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On supra R-open sets and some applications on topological spaces

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Abstract

In the present paper a new class of generalized supra open sets called supra R-open set is introduced. The relationships between some generalized supra open sets and this class are investigated and illustrated with enough examples. Also, new types of supra continuous maps, supra open maps, supra closed maps, and supra homeomorphism maps are studied depending on the concept of supra R-open sets. Finally, new separation axioms are defined and their several properties are studied.

Keywords and phrases: Supra R-open set, Supra R-continuous (open, closed, homeomorphism) maps, $SR - T_i$-spaces ($i = 1,\ 2$) and supra topological spaces.

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1. Introduction

In 1983, Mashhour et al. [4] introduced the concept of supra topological spaces. They also introduced the notions of S-continuous maps, S^*-continuous maps and some separation axioms in supra topological spaces. In 1990, Kar and Bhattacharyya [3]studied another form of separation axioms in topological spaces called Pre-T_i-spaces($i = 0, 1,\ 2$). In ([2], [7], [8]), the concepts of supra α-open, supra pre-open and supra b-open sets are studied in detail. Mustafa and Qoqazeh [5] studied and investigated separation axioms in supra topological spaces utilizing the notion of supra D-sets. Recently, the notion of supra semi-homeomorphism maps [1] are presented and investigated. In this work, we first present the notion of supra R-open sets as a new class of generalized supra open sets. Then we introduce the concepts of supra R-continuous (R^*-continuous, R-irresolute) maps, supra R-open (R-closed) maps, supra R-homeomorphism maps and explicate the properties of these concepts separately. Last but not least, we introduce new separation axioms, called $SR - T_i$-space ($i = 1,\ 2$) and study enough conditions for some maps to preserve these separation axioms.

2. Preliminaries

Let X be a non-empty set and τ be a topology on X. Then for a subset E of X, the interior and closure of E are denoted by $int\,(E)$ and $cl\,(E)$, respectively. A sub collection μ of 2^X is called a supra topology on X[4] if X belongs to μ and μ is closed under arbitrary union. Every element of μ is called supra open and its complement is called supra closed. For a subset $E$of X, the supra interior and supra closure of E, denoted by $int^{\mu}(E)$ and $cl^{\mu}(E)$, respectively. If the topology τ on X is contained in a supra topology μ on X, then we called μ a supra topology associated with τ. A supra topological space (X,μ) is said to be

(i) $SR-T_0$ [4] if for every $a \neq b$ in X there exists a supra open set containing only one of them.

(ii) $SR-T_1$ [4] if for every $a \neq b$ in X there exist two supra open sets one containing a but not b and other containing b but not a.

(iii) $SR-T_2$ [4] if for every $a \neq b$ in Xthere exist two disjoint supra open sets one containing a and other containing b.

Definition 2.1. A subset E of a supra topological space (X,μ) is called:

(i) supra α-open [2] if $E \subseteq int^{\mu} cl^{\mu} int^{\mu}(E)$.

(ii) supra pre-open [7] if $E \subseteq int^{\mu} cl^{\mu}(E)$

(iii) supra b-open [8] if $E \subseteq cl^{\mu} int^{\mu}(E) \cup int^{\mu} cl^{\mu}(E))$.

(iv) supra semi-open [1] if $E \subseteq cl^{\mu} int^{\mu}(E)$.

3. Supra R-open sets

In this section, we introduce the notion of supra R-open set and illustrate its relationship with some famous generalized supra open sets. Also, we explicate its basic properties.

Definition 3.1. Let (X,μ) be a supra topological space. A subset E of X is called supra R-open set if there exists a non-empty supra open set G such that $G \subseteq cl^{\mu}(E)$. The complement of supra R-open set is called supra R-closed.

Remark 1. The collection of all supra R-open sets in supra topological space (X,μ) is denoted by $SR(X)$.

Theorem3.1. Let (X,μ) be a supra topological space. A subset B of X is supra R-closed iff there exists a supra closed set $F \neq X$such that $int^{\mu}(B) \subseteq F$.

Proof. Suppose that B is a supra R-closed set. Now,B^c is a supra R-open set, then there exists $G \in \mu$ such that $G \subseteq cl^{\mu}(B^c)$ and $G \neq \emptyset$. Therefore $(cl^{\mu}(B^c))^c \subseteq G^c \neq X$. Thus $int^{\mu}(B) \subseteq G^c$. Putting$F = G^c$ and this implies $int^{\mu}(B) \subseteq F \neq X$. Conversely, consider $B \subseteq X$ and there exists a supra closed set $F \neq X$ such that $int^{\mu}(B) \subseteq F$.Then $F^c \subseteq (int^{\mu}(B))^c$and $F^c \neq \emptyset$. Since $(int^{\mu}(B))^c = cl^{\mu}(B^c)$, then B^c is a supra R-open set. Therefore B is a supra R-closed set.

Theorem 3.2. Every supra b-open set is a supra R-open set.

Proof. Suppose that E is a non-empty suprab-open set. Then $E \subseteq cl^{\mu} int^{\mu}(E) \cup int^{\mu} cl^{\mu}(E)) \subseteq cl^{\mu}(E)$. Therefore E is a supra R-open set.

The following example shows that the converse of the above theorem need not be true in general.

Example 3.1. Let $X = \{x,y,z\}$and $\mu = \{\emptyset, X, \{x\}, \{y\}, \{x,y\}, \{y,z\}\}$ be a supra topology on X. Then $\{x,z\}$ is a supra R-open set, but is not supra b-open set.

