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On Q*s - regular spaces and Q*s - normal spaces

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Abstract

The notion of Q* - open sets in a topological space was introduced by Murugalingam and Lalitha [7]. We introduce the notion of Q*s - regular, Q*s - normal, s*Q* - normal and obtain some characterizations Q*s - regularity and Q*s – normality, s*Q* - normal.

Keywords: Q*s - regular; Q*s - normal; s*Q* - normal; Q* - normal.

1. Introduction :

In 1963, Levine introduced the concept of semi - open sets. Since then , a considerable number of papers discussing separation axioms, essentially by replacing open sets by semi-open sets, have appeared in the literature. For instance, Maheshwari and Prasad introduced semi-T_0, semi-T_1, semi-T_2, s - normality and s - regularity as a generalization of T_0, T_1, T_2, regularity and normality axioms respectively, and investigated their properties. The notion of semi-open sets was used by Maheshwari and Prasad to introduce pairwise semi-T_0, pairwise semi-T_1, pairwise semi-T_2, pairwise s - regular and pairwise s-normal spaces. Moreover, s - normal (resp. semi normal) spaces were introduced and studied by Maheshwari and Prasad [6] (resp. Dorsett [3]). The concept of g - closed sets was also considered by Dunham and Levine in 1980. In 2002, Rao and Joseph introduced the concept of s*g - closed sets. The notion of Q* - open sets in a topological space was introduced by Murugalingam and Lalitha [7]. We introduce the notion of Q*s - regular, Q*s - normal and obtain some characterizations Q*s - regularity and Q*s - normality.

2. Preliminaries :

Definition 2.1 [13]: A space X is said to g**s - regular** if for every g - closed set F and a point $x \notin F$, there exist disjoint semi-open sets U and V such that $x \in U$ and $F \subseteq V$.

Definition 2.2: A space X is said to ***s*** **- normal [6] (resp. semi normal [3])** if for any two disjoint closed sets A and B, there exist disjoint semi open sets U and V such that $A \subseteq U$ and $B \subseteq V$.

Definition 2.3 [5]: A space X is said to s* **- normal** if for any two disjoint semi - closed sets A and B, there exist disjoint semi open sets U and V such that $A \subseteq U$ and $B \subseteq V$.

Definition 2.4 [12]: A space X is said to g***s*** **- normal** if for any two disjoint g - closed sets A and B, there exist disjoint semi open sets U and V such that $A \subseteq U$ and $B \subseteq V$.

Definition 2.5 [7] - Let (X , τ) be a topological space . The set of all Q* - closed sets with X is a topology. It is denoted by $\tau_{Q^*} = \sigma^*$.

3. Q*s - regular spaces

Definition 3.1 : A space X is said to **Q*s - regular** if for every Q* - closed set F and a point $x \notin F$, there exist disjoint semi-open sets U and V such that $x \in U$ and $F \subseteq V$.

Clearly, every Q*s - regular space is gs - regular but converse is not true.

Example 3.1 : Let X = { a , b , c }, τ = { ϕ , X , { a }, { b } , { a , b } }

SO (X , τ) = { ϕ , X , { a } , { b } , { b , c } ,{a, c}, {a , b} } and σ^* = { ϕ , X , { a , b } }.Then the space X is Q***s -** regular **.**

Example 3.2 : Let X = { a , b , c } , τ = { ϕ , X , { c } , { a , c } } . SO (X , τ) = { ϕ , X , { b , c } , { a , c } , { c }. GC (X , τ) = { ϕ , X , { b } { a , b }, { b , c } } . Hence the space is gs - regular but not Q*s - regular. Here { b , c } is not Q* closed.

Theorem 3.1 : For a space X , the following are equivalent:

a) X is Q*s - regular.

b) For each $x \in X$ and every Q* - open set U containing x , there exists a semi - open set G such that $x \in G \subseteq scl\, G \subseteq U$.

c) For every Q* - closed set F, the intersection of all semi - closed semi neighborhoods of F is exactly F.

d) For every set A and a Q* - open set B such that $A \cap B \neq \phi$, there exists a semi open set G such that $A \cap G \neq \phi$ and $scl\, G \subseteq B$.

e) For every non empty set A and any Q* - closed set B satisfying $A \cap B = \phi$, there exist disjoint semi - open sets G and M such that $A \cap G \neq \phi$, and $B \subseteq M$.

