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A comparative effectiveness of stochastic approximation method and pseudo inversion method for solution to PDE with financial application

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Abstract.

This paper presents a comparative effectiveness of stochastic approximation method and pseudo inversion method for American option valuation under the Black-Scholes model. The stochastic approximation method and pseudo inversion method base its analysis on a drifted financial derivative system. With finer discretization, space nodes and time nodes, we demonstrate that the drifted financial derivative system can be efficiently and easily solved with high accuracy, by using a stochastic approximation method and pseudo inversion method. The stochastic approximation method proves to be faster in pricing an American options than the pseudo inversion method which needs the system to be stabilized for its accuracy. An illustrative example is given in concrete setting.

Keywords: Financial PDE; Stochastic algorithm; Drifted system; Option pricing; Spatial discretization; Pseudo inversion matrix.

MSC: 65C05, 65D30, 98B28.

1. Introduction

In this paper we compare the effectiveness of pricing an American options via two methods: Stochastic approximation and pseudo inversion method due to the use of the same system (drifted financial derivative system). During the last couple of decades, the trading of options has grown to tremendous scale. The most basic options give either the right to sell (put) or buy (call) the underlying asset with the strike price. European options can be exercised only at the expiry time while American options can be exercised any time before the expiry. Usually American options need to be priced numerically due to the early exercise possibility. One approach is to formulate a linear complementarity problem (LCP) or variational inequality with a partial (integro-)differential operator for the price and then solve it numerically after discretization. Another way is to discretize the Black-Scholes differential equation into system of ordinary differential equation and further transform into a drifted financial derivative system and then solve numerically using stochastic approximation method and pseudo inversion method.

For pricing options, a model is needed for the behavior of the value of the underlying asset. Many such models of varying complexity have been developed. More complicated models reproduce more realistic paths for the value and match between the market price and model prices of options, but they also make pricing more challenging. Black and Scholes (1973) discovered the partial differential equation which financial derivatives (the underlying assets of which are stocks) have to satisfy; furthermore, they found the evaluation formula when the financial derivative is a European call option. The partial differential equation is known as the Black-Scholes equation. Scholes obtained a Nobel Prize for economics in 1997 for this contribution. In this paper, we use Black-Scholes model to ascertain the behavior of the underlying asset.

The finite difference method for pricing American options was first presented in[1, 2, 3]. Jaillet et al. [4] showed the convergence of the finite difference method. Generally, there still exist some difficulties in using these numerical methods. For finite difference method, the difficulty arises from the early exercise property, which changes the original Black-scholes equation to an inequality that cannot be solved via fractional finite difference process. Therefore, finding the early exercise boundary prior to spatial discretization (discretization on underlying asset) is a must in each time step. Horng et al. [5] proposed a simple numerical method base on finite difference and method of lines to overcome this difficulty in American option valuation. Also,Osu and Solomon [6]proposed a stochastic approximation method for American option valuation based on the fact that financial derivative experience a drift which hardly can be brought to equilibrium state. By discretization of Black-Scholes equation using central finite-difference approximation into first-order ordinary differential equation and later transformed to a drifted financial derivative system and solve the resulting drifted financial derivative system by employing a stochastic algorithm described and analyzed in [6] where each iteration requires the adjustment of the drift parameter based on the dividend yield.

Although a comparism of different numerical methods for American options pricing have been discussed in [7, 8], a comparative effectiveness of stochastic approximation method and pseudo inversion method for American option valuation is proposed in this paper because the stochastic approximation method and pseudo inversion method base its' analysis on a drifted financial derivative system.

The outline of the paper is the following: In section 2 we review modeling of Black-Scholes, the partial differential equation which financial derivative have to satisfy and formulate Linear Complementary Problem (LCP) for an American option. In section, 3, we discretize the generic PDE into LCP and drift financial derivative system. Controllability and stability of a financial derivative system is presented in section 4. Numerical experiments are presented in section 5 and conclusions are given in section 6.