Remark 2. In [2] and [8], the authors proved that every supra open set, supra α-open set, supra semi-open set and supra pre-open set are supra b-open set. Then we can deduce that they are supra R-open set.

The relationships that discussed in the previous theorems and examples are illustrated in the following Figure.

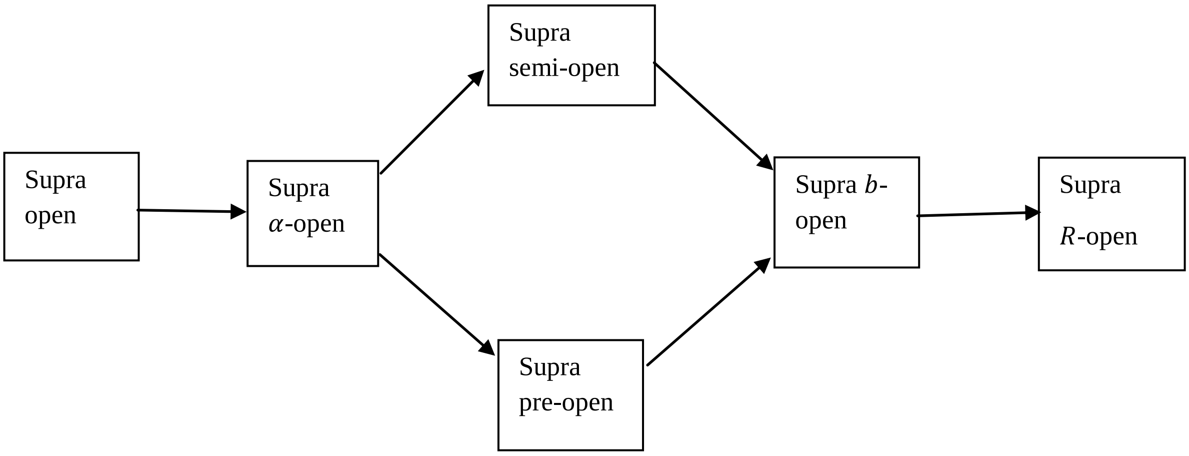

Figure 1: The relationships between supra R-open set and some generalized supra open sets

Now, it is easy to prove the following two propositions.

Proposition 3.1. Every supra neighborhood of any point in a supra topological space (X, μ) is a supra R-open set.

Proposition 3.2. If E is a supra R-open set in supra topological space (X, μ), then every proper superset of E is supra R-open.

The converse of the above two proposition need not be true as shown in the following example.

Example 3.2. Let the supra topology $\mu = \{\emptyset, X, \{x, y\}, \{x, w, z\}\}$ on$X = \{x, y, w, z\}$. Then

$(i)\{x\}$ is supra R-open, but is not supra neighborhood of any point.

(ii) For any proper superset A of $\{y\}$, then A is supra R-open. But $\{y\}$ is not supra R-open.

Theorem 3.3. Let (X, μ) be a supra topological space.The the union of an arbitrary supra R-open sets is a supra R-open set.

Proof. Let $\{E_i : i \in I\}$ be a family of supra R-open sets. Then there exist $i_\circ \in I$and $G \in \mu$ such that $G \subseteq cl^\mu(E_{i_\circ}) \subseteq cl^\mu(\bigcup_{i \in I} E_i)$. Therefore $\bigcup_{i \in I} E_i$ is a supra R-open set.

Remark 3. The intersection of a finite supra R-open sets may not be supra R-open as shown in the following example.

Example 3.4. Let $X = \{x, y, z\}$and$\mu = \{\emptyset, X, \{x, z\}, \{y, z\}\}$ be a supra topology on X. Now, $\{x, z\}$ and $\{x, y\}$ are supra R-open sets, but the intersection of them $\{x\}$is not a supra R-open set.

Theorem 3.6. Let (X, μ) be a supra topological space. Then the intersection of an arbitrary supra R-closed sets is a supra R-closed set.

Proof. Let $\{B_i : i \in I\}$ be a family of supra R-closed sets. Then $\{B_i^c : i \in I\}$ is a family of supra R-open sets. Therefore $\bigcup_{i \in I} B_i^c$is a supra R-open set. Hence $\bigcap_{i \in I} B_i$is supra R-closed.

Remark 4. The union of a finite supra R-closed sets may not be supra R-closed set as shown in the following example.

Example3.4. Let $X = \{x, y, z\}$and$\mu = \{\emptyset, X, \{x, y\}, \{y, z\}\}$ be a supra topology on X. Now, $\{y\}$ and $\{z\}$ are supra R-closed sets, but the union of them $\{y,\ z\}$ is not supra R-closed set.

Definition 3.2. Let (X, μ)be a supra topological space and A be a subset of X. Then:

(i)The supra R-interior of A (denoted by $int_R^\mu(A)$) is the union of all supra R-open sets included in A.

(ii)The supra R-closure of A (denoted by $Cl_R^\mu(A)$) is the intersection of all supra R-closed sets including A.

Proposition3.3. Let (X, μ)be a supra topological space. Then:

(i) $A \subseteq\ cl_R^\mu(A)$; and $A =\ cl_R^\mu(A)$ iff A is a supra R-closed set.

(ii) $int_R^\mu(A) \subseteq A$; and $A =\ \ int_R^\mu(A)$ iff A is a supra R-open set.

