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Some results on integer cordial graph

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Abstract:

An *integer cordial labeling* of a graph G(V, E) is an injective map f from V to $\left[-\frac{p}{2}..\frac{p}{2}\right]^*$ or $\left[-\left\lfloor\frac{p}{2}\right\rfloor..\left\lfloor\frac{p}{2}\right\rfloor\right]$ as p is even or odd, which induces an edge labeling $f^*: E \to \{0, 1\}$ defined by $f^*(uv) = 1$ if $f(u) + f(v) \geq 0$ and 0 otherwise such that the number of edges labeled with1 and the number of edges labeled with 0 differ atmost by 1. If a graph has integer cordial labeling, then it is called ***integer cordial graph***. In this paper, we introduce the concept of integer cordial labeling and prove that some standard graphs are integer cordial.

Key Words: Cordial labeling; integer cordial labeling.

AMS Mathematical Subject Classification (2010): 05C78.

1. INTRODUCTION

By a graph we mean a finite undirected graph without loops and multiple edges. For terms not defined here we refer to Harary [9].

An *integer cordial labeling* of a graph G (V, E) is an injective map f from V to $\left[-\frac{p}{2}..\frac{p}{2}\right]^*$ or $\left[-\left\lfloor\frac{p}{2}\right\rfloor..\left\lfloor\frac{p}{2}\right\rfloor\right]$ as p is even or odd, which induces an edge labeling $f^*: E \to \{0, 1\}$ defined by $f^*(uv) = 1$ if $f(u) + f(v) \geq 0$ and 0 otherwise such that the number of edges labeled with 1 and the number of edges labeled with 0 differ at most by 1. If a graph has integer cordial labeling, then it is called ***integer cordial graph***. The concept of cordial graph originated from I.Cahit [1,2] in 1987 as a weaker version of graceful and harmonious graphs and was based on {0,1} binary labeling of vertices.

Let f: V → {0, 1} be a mapping that induces an edge labeling $\overline{f}$: E → {0, 1} defined by $\overline{f}(uv) = |f(u) - f(v)|$. Cahit called such a labeling cordial if the following condition is satisfied: $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$, where $v_f(i)$ and $e_f(i),\ i = 0, 1$ are the number of vertices and edges of G respectively with label i (under f and $\overline{f}$ respectively). A graph G is called cordial if it admits cordial labeling.

In [1], Cahit showed that (i) every tree is cordial (ii) K_n is cordial if and only if n ≤ 3 (iii) $K_{r,s}$ is cordial for all r and s (iv) the wheel W_n is cordial if and only if n ≡ 3(mod4) (v) C_n is cordial if and only if n ≢ 2(mod4) (vi) an Eulerian graph is not cordial if its size is congruent to 2 modulo 4.

Du [4] investigated cordial complete k-partite graphs. Kuo et al. [13] determined all m and n for which mK_n is cordial. Lee et al. [14] exhibited some cordial graphs. Generalised Peterson graphs that are cordial are characterised in [7]. Hoet.al [6] investigated the construction of cordial graphs using Cartesian products and composition of graphs. Shee and Ho [7] determined the cordiality of $C_m^{(n)}$; the one–point union of n copies of C_m. Several constructions of cordial graphs were proposed in [10-12, 15-18]. Other results and open problems concerning cordial graph are seen in [2, 6]. Other types of cordial graphs were considered in [3, 4, 8, 20]. Vaidya et.al [21] have also discussed the cordiality of various graphs.

Definition 1.1[25]

Let f be a binary edge labeling of graph G = {V, E}and the induced vertex labeling is given by $f(v) = \sum_{\forall u} f(u,v)(mod 2)$ where v ϵVand {u,v} ϵ E. f is called an ***E-cordial labeling*** of G if $|\ e_f(0) - e_f(1)| \leq 1$ and $|\ v_f(0) - v_f(1)| \leq 1$, where $e_f(0)$ and $e_f(1)$ denote the number of edges, and $v_f(0)$ and $v_f(1)$ denote the number of vertices with $0's$ and $1's$ respectively. The graph G is called ***E-cordial*** if it admits E-cordial labeling.