2. **Option Pricing Model**

Here, we consider the Black and Scholes Model [9] and Merton [10] and the partial differential equation which financial derivative (stock) have to satisfy. The Black-Scholes Model assumes a market consisting of a single risky asset (S) and a risky-free bank account (r). This market is given by the equations;

$$dS = \mu S dt + \sigma S dz \tag{1}$$

$$dB = rBdt\,. \tag{2}$$

Here (1) is a geometric Borwnian-Motion and (2) a non-stochastic. S is a Brownian-Motion, Z is a Wiener process μ is a constant parameter called the drift. It is a measure of the average rate of growth of the asset price. Meanwhile,σ , is a deterministic function of time. When σ is constant, (1) is the original Black-Scholes Model of the movement of a security, S. In this form $\mu$is the mean return of S, and σ is a variance. The quantity dZ is a random variable having a normal distribution with mean 0 and variance dt.

$$dZ \propto N(0, (\sqrt{dt})^2)\ .$$

For each interval $dt,\ dZ$ is a sample drawn from the distribution $N(0,(\sqrt{dt})^2)$, this is multiplied by σ to produce the term σdZ. The value of the parameters μ and σ may be estimated from historical data.

Under the usual assumptions, Black and Scholes [9] and Merton [10] have shown that the worth V of any contingent claim written on a stock, whether it is American or European, satisfies the famous Black-Scholes equation:

$$\frac{\partial V}{\partial t}+\frac{1}{2}\sigma^2 S^2\frac{\partial^2 V}{\partial S^2}+(r-q)S\,\frac{\partial V}{\partial S}-rV=0\,. \tag{3}$$

Where volatilityσ, the risk-free rate$r$, and dividend yield q are all assumed to be constants. The value of any particular contingent claim is determined by the terminal and boundary conditions. For an American option, notice that the PDE only holds in the not-yet-exercised region. At the place where the option should be exercised immediately, the equality sign in (3) would turn into an inequality one. That means the option value $V(S,t)$ at each time follows either $V(S,t)\ =\wedge\ (S,t)$ for the early exercised region or (3) for the not-yet-exercised region, where $\wedge\,(S,t)$ is the payoff of an American option at time t.

The generic form of (3) is derived by the change of variable $\tau=T-t$ to

$$\frac{\partial V}{\partial \tau}\ -\ \frac{1}{2}\sigma^2 S^2\frac{\partial^2 V}{\partial S^2}-(r-q)\ S\ \frac{\partial V}{\partial S}\ +\ rV=LV \tag{4}$$

where $V(.,\tau)\equiv V(.,T-\tau),\ \ \sigma\,(.,\tau)\equiv\ \sigma\,(.,T-\tau),\ \tau=0\ to\ \tau=T$

$S_{min}\ <S<S_{max}$, subject to the initial condition $V(S,0)=\ \wedge(S)$.

For the computations, the unbounded domain is truncated to

$$(S,t)\ \epsilon\ \ (0,S)\ x\ \ (0,T] \tag{5}$$

with sufficiently large $S\ \equiv\ S_{max}$.

The worth V of an American option under Black-Scholes model satisfies an LCP

$$\begin{cases} LV\ \geq 0 \\ V\ \geq\wedge \\ (LV)(V-\wedge)=0\,, \end{cases} \tag{6}$$

we impose the boundary conditions

$$\begin{cases} V(0,t)=0 \\ V(S,t)=\wedge\,(S),\ S\epsilon\,(0,S_{max}\,)\ \ . \end{cases} \tag{7}$$

Beyond the boundary $S=\ S_{max}$, the worth V is approximated to be the same as the payoff $\wedge$, that is $V\ (S,t)=\wedge\,(S)$ for $S\geq\ S_{max}$.

3 Discretizing the financial PDE for American option

American options can be exercised at any time before expiry. Formally, the value of an American put option with a strike price k is

$$V(0,k)=\sup(0\leq\tau^*\leq T{:}\,E(e^{-\tau\tau^*}(k-S_{\tau*})^+).$$

The optimal exercise time τ^* is the value that maximizes the expected payoff - any scheme to price an American must calculate this.