(iii) $X -\ int_R^\mu(A)\ \ = cl_R^\mu(X - A)$.

(iv) $X -\ cl_R^\mu(A)\ \ = int_R^\mu(X - A)$.

Proof. The proof of (i) and (ii) come immediately from Definition 3.2.

(iii) $X -\ int_R^\mu(A)\ \ = \left(int_R^\mu(A)\right)^c = \{\cup\, G: G$ is a supra R-open included in $A\}^c = \{\cap\, G^c: G$is a supra R-closed including $A^c\} = cl_R^\mu(A^c)$.

Analogous, one can prove (iv).

Proposition3.4. Let$A$ and B be subsets of a supra topological space(X, μ). Then:

(i) $int_R^\mu(A) \cup int_R^\mu(B)\ \subseteq\ int_R^\mu(A \cup B)$.

(ii) $cl_R^\mu(A)\ \cap cl_R^\mu(B)\ \subseteq\ cl_R^\mu(A \cap B)$.

Proof. It's clear.
In the above theorem, the inclusion may not be replaced by equality relation as shown in the following example.

Example 3.5. Let $X = \{a, b, c\}$and $\mu = \{\emptyset, X, \{a, b\}, \{b, c\}\}$ be a supra topology on X. Let $A = \{b\}$ and $B = \{c\}$.Then $int_R^\mu(A) = \{b\}$, $int_R^\mu(B) = \emptyset$ and $int_R^\mu(A \cup B) = \{b, c\}$. Also, if $C = \{b, c\}$ and $D = \{a, c\}$, then $cl_R^\mu(C) = X$, $cl_R^\mu(D) = X$ and $cl_R^\mu(A \cap B) = \{c\}$.

4. Supra R-continuous maps

The aim of this section is to introduce the notions of supra R-continuous (supra R*-continuous, R-irresolute) maps. Several characterizations of these concepts are investigated with illustrative examples.

Definition 4.1. Let (X, τ)be a supra topological space and (Y, θ)be a topological space. Where ρ is an associated supra topology with θ. A map $g: (X, \tau) \to (Y, \theta)$ is called supra R-continuous (resp. supra R*-continuous, R-irresolute) if the inverse image of each open (resp. supra open, supra R-open) subset of Y is a supra R-open subset of X.

Theorem 4.1. Every supra continuous map is always supra R-continuous.

Proof. Suppose that $g: (X, \tau)\ \to (Y, \theta)$ is a supra continuous map. If E is an open subset of Y, then $g^{-1}(E)$ is a supra open subset of X. Since every supra open set is a supra R-open set, then g is supra R-continuous.

The converse of the above theorem need not be true in general as shown in the following example.

Example 4.1. Consider$\tau = \{\emptyset, X, \{a\}, \{a, b\}\}$ is a supra topology on$X = \{a, b, c\}$, $\theta = \{\emptyset, Y, \{y, z\}\}$ is a topology on $X = \{x, y, z\}$ and $\rho = \{\emptyset, Y, \{x\}, \{x, y\}, \{y, z\}\}$is an associated supra topology with θ. Let the map $g: (X, \tau) \to (Y, \theta)$ be defined as follows: $g(a) = y, g(b) = x$ and $g(c) = z$. Then g is supra R-continuous, but is not supra continuous, since$g^{-1}(\{y, z\}) = \{a, c\} \notin \tau$.

Theorem 4.2. Every supra R*-continuous map is supra R-continuous.

Proof. Straight forward.

The following example shows that the converse of the above theorem is not alwaystrue.

Example4.2. By Example 4.1, $\{x\}$ is supra open subset of Y, but $g^{-1}(\{x\}) = \{b\}$ is not supra R-open set.

Theorem 4.3. Every supra R-irresolute map is supra R*-continuous.

Proof. Obvious.

The converse of the above theorem is not true in general as illustrated in the following example.

Example 4.3. Let $\tau = \mu = \{\emptyset, X, \{a, b\}, \{c, d\}, \{a, b, c\}\}$ is a supra topology on $X = \{a, b, c, d\}$, and $\rho = \{\emptyset, Y, \{x\}, \{y, z\}\}$ be a supra topology on $Y = \{x, y, z\}$. Let the map $g: (X, \tau) \to (Y, \theta)$ be defined as follows: $g(a) = g(b) = xg(c) = y$ and $(d) = z$. Then g is supra R*-continuous map, but is not supra R-irresolute map, since $\{z\}$ is supra R-open and $g^{-1}(\{z\}) = \{d\}$ which is not supra R-open.

Now, the relationships that discussed in the previous theorems and examples in this section are illustrated in the following figure

R-irresolute → Supra R^*-continuous → Supra R-continuous ← Continuous

Figure 2: The relationships between supra R-continuous map and other types of continuous maps

Now, we are ready to prove the main theorem in this section.

Theorem 4.4. Consider $g: (X, \mu) \to (Y, \theta)$ is a map, where (X, μ) is a supra topological space and (Y, θ) is a topological space. The following statements are equivalent:

$(i)g$ is a supra R-continuous map;

(ii) The inverse image of each closed subset of Y is a supra R-closed subset of X;

$(iii) cl_R^\mu(g^{-1}(A)) \subseteq g^{-1}(cl(A))$, for each $A \subseteq Y$;

$(iv) g(cl_R^\mu(E)) \subseteq cl(g(E))$, for each $E \subseteq X$;

(v) $g^{-1}(int(A)) \subseteq int_R^\mu(g^{-1}(A))$, for every $A \subseteq Y$.