In 1997 Yilmaz and Cahit [25] have introduced E-cordial labeling as a weaker version of edge–graceful labeling. They proved that the trees with n vertices,K_n, C_n are E-cordial if and only if n≢2 (mod4) while $K_{m,n}$ admits E-cordial labeling if and only if m + n ≢ 2(mod4).

Definition 1.2 [20]

A ***prime cordial labeling*** of a graph G with vertex set V is a bijection f from V to {1, 2, 3, . . . , |V|} where each edge uv is assigned the label 1 if gcd (f(u), f(v)) = 1 and 0 if gcd (f(u), f(v)) > 1, such that the number of edges having label 0 and edges having label 1differ by at most 1.

Sundaram et.al. [19] introduced the notion of prime cordial labeling. They proved the following graphs are prime cordial: C_n if and only if n ≥ 6; P_n if and only if n ≠ 3 or 5; $K_{1,n}$(n, odd); the graph obtained by subdividing each edge of $K_{1,n}$ if and only if n ≥ 3; bi-stars; dragons; crowns; triangular snakes if and only if the snake has at least three triangles; ladders. J. Babujee and L.Shobana [23] proved the existence of prime cordial labeling for sun graph, kite graph and coconut tree and Y-tree, $< K_{1,n}: 2 >$ (n ≥ 1); Hoffman tree, and $K_2 \Theta\ C_n(C_n)$

In this paper we introduce the concept of integer cordial labeling and we prove that some standard graphs such as cycle C_n, Path P_n, Wheel graph W_n ; n >3, Star graph $K_{1,n}$, Helm graph H_n,Closed helm graph CH_n are integer cordial, K_n is not integer cordial and $K_{n,n}$ is integer cordial iff n is even. It is also proved that $K_{nn} \backslash M$ is integer cordial for any n where M is a perfect matching of $K_{n\,n}$.

Notation: 1.3

(i) $[-x..x] = \{t/t \text{ is an integer and } |t| \leq x\}$
(ii) $[-x..x]^* = [-x..x] - \{0\}$

Definition 1.4

Let G = (V,E) be a simple connected graph with p vertices. Let $f: V \rightarrow \left[-\frac{p}{2}..\frac{p}{2}\right]^*$ or $\left[-\left\lfloor\frac{p}{2}\right\rfloor..\left\lfloor\frac{p}{2}\right\rfloor\right]$ as p is even or odd be an injective map, which induces an edge labeling f^*such that f(uv) = 1, if f(u) + f(v) ≥ 0 and f(uv) = 0 otherwise. Let $e_f(i)$ = number of edges labeled with i, where i = 0 or 1. f is said to be ***integer cordial*** if $|\, e_f(0) - e_f(1)| \leq 1$.

A graph G is called integer cordial if it admits a ***integer cordial labeling***.

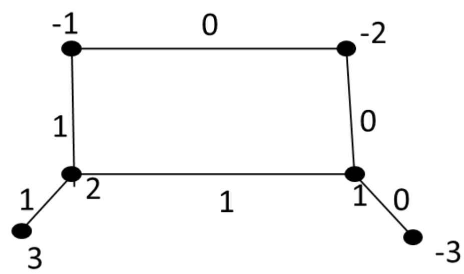

Fig: 1 Integer Cordial Graph

2. Main Results

Theorem 2.1

The cycle C_n is integer cordial graph.

Proof:

Let $v_1, v_2, \ldots, v_n$be the n vertices of the cycle C_n. Here p = n and q = n.

Case (i) p is even. Let p = 2n.