For American options with payoff $\wedge(s)$, the equivalent of equation (4) is

$$\begin{bmatrix} \frac{\partial V}{\partial \tau} - \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} - (r-q)S\,\frac{\partial V}{\partial S} + rV \geq 0 \\ V(S,T) \geq \wedge(S) \end{bmatrix}$$

$$\left[\frac{\partial V}{\partial \tau} - \frac{1}{2}\,\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} - (r-q)s\frac{\partial V}{\partial S} + rV\right][V - \wedge(S)] = 0\,. \tag{8}$$

Consider a uniform spatial mesh on the interval $[s_{min},\ s_{max}]$:

$S_j = S_{min} + j\delta S,\ j = O, 1, \ldots, n+1$, where

$$\delta s = \frac{S_{max} - S_{min}}{n+1},\quad S_{max} = S_0 \exp\left[\left(r - q - \frac{\sigma^2}{2}\right)T + 6\sigma\sqrt{T}\right]. \tag{9}$$

The truncated domain $\widehat{D}$ has the lower bound $S_{min} = 0$ and upper bound S_{max} as in (9).

Replacing all derivatives with respect to S by their central finite-difference approximations, we obtain the following approximation to the Black-Scholes PDE (8)

$$\frac{\partial V(\tau,S)}{\partial \tau} = \frac{1}{2}\ \sigma^2(S)\,S^2 \frac{V(\tau,\ S+\delta S) - 2V(\tau,S)\ +\ V\,(\tau, S-\delta S)}{\delta S^2}$$

$$+ (r-q)\ S\ \frac{V(\tau,\ S+\delta S) - V\,(\tau,\ S-\delta S)}{2\delta S} - rV\,(\tau,S) + O(\delta S^2). \tag{10}$$

Let $V_j(\tau)$ denote the semi-discrete approximation to $V(\tau, S_j)$. Applying (10) at each internal node S_j, we obtain the following system of first-order ordinary differential equations;

$$\frac{dV_j(\tau)}{d\tau} = \frac{1}{2}\left(\left(\frac{\sigma(S_j)S_j}{\delta S}\right)^2 - \frac{(r-q)S_j}{\delta S}\right)V_{j-1}(\tau) - \left(-\left(\frac{\sigma(S_j)S_j}{\delta S}\right)^2 - r\right)V_j(\tau)$$

$$+\ \frac{1}{2}\left(\left(\frac{\sigma(S_j)S_j}{\delta S}\right)^2 + \frac{(r-q)S_j}{\delta S}\right)V_{j+1}(\tau),\quad j = 1, 2, \ldots, n\,; \tag{11a}$$

with discretized form given as

$$\frac{dV_j(\tau)}{d\tau} = L_{j,j-1}V_{j-1}(\tau) - L_{j,j}V_j(\tau) + L_{j,j+1}V_{j+1}(\tau). \tag{11b}$$

System (11) has n equation in $n+2$ unknown functions, $V_0(\tau), V_1(\tau), \dots, V_n(\tau), V_{n+1}(\tau)$. Using the boundary conditions we have the functions $V_0(\tau)$ and $V_{n+1}(\tau)$ which respectively approximate the solution at the boundary nodes $S_0 = S_{min}$ and $S_{n+1} = S_{max}$ .As a result, the system of differential equations (11) can be written as the following matrix-vector differential equation with an n-by-n tri-diagonal coefficient matrix L whose entries are defined in (11)

$$\frac{dV(\tau)}{d\tau} = LV(\tau) + wG(\tau), \tag{12}$$

Subject to the initial condition

$$V(0) = \Lambda := [\Lambda(S_1)\ , \Lambda(S_2), \dots, \Lambda(S_{n)})]^T \ . \tag{13}$$

Here we use the notation:

$$L = \begin{pmatrix} L_{11} & L_{12} & 0 & \cdots & 0 & 0 \\ L_{21} & L_{22} & L_{23} & \cdots & 0 & 0 \\ 0 & L_{32} & L_{33} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & L_{n-1,n-1} & L_{n-1,n} \\ 0 & 0 & 0 & \cdots & L_{n,n-1} & L_{n,n} \end{pmatrix}, V(\tau) = \begin{pmatrix} V_1(\tau) \\ V_2(\tau) \\ \vdots \\ V_{n-1}(\tau) \\ V_n(\tau) \end{pmatrix}.$$

The vector $G(\tau) \epsilon\, R^n$ is given by

$$\left[\left(\frac{\sigma^2(S_0)S_0^2}{2\delta S^2} - \frac{(r-q)S_0}{2\delta S}\right) V_0(\tau), 0\ , \dots, 0, \left(\frac{\sigma^2(S_{n+1})S_{n+1}{}^2}{2\delta S^2} + \frac{(r-q)S_{n+1}}{2\delta s}\right)\ V_{n+1}(r)\right]^T.$$

$G\ (\tau)$ contains boundary values of the mesh solution.

