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On Q*O compact spaces

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Abstract

The aim of this paper is to introduce the new type of compact spaces called Q* compact spaces and study its properties.

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1. Introduction:

Covering spaces with closed sets has its historical back ground in general topology. In 1918, Sierpinski proved [19] that if a connected compact Hausdorff space has a countable cover of pairwise disjoint closed sets , at most one of those sets is nonvoid In 1992, Cater and Daily showed that if a complete , connected , locally connected metric space is covered by countably many proper closed sets, then some two members of these sets must meet. Cater and Daily improved slightly Sierpinski's result by proving that some two members must meet in at least continuum many points. Their new result has applications to several spaces frequently encountered in functional analysis.

Some important results on the topic are contained in [2] and [3]. Let (X , τ) be a topological space . Let A be a subset of (X , τ) . Then A is said to be semi open if $A \subset cl (int (A))$. A is semi closed if $A \subset int(cl (A))$.

Note that every open set is semi open. If every open cover of X has a finite sub cover then X is called a compact space . If every semi open cover has a finite sub cover, then X is a semi compact space. (X , τ) is said to have semi Hausdorff space if $x \neq y$ in X implies existence of semi open neighbourhoods U and V of x and y such that $U \cap V = \phi$.

A function $f : (X , \tau) \rightarrow (Y , \sigma)$ is called a semi continuous function if $f^{-1} (G)$ is a semi open set in X for each open set G in Y .(X , τ) is called an s - normal space if given two disjoint closed sets A and B in X, there exist disjoint semi open neighbourhoods U and V of A and B respectively.

2. Preliminaries:

Definition 2.1: A topological space (X , τ) is said to be **Lindelof** if every open cover of X has a countable sub cover.

Definition 2.2: A subset A of a topological space (X , τ) is said to be **compact** set if every τ - open cover of A has a finite sub cover.

3. Q* O Compact

Definition 3.1: A topological space (X , τ) is said to be **Q* - Lindelof** if every Q* open cover of X has a countable sub cover.

Definition 3.2: A subset A of a topological space (X , τ) is said to be **Q*O - compact** space if every τ - Q* open cover of X has a finite sub cover.

Theorem 3.1: Every Q*O compact space is a Q* - Lindelof space.

Proof: Let X be a Q*O compact space.

To prove that X is Q* Lindelofspace.

Since every finite set is countable and hence in a Q*O compact space every τ - Q* open cover of X has a finite and hence a countable subcover so that it is a Lindelofspace.

Hence every Q*O compact space is Q* - Lindelofspace.

Example 3.1: Q*O - Compactness of discrete & indiscrete topological spaces of X .Consider the discrete topological space (X, p), where p is the power set of X . If X is finite. Then the number of τ - Q* open subset of X is also finite so that every τ - Q* open covering of X is finite so that it is Q*O -compact.

Remark 3.1: Every finite subset of a topological space always Q*O - compact. The following example supports or claim.

Example 3.2: If X is finite. Then $\mathcal{C}$ = { { x } : x ∈ X } is an infinite D - Q* open covering for X as X = ∪{ { x } : x ∈ X } and hence there does not exists any finite sub collection of $\mathcal{C}'$ such that X is the union of that collection . Hence it does not have a finite subcover. Thus, an infinite discrete topological space is not Q*O - compact.

Remark 3.2: Every infinite subset of a topological space is not Q*O - compact.

Example 3.3: If we consider indiscrete topological space (X , τ) then the collection $\mathcal{C}$ = { x } such that X = ∪ { x } , then $\mathcal{C}$ is covering for X which consists of only one set and hence finite. Therefore, (X , τ) is Q*O- compact space.

Definition 3.3: A subset A of a topological space (X , τ) is said to be ***Q* countably compact*** if and only if every countable τ - Q* open covering of X has a finite subcover.

Remark 3.3: Cofinite topological space is Q*O - compact.

Example 3.4: Let $\mathcal{C}$ = { G_α : α∈Λ } be a covering for X so that each G_α is a τ - Q* open set and X = ∪{ G_α : α∈Λ } . $G_\alpha^c$is the complement of G_α is a finite set by definition of co-finite topology . Therefore , G_α^c = { x_1 , x_2 , … , x_n } ie) a finite set . Cover is $\mathcal{C}$ and hence each member of G_α^c is contained in one or other member of G_α. At the most for each x_i∈G_α^c∃ a set G_{α_i} in $\mathcal{C}$ such that x_i∈G_{α_i}. Hence $G_{\alpha_0^c} \subset G_{\alpha_1} \cup G_{\alpha_2} \cup \ldots \cup G_{\alpha_n}$. Above relation shows that the finite collection $\mathcal{C}^*$ = { $G_{\alpha_0}, G_{\alpha_1}, \ldots, G_{\alpha_n}$ } is a finite covering for X & hence (X , τ) is Q*O - compact.