Proof :

(a) → (b) : Let $x \in U$ and U is Q* - open in X. Therefore, $x \notin X - U$ and X – U is Q* - closed in X . Since X is Q*S - regular , there exist disjoint semi - open sets G and H such that $x \in G$ and $X - U \subseteq H$. Now $G \subseteq X - H \subseteq U$. Since H is semi - open , therefore scl (X – H) = X – H. Hence $x \in G \subseteq scl\, G \subseteq U$.

(b) → (c) : Let F be a Q* - closed subset of X and $x \notin F$. Then X – F is a Q* - open set containing x. Therefore, by (b) there exists a semi - open set G such that $x \in G \subseteq scl\, G \subseteq X - F$. Hence, $F \subseteq X - scl\, G \subseteq X - G$ and $x \notin X - G$. Thus X – G is a semi - closed semi - neighborhood of F which does not contain x. Hence, the intersection of all semi - closed semi - neighborhoods of F is exactly F.

(c) → (d) : Let A be a non empty subset of X and B be a Q* - open set such that $A \cap B \neq \phi$. Let $x \in A \cap B$. Then X – B is a Q* - closed such that $x \notin X - B$. Therefore, by (c), there exists a semi - closed semi - neighborhood of X – B, say V, such that $x \notin V$. Thus for the semi - closed set V, there exists a semi - open set U such that $X - B \subseteq U \subseteq V$. Take G = X – V. Then G is a semi - open set containing x. Also $A \cap G \neq \phi$. Now, $scl\, G = scl\,(X - V) \subseteq X - U \subseteq B$. Hence $scl\, G \subseteq B$.

(d) → (e) : Let $A \cap B = \phi$, where A is non empty and B is a Q* - closed , then $A \cap X - B \neq \phi$, where X – B is a Q* - open set. Therefore by (d), there exists a semi open set G such that $A \cap G \neq \phi$, and $G \subseteq scl\, G \subseteq X - B$. Now, put M = X – scl G. Then $B \subseteq M$ and G and M are semi - open sets such that $G \cap M = \phi$.

(e) → (a) : Let F be a Q* - closed subset of X and $x \notin F$. Then { x } and F are disjoint.

Therefore by (e), there exist disjoint semi - open sets G and M such that $\{ x \} \cap G \neq \phi$, and $F \subseteq M$. Thus $x \in G$ and $F \subseteq M$. Hence X is Q*s - regular.

Definition 3.2 : A space X is said to Q*- **symmetric** if { x } is Q* - closed for each $x \in X$.

Theorem 3.2 : Every Q*s - regular Q*- symmetric space is semi - T_2.

Proof : Let x, y be any two distinct points of X. Since X is Q* - symmetric implies { x } is Q* - closed. Also $y \notin \{ x \}$. Since X is Q*s - regular, there exist semi - open sets U and V such that $x \in V$, $y \in U$ and $U \cap V = \phi$. Hence X is semi - T_2.

Theorem 3.3 : Let f : X → Y is a homeomorphism . Then X is Q*s - regular if and only if Y is Q*s - regular.

Proof : Let Y be Q*s - regular and let G be any Q* - closed set in X such that $x \notin G$. Then $y \notin f (G)$, where f (G) is a Q* - closed set in Y. By Q*s - regularity, there exist disjoint semi-open sets U and V in Y such that $y \in U$ and $f (G) \subseteq V$, which implies that $x \in f^{-1}(U)$ and $G \subseteq f^{-1}(V)$. In addition, $f^{-1}(U)$ and $f^{-1}(V)$ are semi - open sets , since f is homeomorphism implies f is semi - homeomorphism implies f is irresolute. Hence X is Q*s - regular. Also $f^{-1}(U) \cap f^{-1}(V) = \phi$. Hence, X is Q*s - regular.