The spatial discretization leads to:

Semi-discrete LCP, according to [11] from (9), (12) and (13), we have

$$\begin{cases} L^j V^{j+1} \geq g^j \\ V^{j+1} \geq \Lambda \\ (V^{j+1} - \Lambda)^T (L^j V^{j+1} - g^j) = 0 \end{cases}, \tag{14}$$

where L is n-by-n tri-diagonal coefficient matrix, g is a vector resulting from the second term in equation (11) V and Λ are vectors containing the grid point values of the worth V and the pay off Λ, respectively. This again must be solved at every time step. A crude approximation is to solve the system $L^j\ X = g^j$, then set $L^{j+1} = \max(X, \Lambda)$.

Drifted financial derivative system:

According to [12], $G(\tau)$ term in (12) can be treated as an enforced input to the financial derivative system, resulted from boundary condition, defined in (7). With zero boundary condition, equation (12) yields.

$$\dot{V} = LV\ , \tag{15}$$

which represents a pfaftian differential constraints (see [13] for pfaftian differential constraints) but not of kinematic nature arises from the conservation on non-zero financial derivatives. The transformed financial derivative system (15) can be re-expressed as

$$L\,V = d \tag{16}$$

System (16) represents a drifted financial derivative system with a drift term d. In such a system the derivative value V can be solved by computing the stochastic algorithm used by Osu and Solomon [6], and the pseudo inversion L^* of the positive definite matrix L expressed as follows

$$V = L^* d \tag{17}$$

in which

$$L^* = L(L^T L)^{-1} \tag{18}$$

4. Controllability and Stability of Financial Derivatives

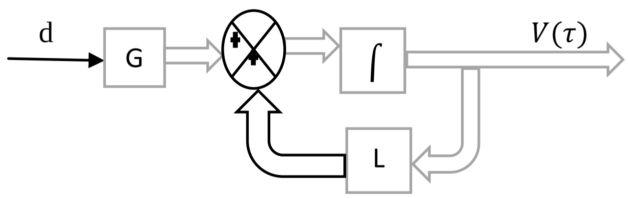

Figure 1. Open-loop financial controlled system

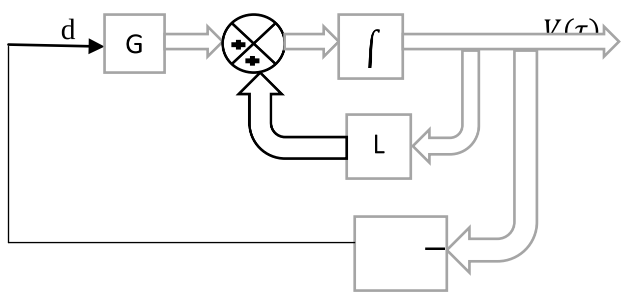

Figure 2. Closed-loop controlled financial system with $w = -Kv$

In the theory of linear time-invariant dynamical control systems the most popular and the most frequently used mathematical model is given by the following differential state equation (12). Stability, controllability, and observability introduced by Kalmanplay an essential role in the development of modern mathematical control theory. There are important relationships between stability, controllability, and observability of linear control systems. Controllability and observability are also strongly connected with the theory of minimal realization of linear time invariant control systems. If a control system is not stable, it is usually of no use in practice.

Consider the continuous-time system shown in figure 1, the system described in Equation (12) is said to be state controllable at $t = t_0$ if it is possible to construct an unconstrained control signal that will transfer an initial state to any final state in a finite time interval $t_0 \leq t \leq t_1$. If every state is controllable, then the system is said to be completely state controllable [14]. The system is said to be controllable if and only if the following nx n matrix is full rank n.

$$[G \quad LG \quad L^2G \quad \cdots \quad L^{n-1}G] \tag{19}$$

This matrix is called controllability matrix. A system is said to be observable at time t_0 , if with the system in state $v(t_0)$, it is possible to determine its state from the observation of the output over a finite time interval. The concept of observability is very important because, in practice, the difficulty encountered with state feed-back control is that some of the state variables are not accessible for direct measurement, with the result that it becomes necessary to estimate the unmeasurable state variables in order to construct the control signals. The system is said to be observable if and only if the following n x nm matrix is of full rank n.

