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Boundedness of singular integrals with oscillating kernels on weighted Morrey space

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Abstract.

In this paper, we obtain some weighted norm inequalities for singular integral with oscillating kernels on Morrey space.

Keywords: Oscillating kernels; Morrey space; weights.

1. Introduction

We first recall the definition of weighted Morrey space [1] as follows

$$\|f\|_{M_{p,k}(w)} = sup_B \left(\frac{1}{(w(B))^k}\int_B |f(x)|^p w(x)\,dx\right)^{\frac{1}{p}},$$

where B denotes any ball in R^n. $1 \le p < \infty, 0 < k < 1$. The weighted Morrey space is a generalization of weighted Lebesgue space. In [1], the author obtained the boundedness of maximal Hardy-Littlewood operator and singular operators on $M_{p,k}(w)$ with $1 \le p < \infty$ and $w \in$ Ap. (the Muckenhoupt classes [2]).

The oscillating kernel was denoted by

$$K_\alpha(x) = e^{i|x|^\alpha}(1+|x|)^{-n}\alpha > 0, \alpha \ne 1.$$

$Tf(x) = K_\alpha * f(x)$ is the singular integral operator with oscillating kernel. It is well known that the operator T was bounded on $L^p(R^n)(1 < p < \infty)$ ([3]). The weighted boundedness of T on Lebesgue spaces can be found in [4].

This paper focuses on the weighted boundedness of T on $M_{p,k}(w)$ following from some ideas which were developed in dealing with Lebesgue spaces. The main results of this paper can be stated as follows

Theorem 1.1. (a) If $w \in A_1$, then the operator T is bounded on the weak weighted Morrey space..

(b) Let $1 < p < \infty$ and $w \in A_p$. Then the operator T is bounded on $M_{p,k}(w)$.

Throughout this paper all definitions and notations are standard.

2. Boundedness of singular integral with oscillating kernels

The following properties of A_p weight classes play important role in the proofs of Theorem 1.1.

Lemma 2.1. [5] Let $1 \le p < \infty$ and $w \in A_p$. Then the following statements are true

(a) There exists a constant C such that

$w(2B) \le Cw(B)$.

(b) There exists a constant $C > 1$ such that

$w(2B) \ge Cw(B)$.

(c) There exist two constants C and $r > 1$ such that the following reverse Hölder inequality holds for every ball $B \subset R^n$

$$\left(\frac{1}{|B|}\int_B |w(x)|^r\, dx\right)^{\frac{1}{r}} \le C\frac{1}{|B|}\int_B |w(x)| \quad dx$$

The following boundedness of T on weighted Lebesgue spaces had proved in [6].

Lemma 2.2. Let $\alpha > 0$ and $\alpha \ne 1$.

(a) If $w \in A_1$, then there is a constant $C > 0$ such that

$$sup_{\lambda>0}\lambda w(\{x \in B: |Tf(x)| > \lambda\}) \le \|f\|_{L^1(w)}.$$

(b) If $w \in A_p$ $(1 < p < \infty)$, then there is a constant $C > 0$ such that

$$\|Tf\|_{L^p(w)} \le C\|f\|_{L^p(w)}.$$

Now we are in a position to give the proof of Theorem 1.1. We first prove (a). Decomposing $f = f_{\chi 2B} + f_{\chi(2B)^c} =: f_1 + f_2$, then for any given $\lambda > 0$, we have

$$w(\{x \in B: |Tf(x)| > \lambda\}) \le w\left(\left\{x \in B: |Tf_1(x)| > \frac{\lambda}{2}\right\}\right) + w\left(\left\{x \in B: |Tf_2(x)| > \frac{\lambda}{2}\right\}\right)$$

$$:= I + II$$

By Lemma 2.1 (a) and Lemma 2.2 (a) , we obtain that

$$I \le \frac{C}{\lambda}\|f\|_{M_{1,k}(w)} w(B)^k$$

For the term II, by an elementary estimate we get

$$w\left(\left\{x \in B: |Tf_2(x)| > \frac{\lambda}{2}\right\}\right) \le \frac{C}{\lambda}\int_{\left\{x\in B: |Tf_2(x)| > \frac{\lambda}{2}\right\}} |Tf_2(x)| \quad w(x)\, dx.$$ After noting that

$$|Tf_2(x)| \le C\sum_{j=1}^{\infty} \frac{1}{|2^j B|}\int_{2^j B} |f \quad (y)| \quad dy\ ,$$

we have

|

$$II \le \frac{C}{\lambda}\|f\|_{M_{1,k}(w)} w(B)^k$$

as desired.

For the proof of (b)**,** it suffices to show that

$$\frac{1}{w(B)^k}\int_B |Tf(x)|^p w(x)dx \le C\|f\|_{M_{p,k}(w)}.$$

Let $f = f_{\chi 2B} + f_{\chi(2B)^c} =: f_1 + f_2$ and hence $Tf\ (x) = Tf_1(x) + Tf_2(x)$. Since T is a linear operator, one has

$$\frac{1}{w(B)^k}\int_B |Tf(x)|^p w(x)dx \le \frac{1}{w(B)^k}\int_B |Tf_1(x)|^p w(x)dx + \frac{1}{w(B)^k}\int_B |Tf_2(x)|^p w(x)dx$$

$$:= J + JJ$$

For the term JJ, by Hölder's inequality,

$$\int_{2^jB} |f\ (y)|\ dy \le C\|f\|_{M_{p,k}(w)} |2^{j+1}B| w(2^{j+1}B)^{\frac{1}{p}(k-1)}$$

Lemma 2.1(b) implies that

$$JJ \le C\|f\|_{M_{p,k}(w)}\ .$$

Therefore, the proof of Theorem 1.2 is completed.

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