Theorem 3.2: If (X , τ) be Q*O - compact & τ* be coarser than τ then (X , τ*) is also Q*O - compact.

Proof: Let (X , τ) be a Q*O - compact space. Let $\mathcal{C}$ be Q* open cover of X. Since X is Q*O -compact, $\mathcal{C}$ has a finite subcover which is τ - Q* open. Now, us τ* be coarser than τ .ie) every τ* - Q* open set is also τ - Q* open. Hence { G_α : α∈Λ } be any τ* - Q* open cover of X then it is also a τ - Q* open cover of X. But as (X , τ) is Q* compact, this τ - Q* open cover has a finite subcover and consequently $\mathcal{C}$ has a finite τ* subcover. Therefore, (X , τ*) is also Q*O- compact .

Remark 3.4: A Q*O - compact space which is not Hausdorff . The following example supports our claim.

Example 3.5: Let X = { a , b , c } , τ = { ϕ , X , { a } , { a , b } }. Since X is finite therefore it is Q*O -compact. But X is not T_2because for distinct points a & b do not have disjoint Q* - open sets containing a & b or disjoint nbds of a & b .Hence it is not Hausdorff.

Remark 3.5: A Q*O - compact subset of a topological space need not be closed. The following example supports our claim.

Example 3.6 : Let X = { a , b , c , d } , τ = { ϕ , X , { a , b, c} , { a , b , d } } . Let A = { a , c , d } . Now A ⊂ { a , d } ∪ { b , c } . Hence by definition A is Q*O - compact set. But A is not Q* closed its complement { b } is not Q* open.

Theorem 3.3: Every Q*O - compact topological space is Q*countably compact.

Proof: Since the space is Q*O - compact, every τ - Q* open covering of X has a finite subcover. Hence every countable τ - Q* open covering of X has a finite subcover and therefore it is countably compact.

Theorem 3.4: Every Q* - closed subsets of a Q*O - compact space is semi compact.

Proof: Suppose that A is a Q*- closed subset of a Q*O- compact space (X , τ).

We shall show that (A , $\tau/_A$) is semi compact.

Let $\mathcal{C}_A$ = { $G_\alpha \cap$ A : α∈Λ) be any relatively Q* open cover of (A , $^{\tau}/_{A}$).

Since every Q* open set is semi open we have $\mathcal{C}_A$ has any relatively semi open cover of (A , $^{\tau}/_{A}$).

Then $\mathcal{C}$ = { G_α : α∈Λ) is a semi open cover of (X , τ).

But (X , τ) is semi compact .

Hence $\mathcal{C}$ contains a finite sub over { $G_{\alpha_1}, G_{\alpha_2}, \dots, G_{\alpha_n}$ } of (X , τ).

Consequently, { $G_{\alpha_1} \cap A$, ... , ... , ... , $G_{\alpha_n} \cap A$ } is a finite sub cover of A.

Hence (A , $^{\tau}/_{A}$) is semi compact.

This completes the proof.

Theorem 3.5: Every Q* - closed subsets of a semi compact space is semi compact.

Theorem 3.6: Q* continuous image of a semi compact space is compact.

Proof: Suppose that (X , τ) is semi compact.

Let f : (X , τ) → (Y , σ) be Q*- continuous surjection.

We shall show that (Y , σ) is compact.

Let $\mathcal{C}$ = { G_α : α∈Λ } be any open cover of (Y , σ).

Then $f^{-1}(G_\alpha)$ is an semi open set in (X , τ) and $\mathcal{D}$ = { $f^{-1}(G_\alpha)$: α∈Λ } a semi open cover of X.

But (X , τ) is semi compact.

Accordingly ,$\mathcal{D}$ contains a finite sub cover { $f^{-1}(G_{\alpha_1}), \dots, f^{-1}(G_{\alpha_n})$ }.

But then { $G_{\alpha_1}, G_{\alpha_2}, \dots, G_{\alpha_n}$ } is a finite open subcover of Y.

Hence (Y , σ) is a compact space.

This finishes the proof.

Theorem 3.7: Suppose that A is a Q* - compact subset of a Q* - Hausdorff space X . Let x ∈ X – A. Then there exist disjoint Q* - open neighborhoods U and V of A and X respectively.

Proof: By hypothesis, X is Q* - Hausdorff.

