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Estimations of Dzjadyk-Tamrazov Type in L_p

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Abstract

In the given paper we give auxiliary results allowing proving the main theorem connecting weight norms of derivatives of polynomials with weight norms of polynomials themselves.

Keywords: Jordan rectifiable curves; weight functions; **a** complex plane.

1. Introduction

In the paper [1] the estimations connecting the norms of derivatives of polynomials with the norms of the polynomials themselves with different weight functions are cited in the terms of singular characteristics.

In the metric C in a complex plane these estimations were obtained in the papers of V.K. Dzjadyk and P.M. Tamrazov. In the metric L_p similar estimations were obtained in the paper of J.I. Mamedkhanov [2]. In the given paper that complements the paper [1] similar estimations were obtained in a more wide class of curves. We give some necessary notions, facts and definitions.

Let Γ be a closed curve in a complex plane with parametric representation $z = z(t)$ ($0 \le t \le l$, l is length of Γ), the function $z = \psi(\omega)$ map the exterior of a unit circle γ_0 onto the exterior of Γ and $z = \psi_0(\omega)$ map the interior of a unit circle γ_0 onto the interior of Γ, the functions be inverse to the functions $w = \varphi(z)$ and $w = \varphi_0(z)$, respectively, $z = \psi(\omega)$ and $z = \psi_0(\omega)$, $\Gamma_{1+\rho}$ be a level of the curve Γ corresponding to the equation $|\varphi(z)| = 1+\rho \quad (\rho > 0)$.

Let t be some fixed point on $\Gamma_{1+\rho}$ $(\rho > 0)$, $d(t,\Gamma) = d$ be a distance from the point t to the curve Γ, $\Gamma_\delta(t) = \{z \in \Gamma : |z-t| < \delta\}$, $\theta_t^*(\delta) = \theta_t^*(\delta,\Gamma) = mes\, \Gamma_\rho(t)$. Obviously, $\theta_t^*(\delta) \uparrow$ as $\delta \uparrow$, $\delta \in (0,d)$.

We denote by J_γ a class of Jordan rectifiable curves Γ for which the relation

$$\tilde{d}^{\,\gamma-1}\left(t,\frac{1}{n}\right) \cdot \int\limits_\Gamma \frac{|dz|}{|z-t|^\gamma} \le C(\Gamma,\gamma)$$

is valid ($\tilde{d}$ is a distance from the point $t \in \Gamma_{1+1/t}$ to the curve Γ) for the given $\gamma > 1$ and all $t \in \Gamma_{1+1/n}$, and J_γ is a class of curves J_γ^* for which $\dfrac{\theta_t(\delta,\Gamma)}{\delta}$ almost decreases.

Let's consider a class of curves Γ for which $\theta_t^*(\delta) \le C(\Gamma)\delta$ for $\delta \ge 2d$. If we denote this class of curves by S_θ^*, it is easy to show that the class S_θ^* coincides with the class S_θ introduced by V.V.Salayev [3], i.e. the following statements [1] are true.

Statement 1. $S_\theta = S_\theta^*$.

Statement 2. If $1<\gamma_1<\gamma_2$, that $J_{\gamma_1} \subset J_{\gamma_2}$.

Statement 3. The imbeddings

$$S_\theta \subset J_\gamma ,$$

$$J_\gamma^* \subset S_\theta .$$

Now, let's consider the quantity

$$\delta\left(z, \frac{1}{n}\right) = \left(\int_{\Gamma_{1+t}} \frac{|dt|}{|z-t|^2} \right)^{-1}, \ z \in \Gamma .$$

The statements 1, 2 and 3 will help us in future to prove the following theorem formulated in [1].