Conversely, Let X be Q*s - regular. Let F be any Q* - closed subset in Y such that $y \notin F$. Then $x \notin f^{-1}(F)$, where $y = f(x)$ and $f^{-1}(F)$ is Q* - closed , since f is homeomorphism. By Q*s -regularity, there exist disjoint semi - open sets U and V such that $x \in U$, $f^{-1}(F) \subseteq V$. Hence, $y \in f(U)$ and $F \subseteq f(V)$. However, f (U) and f (V) are semi - open sets; since f is homeomorphism implies f is semi homeomorphism, implies f is pre-semi open. Also $f(U) \cap f(V) = \phi$. Hence, Y is Q*s - regular.

4. Q*S - normal spaces

By replacing g closed sets by Q *- closed sets in gs – normality due to sharma [shar], we introduce a new concept of Q*s - normality.

Definition 4.1: A space X is said to **Q *- normal** if for any two disjoint Q*- closed sets A and B, there exist disjoint Q*- open sets U and V such that $A \subseteq U$ and $B \subseteq V$.

Definition 4.2 : A space X is said to **Q*S - normal** if for any two disjoint Q*- closed sets A and B, there exist disjoint semi - open sets U and V such that $A \subseteq U$ and $B \subseteq V$.

Example 4.1: Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{a, c\}, \{a, b\}\}$

$SO(X, \tau) = \sigma^* = \{\phi, X, \{a\}, \{a, c\}, \{a, b\}\}$. Then the space X is Q*s - normal.

Example 4.2: Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a, c\}, \{b, c\}\}$. $SO(X, \tau) = \{\phi, X, \{a, c\}, \{b, c\}\}$. Hence the space X is Q*S - normal .

Definition 4.3: A space X is said to be **S*Q* - normal** if for every pair of disjoint semi closed sets A & B in X , there exists disjoint Q* open sets U & V such that $A \subset U$ and $B \subset V$.

Definition 4.4 : (i) A space (X, τ) is called *weakly* **Q*** - *normal* if disjoint **Q*** - closed sets can be separated by disjoint closed sets.

(ii) A function $f : (X, \tau) \to (Y, \sigma)$ is said to be *always* **Q*** - *closed* if the image of each **Q*** - closed set in (X, τ) is **Q*** - closed in (Y, σ) .

Example 4.3: In example 3.2 , X is S*Q* - normal **.**

Theorem 4.1: For a space X the following are equivalent:

a) X is Q*S - normal.

b) For each Q*closed set F and a Q* - open set K containing F, there exists a semi open set U such that $F \subseteq U \subseteq scl\, U \subseteq K$.

c) For every Q* closed set A and a Q* closed set B disjoint from A, there exists a semi – open set U containing A such that $scl\, U \cap B = \phi$.

Proof :

a) → b) : Let X be Q*s - normal and let K be a Q* - open set containing a Q* - closed set F. Then F and X – K are disjoint Q* - closed sets. So by (a), there exists a semi-open set U and a semi-open set V such that $F \subseteq U$, $X - K \subseteq V$ and $U \cap V = \phi$. Thus $U \subseteq X - V$, which implies that $scl\, U \subseteq X - V$. Hence, $F \subseteq U \subseteq scl\, U \subseteq K$.

b) → c) : Let A and B are Q* - closed subsets of X such that $A \cap B = \phi$, which implies $A \subseteq X - B$, a Q* - open set. So by (b) , there exists a semi-open set U such that $A \subseteq U \subseteq scl\, U \subseteq X - B$. Hence, $scl\, U \cap B = \phi$.

c) → a) : Let A and B be disjoint Q* - closed sets. Then, by (c), there is a semi - open set U such that $A \subseteq U$ and $scl\, U \cap B = \phi$. Now scl U is semi-closed. Hence, $B \subseteq X - scl\, U$, let $V = X - scl\, U$. Then V is a semi-open set such that $B \subseteq V$ and $U \cap V = \phi$. Hence, X is Q* s - normal.

Theorem 4.2 : For a space X the following are equivalent:

a) X is S*Q* - normal.

b) For each semi - closed set F and a semi - open set K containing F, there exists a Q* - open set U such that $F \subseteq U \subseteq \sigma^*\text{-}cl(U) \subseteq K$.

c) For every semi - closed set A and a semi closed set B disjoint from A, there exists a Q*- open set U containing A such that σ^* - cl (U) $\cap$ B = ϕ .

