change etc. The important examples of Finsler space are different type of (α, β)-metric are Randers metric, Kropina metric and other special (α, β)-metric. Many authors have shown that the condition for the above spaces to be a Douglas spaces or Douglas type.

In this paper, we consider the one of the special (α, β)-metric metric $F = c_1\alpha + c_2\beta + \frac{\beta^2}{\alpha}; c_2 \neq 0$, and in the first step we prove that F is a Douglas type. Further we apply the conformal change and obtained $\overline{F}$ is Douglas metric if and only if the conformal change is homothetic.

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