We define $f : V \rightarrow [-n\,..n]^*$as follows:

$$f(v_i) = -i \;; \quad 1 \leq i \leq n$$

$$f(v_{n+i}) = i \;; 1 \leq i \leq n$$

$Then$ $f(v_n) = -n$ and $f(v_{n+1}) = 1$, the edge $f(v_n v_{n+1}) = -n + 1$

When n ≥ 2, $f(v_n v_{n+1})$ will be negative. That is, $f(v_n v_{n+1}) = 0$

Similarly, $f(v_1) = -1$ and $f(v_{2n}) = n$ and the edge $f(v_1 v_{2n}) = n - 1$

When $n \geq 2$, $f(v_1 v_n)$ will be positive. That is, $f(v_1 v_n) = 1$

Obviously, the sum of consecutive negative integers is negative and sum of consecutive positive integers is positive. There is $\frac{q}{2}$ such negative integers and $\frac{q}{2}$ positive integers. Thus $e_f(0) = e_f(1) = \frac{q}{2}$.

Case (ii) p is odd. Let p = 2n + 1

We define $f : V \rightarrow [-n\,..n]$as follows:

$$f(v_i) = -i; \quad 1 \leq i \leq n$$

$f(v_{n+i}) = i;\ 1 \leq i \leq n$

$f(v_{2n+1}) = 0$

Since $f(v_n) = -n$ and $f(v_{n+1}) = 1$, we have $f(v_n v_{n+1}) < 0$, which implies the edge $v_n v_{n+1}$ receives the label 0.

Similarly, since $f(v_1) = -1$ and $f(v_{2n+1}) = 0$, we have $f(v_1 v_n) = -1$, hence the edge $v_1 v_{2n+1}$ receives the label 0.This implies that n +1 edges receive label 0 and n edges receive label 1.

Therefore, $e_f(0) = n + 1$ and $e_f(1) = n$.

Thus $|e_f(0) - e_f(1)| = 1$.

Case (iii)

As a special case, we consider a cycle when n = 3

We define the labelling f :V →[-1, 0, 1] as follows:

Let $f(v_1) = -1$; $f(v_2) = 1$; $f(v_3) = 0$

Then $f(v_1 v_2)$ receives label 1, $f(v_1 v_3)$ receives label 0 and $f(v_2\, v_3)$ receives label 0

Therefore $|e_f(0) - e_f(1)| = 1$.

From all the cases $|e_f(0) - e_f(1)| \leq 1$.

Thus C_n is integer cordial graph. ■

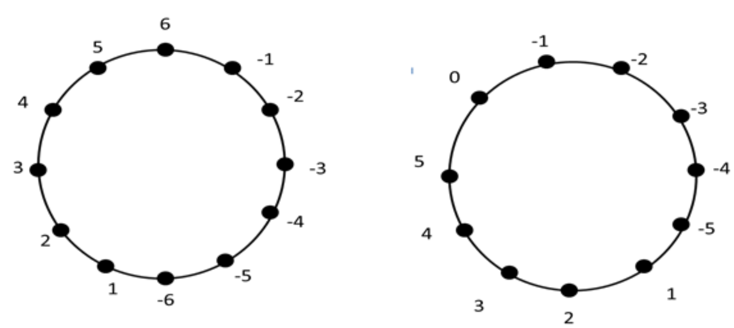

Fig: 2. C_{12} and C_{11} are integer cordial graph

Note: 2. 2 Similar argument proves that Path P_n is also an integer cordial graph.

Theorem 2.3

Complete graph K_n, where n > 3, is not integer cordial.

Proof:

Let $v_1, v_2, \ldots v_n$ be the n vertices of K_n. Here $p = n$ and $q = \frac{n(n-1)}{2}$. Since each vertex is adjacent to every vertex, $\frac{n(n-2)}{4}$ integers are negative and $\frac{n(n-2)}{4} + \frac{n}{2}$ integers will be positive. That is $\frac{n(n-2)}{4}$ edges receive label 0 and $\frac{n^2}{4}$ edges receive label 1.

Thus, $e_f(0) = \frac{n(n-2)}{4}$; $e_f(1) = \frac{n^2}{4}$.

Therefore, $|e_f(0) - e_f(1)| \nleq 1$. Hence, K_n is not an integer cordial. ■

Theorem 2.4

The Wheel graph W_n ; n > 3 is integer cordial.