Proof :

a) → b) : Let X be S*Q* - normal and let K be a semi - open set containing a semi - closed set F. Then F and X – K are disjoint semi - closed sets. So by (a), there exists a Q* - open set U and a Q* - open set V such that F $\subseteq$ U, X – K $\subseteq$ V and U $\cap$ V = ϕ. Thus U $\subseteq$ X – V, which implies that σ^* - cl (U) $\subseteq$ X – V . Hence, F $\subseteq$ U $\subseteq$ σ^* - cl (U) $\subseteq$ K .

b) → c) : Let A and B are semi - closed subsets of X such that A $\cap$ B = ϕ , which implies A $\subseteq$ X – B , a semi - open set. So by (b), there exists a Q* - open set U such that A $\subseteq$ U $\subseteq$ σ^* - cl (U) $\subseteq$ X – B. Hence, σ^* - cl (U) $\cap$ B = ϕ.

c) → a) : Let A and B be disjoint semi - closed sets . Then , by c) , there is a Q* - open set U such that A $\subseteq$ U and σ^* - cl (U) $\cap$ B = ϕ . Now σ^* - cl (U) is Q* - closed. Hence, B $\subseteq$ X – σ^* - cl (U), let V = X – σ^* - cl (U) . Then V is a Q* - open set such that B $\subseteq$ V and U $\cap$ V = ϕ . Hence, X is S*Q* - normal.

Theorem 4.3 : Every Q* s - normal Q* - symmetric space X is Q*s - regular.

Proof: Let F be a Q* - closed subset of X with x $\in$ F. Since X is Q* - symmetric so{ x }is Q* - closed. So { x } and F are disjoint Q* - closed sets in X. Since X is Q*S - normal , there exist disjoint semi - open sets U and V such that { x } $\subseteq$ U, F $\subseteq$ V. Hence X is Q*S - regular.

Theorem 4.4 : Every Q*- normal Q* - symmetric space X is Q* - regular.

Proof: Let F be a Q* - closed subset of X with x $\in$ F. Since X is Q* - symmetric so{ x }is Q* - closed. So { x } and F are disjoint Q* - closed sets in X. Since X is Q*- normal , there exist disjoint Q* - open sets U and V such that { x } $\subseteq$ U, F $\subseteq$ V. Hence X is Q*- regular.

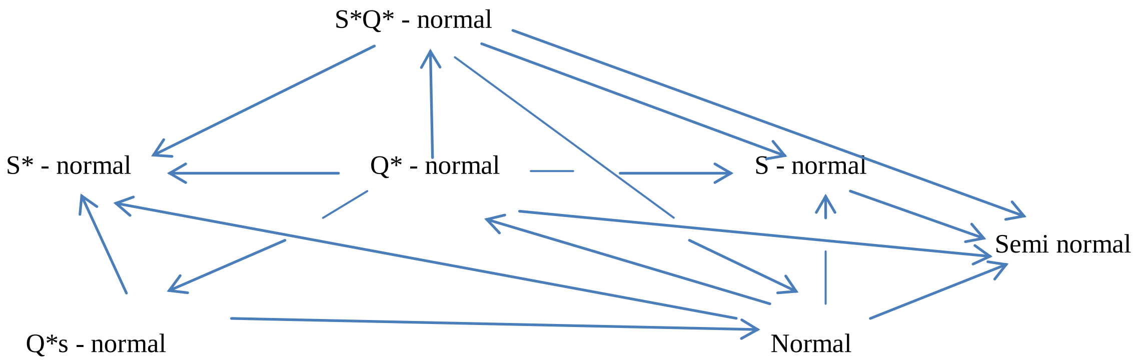
Theorem 4.3 : Let f : X → Y is a homeomorphism. Then X is Q*S - normal if and only if Y is Q*S - normal.