Proof.$(i) \Longrightarrow (ii)$: Suppose that E is a closed subset of Y. Then E^c is an open subset of Y. Therefore $g^{-1}(E^c) = X - g^{-1}(E)$ is a supra R-open subset of X. Thus $g^{-1}(E)$ is a supra R-closed subset of X.

$(ii) \Longrightarrow (iii)$: For any subset A of Y, $cl(A)$ is a closed subset of Y. Since $g^{-1}(cl(A))$ is a supra R-closed subset of X, then $cl_R^\mu(g^{-1}(A)) \subseteq cl_R^\mu(g^{-1}(cl(A))) = g^{-1}(cl(A))$.

$(iii) \Longrightarrow (iv)$: Let E be any subset of X. Then ${}^{\mu}_{R}(E) \subseteq cl^{\mu}_{R}(g^{-1}(g(E)))$. Therefore $cl^{\mu}_{R}(g^{-1}(g(E))) \subseteq g^{-1}(cl(g(E)))$. Thus $g(cl^{\mu}_{R}(E)) \subseteq g\left(g^{-1}\left(cl\big(g(E)\big)\right)\right) \subseteq cl\big(g(E)\big)$.

$(iv) \Longrightarrow (v)$: Let A be any subset of Y. By $(iv) g\left(cl^{\mu}_{R}\big(X - g^{-1}(A)\big)\right) \subseteq cl^{\mu}_{R}\left(g\big(X - g^{-1}(A)\big)\right)$. Therefore $g(X - int^{\mu}_{R}(g^{-1}(A))) = g(cl^{\mu}_{R}(X - g^{-1}(A))) \subseteq cl(g(X) - g(g^{-1}(A)) \subseteq cl(Y - A) = Y - int(A)$. Thus $X - int^{\mu}_{R}\big(g^{-1}(A)\big) \subseteq g^{-1}(Y - int(A)) = g^{-1}(Y) - g^{-1}(int(A))$. Hence, $g^{-1}(int(A)) \subseteq int^{\mu}_{R}(g^{-1}(A))$.

$(v) \Longrightarrow (i)$: Suppose that A is any open subset of A. Since $g^{-1}(int(A)) \subseteq int^{\mu}_{R}\big(g^{-1}(A)\big)$, then $g^{-1}(A) \subseteq int^{\mu}_{R}\big(g^{-1}(A)\big)$. Since $int^{\mu}_{R}\big(g^{-1}(A)\big) \subseteq g^{-1}(A)$, then $g^{-1}(A) = int^{\mu}_{R}\big(g^{-1}(A)\big)$. Therefore $g^{-1}(A)$ is a supra R-open set. Thus$g$ is a supra R-continuous map.

Theorem 4.5. Consider (X,μ) and (Y,σ) are supra topological spaces and let $g:(X,\tau) \to (Y,\theta)$ be a map.

The following statements are equivalent:

$(i)g$is a supra R*-continuous map;

(ii)The inverse image of each supra closed subset of Y is a supra R-closed subset of X;

$(iii)cl^{\mu}_{R}(g^{-1}(A)) \subseteq g^{-1}(cl^{\sigma}(A))$, for each $A \subseteq Y$;

$(iv)g(cl^{\mu}_{R}(E)) \subseteq cl^{\sigma}(g(E))$, for each $E \subseteq X$;

$(v)\ g^{-1}(int^{\sigma}(A)) \subseteq int^{\mu}_{R}(g^{-1}(A))$, for every $A \subseteq Y$.

Proof. Similar to the proof of Theorem 4.4.

Theorem 4.6. Consider (X,μ) and (Y,σ) are supra topological spaces and let $g:(X,\tau) \to (Y,\theta)$be a map.

The following statements are equivalent:

$(i)g$ is a supra R-irresolute map;

(ii)The inverse image of each supra R-closed subset of Y is a supra R-closed subset of X;

$(iii)\ cl^{\mu}_{R}\big(g^{-1}(A)\big) \subseteq g^{-1}(cl^{\sigma}_{R}(A))$,for each $A \subseteq Y$;

$(iv)g(cl^{\mu}_{R}(E)) \subseteq cl^{\sigma}_{R}(g(E))$, for each $E \subseteq X$;

$(v)g^{-1}(int^{\sigma}_{R}(g(A))) \subseteq int^{\mu}_{R}(g^{-1}(A))$, for every $A \subseteq Y$.

Proof. Similar to the proof of Theorem 4.4.

Theorem 4.7. Consider (X,τ) and (Y,θ) are topological spacesand let μ and σ be associated supra topologies with τ and θ, respectively. If one of the following conditions holds, then $g:(X,\tau) \to (Y,\theta)$ is a supra R-continuous map.

$(i)g\big(cl(E)\big) \subseteq cl^{\sigma}_{R}(g(E))$, for each $E \subseteq X$.

$(ii)cl(g^{-1}(A)) \subseteq g^{-1}(cl^{\sigma}_{R}(A))$,for each $A \subseteq Y$.

$(iii)\ g^{-1}(int(A)) \subseteq int^{\mu}_{R}(g^{-1}(A))$, for every $A \subseteq Y$.