Proof:

Let u be the apex vertex and $v_1, v_2, \ldots v_n$ be the rim vertices. Here p = n + 1and q = 2n.

Case (i) p is even

We define $f: V \rightarrow$to$\left[-\frac{p}{2}..\frac{p}{2}\right]^*$ as follows:

$$f(u) = 1$$

$$f(v_i) = -i \; ; \; 1 \le i \le \lceil \tfrac{n}{2} \rceil$$

$$f\left(v_{\lceil \frac{n}{2} \rceil + i}\right) = i + 1 \; ; \; 1 \le i \le \lfloor \tfrac{n}{2} \rfloor$$

It can be checked that $e_f(0) = e_f(1) = \frac{q}{2}$.

Therefore, $\left|e_f(0) - e_f(1)\right| = 0$

Case (ii) p is odd

We define $f: V \rightarrow \left[-\left\lfloor\frac{p}{2}\right\rfloor..\left\lfloor\frac{p}{2}\right\rfloor\right]$ as follows:

$$f(u) = 0$$

$$f(v_i) = -i \; ; \; 1 \le i \le \frac{n}{2}$$

$$f(v_{\frac{n}{2}+i}) = i \; ; 1 \le i \le \frac{n}{2}$$

We observe that $\left(\frac{2n}{2} - 1\right) + 1$ edges receive label 0.

That is, $e_f(0) = e_f(1) = \frac{q}{2}$.

Therefore, $\left|e_f(0) - e_f(1)\right| = 0$.

Thus from both the cases $\left|e_f(0) - e_f(1)\right| \le 1$.

Hence W_n ; n > 3 is integer cordial ■

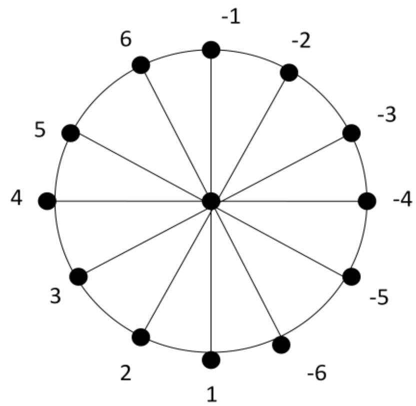

Fig: 3 W_{12} is integer cordial

Note: 2.5

W_3 is not integer cordial.

The result follows from Theorem 2.3.

Theorem 2.6

The Star graph $K_{1,n}$ is integer cordial.

Proof:

Let G = $K_{1,n}$ be the star graph. Let u be the apex vertex and $\{v_1, v_2, \dots v_n\}$ be the pendant vertices. Here p = n + 1 and q = n. We consider two cases:

Case (i) p is odd.

We define $f : V \to \left[-\left\lfloor\frac{p}{2}\right\rfloor .. \left\lfloor\frac{p}{2}\right\rfloor\right]$ as follows:

$$f(u) = 0$$

$$f(v_i) = -i \ ; \ 1 \le i \le \frac{n}{2}$$

$$f(v_{\frac{n}{2}+i}) = i \ ; 1 \le i \le \frac{n}{2}$$

The apex vertex u is given label 0, and edges incident to positive integers receive positive label and edges incident to negative integers receives negative label. There are $\frac{n}{2}$ such edges receiving positive labels and $\frac{n}{2}$ edges receiving negative labels.

That is, $e_f(0) = e_f(1) = \frac{q}{2}$.

Thus, $|e_f(0) - e_f(1)| = 0$.

Case (ii) p is even

We define $f : V \rightarrow$ to $\left[-\frac{p}{2}..\frac{p}{2}\right]^*$ as follows:

$f(u) = 1$

$f(v_i) = -i\ ;\ 1 \leq i \leq \lceil \frac{n}{2} \rceil$

$f(v_{\lceil \frac{n}{2} \rceil + i}) = i + 1\ ;\ 1 \leq i \leq \lfloor \frac{n}{2} \rfloor$

Since the apex vertex is labeled as 1 and $\lfloor \frac{q}{2} \rfloor$ edges receive label 0 and $\lceil \frac{q}{2} \rceil$ edges receive label 1.