Let a ∈ A arbitrarily.

Then there exist disjoint Q* - open neighborhoods U_a and V_x of a and x respectively.

The collection $\mathcal{C}$ = { U_a : a ∈ A } is a Q* - open cover of A.

But A is Q* - compact.

Accordingly, this collection $\mathcal{C}$ has a finite sub cover { $U_{a_1}, U_{a_2}, \dots, U_{a_n}$ } .

Let U = $U_{a_1} \cup \dots \cup U_{a_n}$.

Put V = $V_{a_1} \cap \dots \cap V_{a_n}$.

Then A ⊂ U and x ∈V.

Also U and V are Q* - open .

Since $U_{a_i} \cap V_{a_i} = \phi \, for \, 1 \le i \le n$.

We obtain that U ∩ V = ϕ.

We have proved the result.

Theorem 3.8: Every Q* - compact subset of a Q* - Hausdorff space is Q* - closed.

Proof: Let X be a Q* - Hausdorff Space .

Suppose that A is a Q* - compact subset of X .

Let x ∈ X – A.

By theorem 3.7, there exist disjoint Q* - open sets U_x and V_x containing x and A respectively.

Therefore, x ∈U_x⊂ X – V_x⊂ X – A.

It follows that X – A is Q* - open.

Consequently, A is Q* - closed.

This proves the theorem.

Theorem 3.9: Every Q*O - compact , Q* Hausdorff space is s - normal.

Proof: Suppose that X is a Q* - compact , Q* - Hausdorff space.

Let A and B be two disjoint Q* - closed subsets of X.

Since every Q* compact space is semi compact we have A is semi compact.

It follows , by theorem 3 , that for each x ∈ B there exist disjoint semi open sets U_x and V_x such that x ∈U_x and B ⊂V_x.

The collection $\mathcal{C}$ = { U_x : x ∈ B } is a semi open cover of B.

But B is semi compact.

Hence the collection $\mathcal{C}$ has a sub cover { $U_{x_1}, U_{x_2}, \ldots, U_{x_n}$}.

Let U = $U_{x_1} \cup \ldots \cup U_{x_n}$.

Put V = $V_{x_1} \cap \ldots \cap V_{x_n}$.

Then U and V are semi open with A ⊂ U and B ⊂V.

Since $U_{x_i} \cap V_{x_i} = \phi \; for \; 1 \le i \le n$.

It follows that U ∩ V = ϕ.

Hence theorem holds.

Theorem 3.10: Let X and Y be non empty topological spaces . The product space X × Y is Q*O - compact if both X and Y are Q*O - compact.

Proof: Suppose that X and Y are Q*O - compact.

Let $\mathcal{C}$ be a Q* - open cover of X ×Y , consisting of basic Q* - open sets of the form U × V , where U is a Q* - open set in X and V is a Q* - open set in Y.

Let x ∈X.

Then for each y ∈ Y there exists a set (U_y×V_y) in $\mathcal{C}$ containing (x , y).

The collection { V_y : y ∈Y } is a Q* - open cover of Y.

But Y is Q* -compact.

Consequently, this collection has a finite sub cover {$V_{y_1}, V_{y_2}, \ldots, V_{y_n}$}.

Consider, the corresponding sets $U_{y_1}, U_{y_2}, \ldots, U_{y_n}$.

Put U_x= $U_{y_1} \cap \ldots \cap U_{y_n}$.

Then { x } × Y = U_x× Y

= U_x×{ $V_{y_1} \cup \ldots \cup V_{y_n}$}

= ($U_{y_1} \times V_{y_1}$)∪ … … … ∪ ($U_{y_n} \times V_{y_n}$).

Thus for each x in X there is a set U_x such that { x } × Y ⊂U_x× Y and that U_x× Y is contained in a finite number of sets in $\mathcal{C}$.

But the collection { U_x : x ∈ X } covers X.

Since X is Q*O -compact , this collection has a finite sub cover { $U_{x_1}, U_{x_2}, \dots , U_{x_m}$ } .

Then X × Y $= (U_{x_1} \cup \dots \cup U_{x_m}) \times Y$

$= (U_{x_1} \times Y) \cup \dots \dots \dots \cup (U_{x_m} \times Y)$

But ($U_{x_i} \times$Y)⊂ union of a finite number of sets in $\mathcal{C}$ for each i with 1 ≤ i ≤ m .

It follows that X × Y = union of a finite number of sets in $\mathcal{C}$.

Hence $\mathcal{C}$ has a finite sub cover.

Therefore , X × Y is Q*O – compact.

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