Proof. (i)It is clear that $cl_R^{\mu}(E) \subseteq cl(E)$, for each $E \subseteq X$. If condition $(i)$is satisfied, then $g(cl_R^{\mu}(E)) \subseteq g(cl(E)) \subseteq cl_R^{\sigma}(g(E)) \subseteq cl(g(E))$. Therefore $g(cl_R^{\mu}(E)) \subseteq cl(g(E))$. Thus g is a supra R-continuous map.

(ii) It is clear that $cl_R^{\sigma}(A) \subseteq cl(A)$, for each $A \subseteq Y$. If condition (ii) is satisfied, then $cl_R^{\mu}(g^{-1}(A)) \subseteq cl(g^{-1}(A) \subseteq g^{-1}(cl_R^{\sigma}(A)) \subseteq g^{-1}(cl(A))$. Therefore $cl_R^{\mu}(g^{-1}(A)) \subseteq g^{-1}(cl(A))$. Thus g is a supra R-continuous map.

(iii)It is clear that $int(A) \subseteq int_R^{\sigma}(A)$, for every $A \subseteq Y$. If condition (iii) is satisfied, then $g^{-1}(int(A)) \subseteq g^{-1}(int_R^{\sigma}(A)) \subseteq g^{-1}(A) \subseteq int_R^{\mu}(g^{-1}(A))$. Therefore $g^{-1}(int(A)) \subseteq int_R^{\mu}(g^{-1}(A))$. Thus g is a supra R-continuous map.

The converse of the above theorem need not be true in general as illustrated in the following example.

Example 4.4. Consider $X = \{a, b\}$, τ and μ are indiscrete topologies on X, $Y = \{x, y, z\}$, $\theta = \sigma = \{\emptyset, Y, \{z\}\}$ and the map $g: (X, \tau) \rightarrow (Y, \theta)$ is defined as follows $g(a) = x$ and $g(b) = y$. Then g is a supra R-continuous map, but the three conditions, which mentioned in the above theorem are not satisfied as pointed out in the following:

(i) Let $E = \{a\}$. Then$g\big(cl(E)\big) = \{x, y\}$ and $cl_R^{\sigma}(g(E)) = \{x\}$.Therefore $g\big(cl(E)\big) \nsubseteq cl_R^{\sigma}(g(E))$.

(ii) Let $A = \{y\}$. Then$cl\big(g^{-1}(A)\big) = X$and $g^{-1}(cl_R^{\sigma}(A)) = \{b\}$. Therefore $cl\big(g^{-1}(A)\big) \nsubseteq g^{-1}(cl_R^{\sigma}(A))$.

(iii) Let $A = \{y, z\}$. Then $g^{-1}(int_R^{\sigma}(A)) = \{b\}$ and $int\big(g^{-1}(A)\big) = \emptyset$. Therefore $g^{-1}(int_R^{\sigma}(A)) \nsubseteq int\big(g^{-1}(A)\big)$.

In a similar way, one can prove the following two theorems.

Theorem 4.7. Consider (X, μ) and (Y, σ) are supra topological. If one of the following conditions holds, then $g: (X, \mu) \rightarrow (Y, \sigma)$ is a supra R*-continuous map.

$(i)g\big(cl^{\mu}(E)\big) \subseteq cl_R^{\sigma}(g(E))$, for each $E \subseteq X$.

$(ii)cl^{\mu}(g^{-1}(A)) \subseteq g^{-1}(cl_R^{\sigma}(A))$,for each $A \subseteq Y$.

$(iii)g^{-1}(int^{\mu}(A)) \subseteq int_R^{\mu}(g^{-1}(A))$, for every $A \subseteq Y$.

Theorem 4.8. Consider (X, μ) and (Y, σ) are supra topological. If one of the following conditions holds, then $g: (X, \mu) \rightarrow (Y, \sigma)$ is a supra R-irresolute map.

$(i)g\left(cl_R^{\mu}(E)\right) \subseteq cl_R^{\sigma}(g(E))$, for each $E \subseteq X$.

$(ii)cl_R^{\mu}(g^{-1}(A)) \subseteq g^{-1}(cl_R^{\sigma}(A))$,for each $A \subseteq Y$.

$(iii)g^{-1}(in_R^{\mu}(A)) \subseteq int_R^{\mu}(g^{-1}(A))$, for every $A \subseteq Y$.

Theorem 4.8. Let (X, τ), (Y, θ) and (Z, σ) be topological spaces and $\tau^*(resp.\ \mu^*, \sigma^*)$be an associated supra topology with $\tau\,(resp. \theta,\ \sigma)$. If $g: (Y, \theta) \rightarrow (Z, \sigma)$is a continuous map and$f: (X, \tau) \rightarrow (Y, \theta)$is a supra R-continuous map, then $g \circ f$is a supra R-continuous map.

Proof. Let G be an open subset of Z. Since $g$is a continuous map, then$g^{-1}(G)$ is an open subset of Y . Since $f$is a supra R-continuous, then$(g \circ f)^{-1}(G) = f^{-1}(g^{-1}(G))$is a supra R-open subset of X.

Theorem 4.9. Let (X, τ), (Y, θ) and (Z, σ) be topological spaces and $\tau^*(resp.\ \mu^*, \sigma^*)$ be an associated supra topology with $\tau\,(resp.\ \theta,\ \sigma)$. If $g: (Y, \theta) \rightarrow (Z, \sigma)$is a supra R-irresolute map and$f: (X, \tau) \rightarrow (Y, \theta)$ is a supra R-continuous map, then $g \circ f$is a supra R-continuous map.