That is, $e_f(0) = \lfloor \frac{q}{2} \rfloor$ and $e_f(1) = \lceil \frac{q}{2} \rceil$

Hence, | $e_f(0)$ - $e_f(1)$| =1.

From both the cases | $e_f(0)$ - $e_f(1)$| ≤ 1

Thus, $K_{1,n}$ is integer cordial. ■

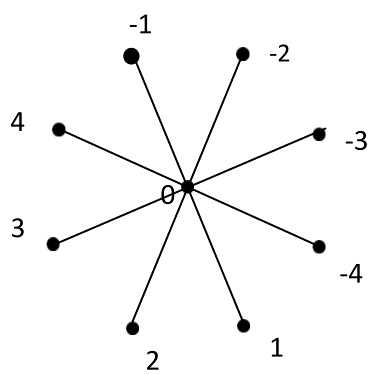

Fig: 4. $K_{1,8}$ is integer cordial

Theorem 2. 7

Helm graph H_n is integer cordial.

Proof:

Helm graph H_n is always of odd order. Let v be the apex vertex, $v_1, v_2, \ldots v_n$ be the vertices of inner cycle and $u_1, u_2, \ldots u_n$ be the pendant vertices. Let $H_n = G$, then p = 2n + 1 and q = 3n.

Case (i).n is odd (p is odd)

We define $f : V \rightarrow \left[-\left\lfloor\frac{p}{2}\right\rfloor .. \left\lfloor\frac{p}{2}\right\rfloor\right]$ as follows:

$f(v) = 0$

$f(v_i) = -i;\ 1 \leq i \leq \lceil \frac{n}{2} \rceil$

$$f\left(v_{\lceil\frac{n}{2}\rceil+i}\right) = i;\ 1 \le i \le \lfloor\frac{n}{2}\rfloor$$

$$f(u_i) = -\left(\lceil\frac{n}{2}\rceil + i\right)\ ;\ 1 \le i \le \lfloor\frac{n}{2}\rfloor$$

$$f\left(u_{\lfloor\frac{n}{2}\rfloor+i}\right) = \left(\lfloor\frac{n}{2}\rfloor + i\right)\ ;1 \le i \le \lfloor\frac{n}{2}\rfloor$$

Since the apex vertex is given the label 0, out of n edges incident to v, $\lceil\frac{n}{2}\rceil$ edges receive label 0 and $\lfloor\frac{n}{2}\rfloor$ edges receive label 1. Similarly, $\lceil\frac{n}{2}\rceil$ edges receive label 0 and $\lfloor\frac{n}{2}\rfloor$ edges receive 1 from the cycle. The pendant vertices have n edges. Out of n edges, $\lfloor\frac{n}{2}\rfloor$ edges receive 0 and $\lceil\frac{n}{2}\rceil$ edges receive 1. That is, $e_f(0) = \frac{3n+1}{2} = \lceil\frac{q}{2}\rceil$ and $e_f(1) = \frac{3n-1}{2} = \lfloor\frac{q}{2}\rfloor$.

Thus $|\ e_f(0) - e_f(1)|\ = 1$.

Case (ii) n is even (p is odd)

$$f(v) = 0$$

$$f(v_i) = -i;\ 1 \le i \le \frac{n}{2}$$

$$f(v_{\frac{n}{2}+i}) = i\ \ ;\ 1 \le i \le \frac{n}{2}$$

$$f(u_i) = -(\frac{n}{2} + i)\ ;\ 1 \le i \le \frac{n}{2}$$

$$f(u_{\frac{n}{2}+i}) = \left(\frac{n}{2} + i\right)\ ;\ 1 \le i \le \frac{n}{2}$$

The apex vertex is labeled 0, so out of n edges incident to v, $\frac{n}{2}$ edges receive label 1 and $\frac{n}{2}$ edges receive label 0. Similarly out of n edges in the cycle, $\frac{n}{2}$ edges receive label 1 and $\frac{n}{2}$ edges receive label 0.