This following analysis presents a design method commonly called the pole-placement technique. We assume that all state variables are measureable and are available for feedback. It is shown that if the system considered is completely state controllable, then poles of the closed- loop system may be placed at any desired locations by means of state feedback through an appropriate state feed-back gain matrix as displayed in Figure 2. Let us assume the desired closed-poles are to be at$s_1 = u_1, s_2 = u_2, \dots, s_n = u_n$.

We shall choose the control signal to be

$$w = -Kv \tag{20}$$

This means that the control signal is determined by an instantaneous state. Such a scheme is called state feed- back. The 1 x n matrix K is called the state feedback gain matrix. Substituting (20) into Equation (12) gives

$$\dot{v}(t) = (L - GK)v(t) \tag{21}$$

The solution of this equation is given by

$$v(t) = v(0)e^{(L-GK)t} \tag{22}$$

where v(0) is the initial state caused by external disturbances. The stability and transient response characteristics are determined by the eigenvalues of matrix $-GK$. If matrix K is chosen properly, the matrix $L - GK$ can be made asymptotically stable matrix. Define a transformation matrix T by

$$T = MW \tag{23}$$

where M is the controllability matrix (19) and

$$W = \begin{pmatrix} a_{n-1} & a_{n-2} & \cdots & a & 1 \\ a_{n-2} & a_{n-3} & \vdots & 1 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_1 & 1 & \dots & 0 & 0 \\ 1 & 0 & \dots & 0 & 0 \end{pmatrix}, \tag{24}$$

where the a_i's are the coefficients of the characteristic polynomial

$$|sI - L| = s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + a_n. \tag{25}$$

Let us choose a set desired eigenvalues as$s_1 = u_1, s_2 = u_2, \dots, s_n = u_n$.Then the desired characteristic equation becomes

$$(s - u_1)(s - u_2)\cdots(s - u_n) = s^n + \alpha_1 s^{n-1} + \cdots + \alpha_{n-1}s + \alpha_n. \tag{26}$$

The sufficient condition for the system to be completely controllable with all eigenvalues arbitrarily placed is by choosing the gain matrix

$$K = [(\alpha_n - a_n)(\alpha_{n-1} - a_{n-1})\cdots(\alpha_2 - a_2)(\alpha_1 - a_1)]T^{-1} \tag{27}$$

5. Numerical Experiment:

In our numerical example, we compare the effectiveness of stochastic approximation method and pseudo inversion method for pricing American put options. The parameters for the Black-Scholes model are the same as in [11] and they are defined below:

Table 1: Estimated parameters for the Black-Scholes model

Parameter	Notation	Value
Risk free interest rate	r	0.2
Dividend yield	q	0.1
Strike price	k	7
Volatility	σ	0.3
Time to expiry	T	2
Spot price	S_0	10
Ratio of Nodes	ϑ	30

We illustrate the method in a concrete setting, using the parameter in table 1 and substitute in (10 and 11), with time nodes $3x10^3$ and space nodes $9x10^4$ satisfying the ratio of nodes ϑ as stipulated, we have the financial matrix (3 by 3 tri-diagonal coefficient matrix) .

$$L = \begin{pmatrix} 0.2 & 0.05 & 0 \\ -0.1 & 0.2 & 0.1 \\ 0 & -0.15 & 0.2 \end{pmatrix} . \tag{28}$$

By using the equation of total investment return;

$$r = d + q \tag{29}$$

where$r$ is the risk adjusted discount rate for V (the worth); q is the dividend yield (or convenience yield in case of commodities) and d is the drift (or capital gain rate). Hence $d = 0.1$ for $q = 0.1$ and $d = 0.2$ for $q = 0.0$ (No dividend yield).