Proof. Similar to the proof of Theorem 4.8.

5. Supra R-open and supra R-closed maps

In this section, we introduce the definitions of supra R-open maps and supra R-closed maps and study their properties. Also, we investigate some relations between supra R-open (R-closed) maps and supra R-continuous (R*-continuous, R-irresolute) maps.

Definition 5.1. Let (X,τ) be a topological space and (Y,μ)be a supra topological space. A map $g:(X,\tau)\to(Y,\mu)$is said to be supra R-open (resp. supra R-closed) if the image of each open (resp. closed) subset of X is a supra R-open (resp. supra R-closed) subset of Y.

Theorem 5.1. Let (X,τ) be a topological space and (Y,μ) be a supra topological space. A map $g:(X,\tau)\to(Y,\mu)$ is supra R-open iff $g(int(E))\subseteq int_R^{\mu}(g(E))$, for every subset E of X.

Proof. " $\Longrightarrow$ " Suppose that g is a supra R-open map. Then $g(int(E))$ is a supra R-open set. We know that $g(int(E))\subseteq g(E)$, and so$g(int(E))\subseteq int_R^{\mu}(g(E))$.

" $\Longleftarrow$ " Suppose that E is an open subset of X. Then $g\big(int(E)\big)=g(E)\subseteq int_R^{\mu}(g(E))$. Therefore $g(E)=int_R^{\mu}(g(E))$. Thus g is a supra R-open map.

Theorem 5.2. Let (X,τ) be a topological space and (Y,μ) be a supra topological space. A map $g:(X,\tau)\to(Y,\mu)$ is supra R-closed iff $cl_R^{\mu}(g(E))\subseteq g(cl(E))$, for every subset E of X.

Proof. " $\Longrightarrow$ " Suppose that g is a supra R-closed map. Then $g(E)\subseteq g(cl(E))$and $g(cl(E))$ is a supra R-closed set. Therefore $cl_R^{\mu}(g(E))\subseteq g(cl(E))$.

" $\Longleftarrow$ " Suppose that E is a closed subset of X. Since$g(E)\subseteq cl_R^{\mu}(g(E))\subseteq g(cl(E))$, then $g(E)$ is supra R-closed. Thus g is a supra R-closed map.

Theorem 5.3. For a bijective map $g:(X,\tau)\to(Y,\theta)$, where μ and σ are associated supra topologies with τ and θ, respectively. The following statements are equivalent:

$(i)$$g$ is a supra R-open map;

$(ii)$$g$ is a supra R-closed map;

$(iii)$$g^{-1}$ is a supra R-continuous map.

Proof. $(i)\Longrightarrow(ii)$: Suppose that E is a closed subset of X. Then E^c is an open subset of X and $g(E^c)$ is a supra R-open subset of Y. Since g is bijective, then$g(E^c)=Y-g(E)$. Therefore $g(E)$ is a supra R-closed subset of Y. Thus$g$ is a supra R-closed map.

$(ii)\Longrightarrow(iii)$: Suppose that g is a supra R-closed map and $E$is a closed subset of X. Then $g(E)$ is supra R-closed. Since g is a bijective supra R-closed map, then $(g^{-1})^{-1}(E)=g(E)$. Therefore g^{-1} is a supra R-continuous map.

$(iii)\Longrightarrow(i)$: Suppose that G is an open subset of X. Since g^{-1} is supra R-continuous, then $(g^{-1})^{-1}(G)$ is a supra R-open subset of Y. Since $g$is bijective supra R-continuous, then $g(E)=(g^{-1})^{-1}(G)$. Thus g is a supra R-open map.

Theorem 5.4. Let $f:(X,\tau)\to(Y,\theta)$ and $g:(Y,\theta)\to(Z,\sigma)$ be two maps and$\tau^*(resp.\ \theta^*,\sigma^*)$ be an associated supra topology with τ $(resp.\theta,\ \sigma)$. Then:

(i)If f is a continuous surjective map and $g\circ f$ is a supra R-open map, then g is a supra R-open map.

(ii)If g is a supra R-continuous injective map and $g\circ f$ is an open map, then f is a supra R-open map.

(iii)If g is a supra R*-continuous injective map and $g\circ f$ is a supra open map, then f is a supra R-open map.

(iv)If g is a supra R-irresolute injective map and $g \circ f$ is a supra R-open map, then f is a supra R-open map.

Proof. (i)Let G be an open subset of Y. Then $f^{-1}(G)$ is an open subset of X. Therefore $(g \circ f)(f^{-1}(G))$ is a supra R-open subset of Z. Since f is surjective, then $(g \circ f)\left(f^{-1}(G)\right) = g\left(f\left(f^{-1}(G)\right)\right) = g(G)$. Therefore g is a supra R-open map.

(ii)Let G be an open subset of X. Then$(g \circ f)(G)$is an open subset of Z. Therefore $g^{-1}(g \circ f(G))$ is supra R-open. Since g is injective, then$g^{-1}\left(g \circ f(G)\right) = (g^{-1} \circ g)(f(G)) = f(G)$. Thus f is a supra R-open map.

A similar proof can be given for (iii) and (iv).

6. Supra R-homeomorphism maps

In this section, we introduce a concept of supra R-homeomorphism maps and study its basic properties.