That is, $e_f(0) = e_f(1) = \frac{q}{2}$.

Thus $|e_f(0) - e_f(1)| = 1$.

Hence, H_n is integer cordial. ■

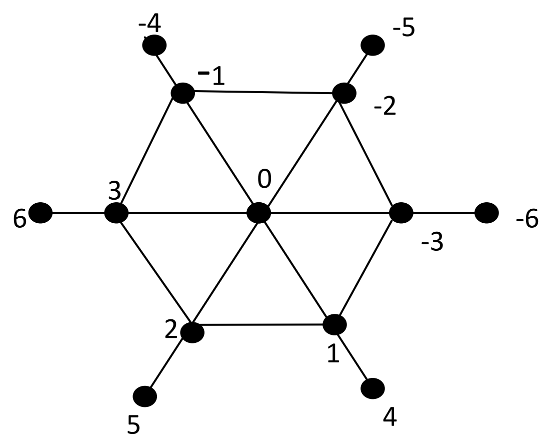

Fig: 5. H_6 is integer cordial

Theorem 2. 8

The closed helm graph CH_n is integer cordial.

Proof:

Let v be the apex vertex, $\{v_1, v_2, \ldots v_n\}$ be the vertices of inner cycle and $\{u_1, u_2, \ldots u_n\}$ be the rim vertices.

Let $CH_n = G$. Then p = 2n + 1andq = 4n.We define $f: V \to \left[-\left\lfloor\frac{p}{2}\right\rfloor..\left\lfloor\frac{p}{2}\right\rfloor\right]$ as follows:

$f(v) = 0$

$f(v_i) = -i;\ 1 \leq i \leq n$

$f(u_i) = i;\ \ 1 \leq i \leq n$

From the above labeling we observe that $\frac{q}{2}$ edges receive label 1 and $\frac{q}{2}$ edges receive label 0.

That is, $e_f(0) = e_f(1) = \frac{q}{2}$.

Thus $|e_f(0) - e_f(1)| = 0$.

Hence, $CH_n$is integer cordial. ■

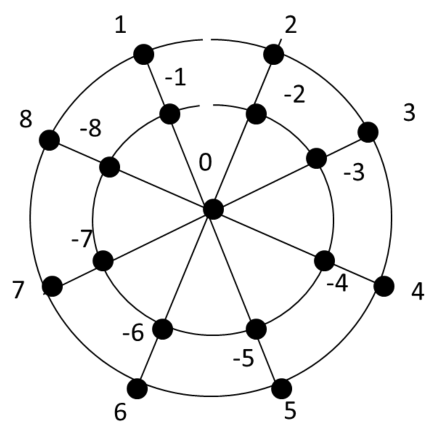

Fig: 6 CH$_8$ is integer cordial

Theorem 2.9

The complete bipartite graph $K_{n,n}$ is integer cordial if and only if n is even.

Proof:

Let $G = K_{n,n}$ be a complete bipartite graph with the partitions {U, V} where $U = \{u_1, u_2, \ldots, u_n\}$ and $V = \{v_1, v_2, \ldots., v_n\}$. Then $p = 2n$ and $q = n^2$. We define $f : V \to \left[-\frac{p}{2}..\frac{p}{2}\right]^*$as follows:

$$f(u_i) = \begin{cases} -\left(\frac{i+1}{2}\right) \; if\ i\ is\ odd\ ; 1 \le i \le n \\ \frac{i}{2} \;\; if\ i\ is\ even\ \ ; 1 \le i \le n \end{cases}$$

$$f(v_i) = \begin{cases} -\left(\frac{n+i+1}{2}\right) \;\; if\ i\ is\ odd\ ; 1 \le i \le n \\ \left(\frac{n+i}{2}\right) \;\; if\ i\ is\ even\ ; \;\; 1 \le i \le n \end{cases}$$

From the above labeling, we observe that there are $\frac{n^2}{2}$ edges receive label 0 and also $\frac{n^2}{2}$ edges receive label 1.