From (16), we have

$$\begin{pmatrix} 0.2 & 0.05 & 0 \\ -0.1 & 0.2 & 0.1 \\ 0 & -0.15 & 0.2 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} 0.2 \\ 0.2 \\ 0.2 \end{pmatrix}, \tag{30}$$

the actual solution by [11] is $V(S,t) = 1.171339$, the PDE result is 0.14459568, which Bjerksunet Stensland (2002) gives 0.14275. Approximations such as Bjerksunet and Stensland (2002) [23] are not accurate enough to test the accuracy of the finite different scheme. The stochastic approximation method in [16] starting at $V_0 = (0\ \ 0\ \ 0)$ gives after one iteration

$V_1 = (1.2\ \ 1.2\ \ 1.2), and\ \ V^*(S,t) = \ 1.2,$ for both values of the drift, which gives a fixed point. This solution is the same as in [11]. Using the financial matrix (28) in the pseudo inversion method (18)we have

$$L^* = \begin{pmatrix} 0.3 & 0.8 & 1.3 \\ 1.3 & 0.03 & 0.3 \\ 0.4 & 0.8 & -0.6 \end{pmatrix}, \tag{31}$$

applying the inversion matrix L^* to (17) gives $V_1 = (0.48\ \ 0.33\ \ 0.12)$, and$V^*(S,t) = \ 0.31$ for both values of the drift, and is not a fixed point and also not equal to the solution in [16]. It is desired to check the controllability condition (19). It can be easily validated that the controllability matrix

$$M = \begin{pmatrix} 1 & 0.25 & 0.06 \\ 1 & 0.2 & 0.02 \\ 1 & 0.05 & -0.02 \end{pmatrix} \tag{32}$$

is of full rank 3. Since the system is controllable the pole placement design can be implemented to stabilize the system. Placing the pole $s_1 = -1, s_2 = -2, and\ s_3 = -3$ alongside with the original system (28) we have the original and desired characteristic equations from (25) as

$$a(s) = |sI - L| = s^3 - 0.6s^2 + 0.14s - 0.01 \tag{33}$$

$$\alpha(s) = |sI - A| = s^3 + 6s^2 + 11s + 6\ , \tag{34}$$

where A is the pole placement diagonal matrix. The transformation matrix(23) is

$$T = \begin{pmatrix} 1 & 0.25 & 0.06 \\ 1 & 0.2 & 0.02 \\ 1 & 0.05 & -0.02 \end{pmatrix}\begin{pmatrix} 0.14 & -0.6 & 1 \\ -0.6 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0.05 & 0.35 & 1 \\ 0.04 & -0.4 & 1 \\ 0.09 & -0.55 & 1 \end{pmatrix}. \tag{35}$$

The sufficient condition for the system to be completely controllable with all eigenvalues arbitrarily placed is by using the gain matrix (27)

$$K = [(6 + 0.01)(11 - 0.14)(6 + 0.6)]\begin{pmatrix} 37.5 & -50 & 12.5 \\ 12.5 & -10 & -2.5 \\ 3.5 & -1 & -1.5 \end{pmatrix}$$

$$K = [384.23\ \ -415.7\ \ \ 38.08]. \tag{36}$$

Normalizing the gain matrix yield

$$K^* = \ [3.8423\ \ -4.157\ \ \ 0.3808] \tag{37}$$

For a negative feedback controlled financial system as shown in Figure 2, according to [12] it implies that to stabilize such a system, the drift parameter d should increase the stock by 4.157 times (from 0.2 to 0.8), the risk free rate r should be decreased by 3.8423 times (from 0.2 to −0.77) and the volatility should also be decrease by 0.3808 times (from 0.3 to -0.11). From physical point of view, the negative sign is to balance the increase of the stock and comply with the conservation of financial money. Some systems reveal a conservation nature such as mechanical systems which comply with the principle of conservation energy.

Applying the stability condition to the pseudo inversion method (17) using (31) we have

$V_1 = (1.92\ \ 1.304\ \ 0.48)$, and $V^*(S,t) =\ 1.2347(V^*(S,t) \approx 1.2)$.This shows that a stochastic approximation method and pseudo inversion method can be used on a discretized financial PDE to price an American option and European option with a considerable success.

6. Conclusion

In this paper we compare the effectiveness of stochastic approximation and pseudo inversion methodon a drifted financial derivative system for pricing American options under the Black-Scholes model.For the Black-Scholes partial derivative, we employed central finite-difference approximation into first-order ordering differential equation and later transformed to a drifted financial derivative system. In numerical experiment, we formed a financial matrix and the value of the drift parameter using Table 1.With finer discretization, space nodes and time nodes, we demonstrate that the drifted financial derivative system can be efficiently and easily solved with high accuracy, by using a stochastic approximation method and pseudo inversion method. The stochastic approximation method proves to be faster in pricing an American options than the pseudo inversion method which needs the system to be stabilized for its accuracy.

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