Definition 6.1. Let (X,τ), (Y,θ) be a topological space and μ and ν be associated supra topologies with$\tau$and θ, respectively.A bijective map $g: (X,\tau) \to (Y,\theta)$is said to be supra R-homeomorphism if g is supra R-continuous and supra R-open.

Since every supra open set is a supra R-open set, then every supra homeomorphism map is a supra R-homeomorphism map. But the converse is not always true as illustrated in the following example:

Example6.1. Consider $\tau = \{\emptyset, \ \Re, \ E_a = (-\infty, \ a): a \in \Re\}$ is the left hand topology on $\Re$, the usual topology u is associated supra topology with τ and let the cofinite topology C is associated supra topology with itself on $\Re$. The identity map$g: (\Re,\tau) \to (Y,C)$ is a supra R-homeomorphism map, but is not a supra homeomorphism map.

Theorem 6.1. Let (X,τ),(Y,θ) be topological spaces and μ and ν be associated supra topologies with τ and θ, respectively. Consider $g: (X,\tau) \to (Y,\theta)$ is a bijective and supra R-continuous map. Then the following statements are equivalent:

$(i)g$ is supra R-homeomorphism;

$(ii)g^{-1}$ is supra R-continuous;

$(iii)g$is supra R-closed.

Proof. Straight forward.

Theorem 6.2. Let (X,τ),(Y,θ) be topological spaces and μ and ν be associated supra topologies with τ and θ, respectively. A bijective map$g: (X,\tau) \to (Y,\theta)$is supra R-homeomorphism iff $g(cl_R^{\mu}(E)) \subseteq cl(g(E))$ and $cl_R^{\nu}(g(E)) \subseteq g(cl(E))$, for any $G \subseteq X$.

Proof." $\Longrightarrow$ " Consider a map g is supra R-homeomorphism. Then g is supra R-continuous and supra R-closed. Therefore $g(cl_R^{\mu}(E)) \subseteq cl(g(E))$ and $cl_R^{\nu}(g(E)) \subseteq g(cl(E))$.

" $\Leftarrow$ " If a bijective map g satisfies that $g(cl_R^{\mu}(E)) \subseteq cl(g(E))$ and $cl_R^{\nu}(g(E)) \subseteq g(cl(E))$, then g is supra R-continuous and supra R-closed. Therefore g is supra R-homeomorphism.

7. New separation axioms

The aim of this section is to define new separation axioms in supra topological spaces called $SR - T_i$ for $i = 1{,}2$ by using supra R-open sets and study some properties of these separation axioms.

Definition 7.1. A supra topological space$(X,\mu)$is said to be

$(i)SR - T_1$if for every distinct points $a, b \in X$, there exist two supra R-open sets one containing a but not b and other containing b but not a.

$(ii) SR - T_2$if for every distinct points $a, b \in X$, there exist two disjoint supra R-open sets one containing $a$and other containing b.

Definition 7.2. A subset W of a supra topological space $(X, \mu)$is said to be supraR-neighborhood of $a \in X$ iff there exists a supra R-open set Gsuch that $a \in G \subseteq W$.

Theorem 7.1. A supra topological space (X, μ) is a $SR - T_1$-space iff every singleton set is a supra R-closed set.

Proof. " $\Longrightarrow$ " Consider (X, μ) is $SR - T_1$-space and let$\{a\} \subseteq X$. For all$b \in X$ such that $a \neq b$ in X, there exists a supra R-open set G containing b such that $G \cap \{a\} = \emptyset$. Then $b \notin cl_R^\mu(\{a\})$. Therefore $cl_R^\mu(\{a\}) = \{a\}$. Thus $\{a\}$ is a supra R-closed set.
" $\Leftarrow$ " Consider (X, μ) is a supra topological space such that every singleton set is a supra R-closed set. Since $\{a\}$ and $\{b\}$are supra R-closed sets, then $X - \{a\}$ and $X - \{b\}$ are supra R-open sets. Thus (X, μ) is a $SR - T_1$-space.

Theorem 7.2. A supra topological space (X, μ) is a $SR - T_2$-space iff for each $a \in X$, we have $\{a\} = \cap \{F_i : F_i$ is a supra R-closed neighborhood of $a\}$.

Proof." $\Longrightarrow$ " Consider (X, μ) is $SR - T_2$-space. Then for $a \neq b$ in X, there exist two disjoint supra R-open sets G_i, H_i such that $a \in G_i$ and $b \in H_i$. Since $G_i \cap H_i = \emptyset$, then $G_i \subseteq H_i^c$. Therefore $cl_R^\mu(G_i) \subseteq H_i^c$. Putting $cl_R^\mu(G_i) = F_i$. Thus$\{a\} = \cap \{F_i : F_i$is a supra R-closed neighborhood of $a\}$.

" $\Leftarrow$ " Consider (X, μ) is a supra topological space such that for each $a \in X$, we have $\{a\} = \cap \{F_i : F_i$ is a supra R-closed neighborhood of $a\}$. Then for $a \neq b$ in X, there exists a supra R-closed neighborhood F_{i_0} of a and $b \notin F_{i_0}$. Therefore there exists a supra R-open set G containing $a$such that $G \subseteq F_{i_0}$. Putting $H = {F_{i_0}}^c$. Obviously, H is a supra R-open set containing b and$G \cap H = \emptyset$. Thus(X, μ) is a $SR - T_2$-space.