That is, $e_f(0) = e_f(0) = \frac{n^2}{2} = \frac{q}{2}$.

Hence $|e_f(0) - e_f(0)| = 0$.

Thus G is integer cordial when n is even.

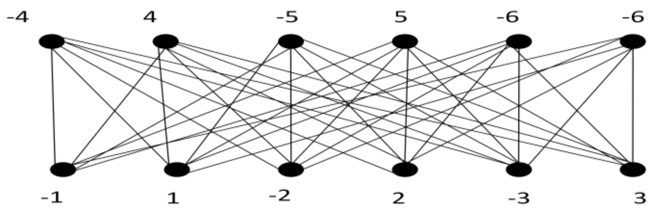

Fig: 7. $K_{6,6}$ is integer cordial

Conversely, when n is odd

Suppose $K_{n,n}$ is integer cordial graph let us consider the labeling

$f(u_i) = -i\ ; 1 \le i \le n$ and $f(v_i) = i\ ; 1 \le i \le n$. $K_{n,n}$ being a complete bipartite graph there are $\frac{n(n-1)}{2}$ possibilities of getting negative integers and $\frac{n(n+1)}{2}$ possibilities of getting positive integers. That is, $e_f(0) = \frac{n(n-1)}{2}$ and $e_f(1) = \frac{n(n+1)}{2}$.

Therefore, $|e_f(0) - e_f(1)| = n \nleq 1$. Similar proof holds if we consider any labeling.

Hence $K_{n,n}$ is not an integer cordial graph when n is odd.■

Theorem: 2.10

The graph $K_{n,n} \backslash M$ is an integer cordial graph for every n, where M is a perfect matching.

Proof:

Consider the graph $K_{nn} \backslash M$ with vertex set V(G)={ X, Y}where $X = \{x_1, x_2, x_3, \dots x_n\}$ and $Y = \{y_1, y_2, y_3, \dots, y_n\}$. Without loss of generality, let M = $\{x_1y_1, x_2y_2, x_3y_3, \dots, x_ny_n\}$. Let $G = K_{n,n} \backslash M$. Here p = 2n and q = n(n-1). We define $f: V \to \left[-\frac{p}{2} .. \frac{p}{2}\right]^*$ as follows:

$f(x_i) = -i\ ; 1 \le i \le n$

$f(y_i) = i\,; 1 \leq i \leq n$

In particular, let us consider x_n and $y_{n-1.}$ Then $f(x_n) = -n$ and $f(y_{n-1}) = n-1$. Therefore, $f(x_n y_{n-1}) = -n + n - 1 = -1$, which is negative. Hence the edge $x_n y_{n-1}$ receives label 0.

Similarly, let us consider, the edges y_n and x_1. Here $f(y_n) = n$ and $f(x_1) = -1$.

Then $f(y_n x_1) = n - 1$. When n > 1, the term will be positive. Hence we give the label 1.

Similarly, the other $\left(\frac{q}{2} - 1\right)$ edges receives label 0 and also $\left(\frac{q}{2} - 1\right)$ edges receives label 1.

That is, $e_f(0) = e_f(1) = \frac{q}{2}$.

Hence $|e_f(0) - e_f(1)| = 0$.■

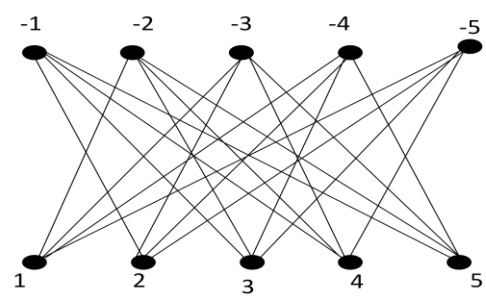

Fig: 8. $K_{5,5} \setminus M$ is integer cordial

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