Proof : Let Y be Q*s - normal. Let A and B be two disjoint Q* - closed sets in X. Then f (A) and f (B) are Q*- closed sets in Y. Since Y is Q*s - normal , there exist disjoint semi - open sets U and V in Y such that f (A) $\subseteq$ U , f (B) $\subseteq$ V. Hence, A $\subseteq$ f^{-1} (U) , B $\subseteq$ f^{-1} (V) , and f^{-1} (U) $\cap$ f^{-1} (V) = ϕ as U $\cap$ V = ϕ . Moreover, f^{-1} (U) and f^{-1} (V) are semi - open sets; since f is homeomorphism implies f is semi homeomorphism implies f is an irresolute map. Hence X is Q*s - normal.

Conversely, Let X is Q*s - normal . Let A and B be two disjoint Q* - closed sets in Y. Then f^{-1} (A) and f^{-1} (B) are Q* - closed sets in X . Since X is Q*s - normal , there exist disjoint semi-open sets U and V in X such that f^{-1} (A) $\subseteq$ U, f^{-1} (B) $\subseteq$ V. Hence A $\subseteq$ f (U) , B $\subseteq$ f (V) , and f (U) $\cap$ f (V) = ϕ as U $\cap$ V = ϕ . However , f (U) and f (V) are semi - open sets ; since f is homeomorphism implies f is semi-homeomorphism implies f is pre - semi open. Hence, Y is Q*s -normal.

5. Comparison

Remark 5.1 : We summarize the relationship between various special types of normal spaces in the following diagram . None of the implications is reversible.



Theorem 5.1 : Every Q*S - normal space is S - normal.

Proof : Let X be a Q*s - normal space . To show that X is S - normal . Let A and B be two disjoint Q* closed sets. Since X is Q*s - normal, there exists disjoint semi - open sets U and V such that A $\subseteq$ U and B $\subseteq$ V. Since every Q* closed set is closed we have A and B are closed sets. Hence X is s - normal.

Remark 5.2 : Converse of the above theorem need not be true in general.

Example 5.2 : Let X = { a , b , c }, τ = { ϕ , X , { a } , { b } , { a , b } } .

SO (X , τ) = { ϕ , X , { a }, { b } , { a , c } , { a , b } , { b , c } } . Here { b , c } is closed but not Q* - closed . Hence the space X is S - normal but not Q*S - normal.

Theorem 5.2 : Every S*Q* - normal space is Semi - normal.

Proof : Let X be a S*Q* - normal space . To show that X is S - normal . Let A and B be two disjoint semi - closed sets . Since X is S*Q* - normal, there exists disjoint Q* - open sets U and V such that A $\subseteq$ U and B $\subseteq$ V. Since every Q* - open set is semi - open, there exists disjoint semi - open sets U and V such that A $\subseteq$ U and B $\subseteq$ V. Hence X is semi - normal.

Remark 5.3 : But the converse of the above theorem need not be true in general.

Example 5.3 : In example 5.2 , { b } is semi open but not Q* - open . Hence the space X is semi normal but not Q*s normal.

Theorem 5.3 : Every Q* - normal space is Q*s - normal.

Proof : Let X be a Q* - normal space . To show that X is Q*s - normal . Let A and B be two disjoint Q* - closed sets . Since X is pairwise Q* - normal , there exists disjoint Q* - open sets U and V such that A $\subseteq$ U and B $\subseteq$ V. Since every Q* - open set is semi - open , there exists disjoint semi - open sets U and V such that A $\subseteq$ U and B $\subseteq$ V. Hence X is Q*s - normal .

Remark 5.4 : But the converse of the above theorem need not be true in general.

Theorem 5.4 : Every Q*s - normal space is gs - normal.

Proof : Let X be a Q*s - normal space . To show that X is gs - normal . Let A and B be two disjoint Q* closed sets . Since X is Q*s - normal , there exists disjoint semi - open sets U and V such that A $\subseteq$ U and B $\subseteq$ V. Since every Q* closed set is g - closed we have A and B are g - closed sets . Hence X is gs - normal.

Remark 5.5 : Converse of the above theorem need not be true in general.

Example 5.4 : In example 3.2 , { b, c } is g closed but not Q* closed . Hence the space X is gs normal but not Q*s normal.

Conclusion :

The notion of Q*s - regular , Q*s - normal , s*Q* - normal in topological space has been generalized and obtain some characterizations Q*s - regularity and Q*s – normality , s*Q* - normal . These notions can be applied for investigating many other properties.

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