Theorem 7.3. Every is $S - T_i$-space a $SR - T_i$-space for $i = 1, 2$.

Proof. Since every supra open set is a supra R-open set, then the theorem is satisfied.

The following example illustrates that the converse of the above theorem is not always true.

Example 7.1. Consider$X = \{a, b, c\}$ and $\mu = \{\emptyset, X, \{a, b\}\}$ is a supra topology on X. Then $SR(X) = \{\emptyset, X, \{a, b\}, \{a\}, \{b\}, \{a, c\}, \{b, c\}\}$. Thus (X, μ) is $SR - T_2$-space, but not $S - T_1$-space.

Theorem 7.4. Everyis $SR - T_2$-space a $SR - T_1$-space.

Proof. It is clear.

The following examples illustrates that the converse of the above theorem is not always true.

Example 7.2. Consider $X = \{a, b, c\}$ and $\mu = \{\emptyset, X, \{a, b\}, \{a, c\}\}$ is a supra topology on X. Then (X, μ) is a $SR - T_1$-space, but not $SR - T_2$-space.

Lemma 7.1. Let(X, μ)be a supra topological space. Then every singleton set in X is a supra R-open set or a supra R-closed set.

Proof. Let $a \in X$. Assume that $\{x\}$ is not a supra R-open set and not a supra R-closed set. Then there are not supra open set $G \neq \emptyset$such that $G \subseteq cl^\mu(\{x\})$ and not supra closed set $F \neq X$ such that $int^\mu(\{x\}) \subseteq F$.Therefore $int^\mu(\{x\}) = \emptyset$and X is the only closed set containing $int^\mu(\{x\})$. Thus μ is indiscrete and this is a contradiction. Hence $\{x\}$ is a supra R-open set or a supra R-closed set.

The following theorem explain why we did not define $SR - T_0$-space.

Theorem 7.5. Let(X,μ)be asupra topological space. Then $cl_R^\mu(\{a\}) \neq cl_R^\mu(\{b\})$, for every distinct points $a, b \in X$.

Proof. For all $a, b \in X$ such that $a \neq b$, we have two cases:

(i)If $\{a\}$ is a supra R-open set, then $\{a\} \cap \{b\} = \emptyset$. Therefore $a \notin cl_R^\mu(\{b\})$. Thus $cl_R^\mu(\{a\}) \neq cl_R^\mu(\{b\})$.

(ii)If $\{a\}$ is not a supra R-open set, then $\{a\}$ is a supra R-closed set. Therefore $X - \{a\}$ is a supra R-open set. Now, $b \in X - \{a\}$ and $X - \{a\} \cap \{a\} = \emptyset$. Thus $b \notin cl_R^\mu(\{b\})$. Hence $cl_R^\mu(\{a\}) \neq cl_R^\mu(\{b\})$.

Theorem 7.6. Consider$(X,\tau),(Y,\theta)$are topological spaceand μ is an associated supra topology with τ. Let $g:(X,\tau) \to (Y,\theta)$be an injective supra R-continuous map. If (Y,θ) is a T_i-space, then (X,μ) is a $SR - T_i$-space for $i = 1,2$.

Proof. Consider (Y,θ) is a T_2-space and let $a \neq b$ in X. Then $g(a) \neq g(b)$. Therefore there are an open set G containing$g(a)$ and an open set Hcontaining$g(b)$ such that $G \cap H = \emptyset$. Thus $g^{-1}(G)$ and $g^{-1}(H)$ are supra R-open sets such that $\in g^{-1}(G)$, $b \in g^{-1}(H)$and $g^{-1}(G) \cap g^{-1}(H) = \emptyset$. Hence (X,μ) is a $SR - T_2$-space.

If $i = 1$, the proof is similar.

Theorem 7.7. Consider$(X,\tau),(Y,\theta)$are supra topological space. Let $g:(X,\tau) \to (Y,\theta)$ be an injective supra R*-continuous(R-irresolute) map. If (Y,θ) is a $S - T_i$ $(SR - T_i)$-space, then (X,τ) is a $SR - T_i$-space for $i = 1,2$.

Proof. The proof is similar to the Theorem 7.6.

Theorem 7.8. Consider$(X,\tau),(Y,\theta)$are topological space and μ is an associated supra topology with θ. Let $g:(X,\tau) \to (Y,\theta)$be a bijective supra R-open map. If (X,τ) is a T_i-space, then (Y,μ) is a $SR - T_i$-space for $i = 1,2$.

Proof. Consider (X,τ) is a T_2-space and let $x \neq y$ in Y. Then $g^{-1}(x) \neq g^{-1}(y)$. Therefore there are disjoint open sets U and Vcontaining $g^{-1}(x)$ and $g^{-1}(y)$, respectively. Thus $g(U)$ and $g(V)$ are supra R-open sets such that $x \in g(U)$, $y \in g(V)$ and $g(U) \cap g(V) = \emptyset$. Hence (Y,μ) is a $SR - T_2$-space.

If $i = 1$, the proof is similar.

Remark 5. The property of being $SR - T_i$-space $(i = 1,2)$ is not hereditary property as pointed out by the following example.

Example7.3. Consider $X = \{a,b,c\}$ and $\mu = \{\emptyset, X, \{a,b\}\}$ is a supra topology on X and let $A = \{b,c\}$. Then (X,μ) is a $SR - T_2$-space, but the subspace (A,μ_A) is not $SR - T_2$-space.

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