2. Results and Discussion

Theorem 1. Let Γ be an arbitrary rectifiable Jordan curve on which for any $s \in (0, \infty)$ and any natural j it is valid the estimation

$$\tilde{\delta}^s\left(t, \frac{1}{n}\right) \int_\Gamma \frac{\delta^{j-s}\left(z, \frac{1}{n}\right)|dz|}{|z-t|^{j+1}} \preccurlyeq 1, \ t \in \Gamma_{1+\frac{1}{n}} ,$$

where

$$\tilde{\delta}\left(t, \frac{1}{n}\right) = \left(\int_\Gamma \frac{|dz|}{|z-t|^2} \right)^{-1} .$$

Then

$$\left\| \delta^{j-s}\left(z, \frac{1}{n}\right) P_n^{(j)}(z) \right\|_{L_p(\Gamma)} \preccurlyeq \left\| \delta^{-s}\left(z, \frac{1}{n}\right) P_n(z) \right\|_{L_p(\Gamma)} ,$$

where $P_n(z)$ is an algebraic polynomial of degree $n \in N$, $p \ge 1$. (The sings $\asymp$, $\preccurlyeq$ determine ordinal relation).

To prove this theorem we'll need the following theorem of independent interest.

Theorem 2. Under the conditions of theorem 1 on the curve Γ, whatever were natural number j and $s \in (-\infty, \infty)$ for the j-th order derivative of the polynomial $P_n(z)$ of degree $\le n$ for $p \ge 1$ it is valid the inequality

$$\left\| \frac{P_n^{(j)}(t)}{\tilde{\delta}^{s-j}\left(t, \frac{1}{n}\right)} \right\|_{L_p\left(\Gamma_{1+\frac{1}{n}}\right)} \le C(\Gamma, p, j, s) \left\| \frac{P_n(z)}{d^s\left(z, \frac{1}{n}\right)} \right\|_{L_p(\Gamma)} .$$

To prove this theorem we'll need the following lemma that we'll prove in the given paper.

Lemma. Under the conditions of theorem 1 on the curve Γ for the polynomial $P_n(z)$ of defree $\le n$ and $s \in (-\infty, \infty)$ it is valid the inequality

$$\left\| \frac{P_n(t)}{\tilde{\delta}^{s}\left(t, \frac{1}{n}\right)} \right\|_{L_p\left(\Gamma_{1+\frac{1}{n}}\right)} \le C(\Gamma, p, s) \left\| \frac{P_n(z)}{\delta^s\left(z, \frac{1}{n}\right)} \right\|_{L_p(\Gamma)} .$$

Proof. At first we consider the case $s \ge 0$. We introduce the auxiliary function

$$s(z)=\frac{P_n(z)}{(\tilde{z}-z)[\varphi(z)]^n}$$

where $\tilde{z}=\tilde{z}\left(\frac{1}{n}\right)=\psi\left[\left(1+\frac{1}{n}\right)\varphi(z)\right]$.

Since $|\tilde{z}-z|\to\infty$ as $z\to\infty$, then $s(z)\to 0$ as $z\to\infty$. Therefore, each branch is analytic and one-valued out of the domain G. Consequently, for $s(z)$ we'll have

$$\left\|\frac{P_n(t)}{(\tilde{t}-t)^s\varphi^n(t)}\right\|_{L_p\left(\Gamma_{1+\frac{1}{n}}\right)}\le C(p)\left\|\frac{P_n(z)}{(\tilde{z}-z)^s\varphi^k(z)}\right\|_{L_p(\Gamma)}.$$

Now, if we take into account $\left|\varphi^n(t)\right|^n=\left(1+\frac{1}{n}\right)^n\asymp 1$ and relation $|\tilde{z}-z|\asymp\delta\left(z,\frac{1}{n}\right)$, then for proving the lemma it suffices to show the validity of the relation

$$|\tilde{t}-t|\asymp\tilde{\delta}\left(t,\frac{1}{n}\right),$$

where $t\in\Gamma_{1+1/n}$.

Show that validity of this relation follows immediately from the arguments given in the paper [2]. Indeed, let

$$\omega=\varphi(t),\ \ \tilde{\omega}=\varphi(\tilde{t}),\ \ t\in\Gamma_{1+\frac{1}{n}},\ t=\psi\left(\left(1+\frac{1}{n}\right)^{-1}\varphi(t)\right),\ \ t\in\Gamma,\ \ \underset{\sim}{\omega}=\varphi(t).$$

It will follow from the obvious relation $|\tilde{\omega}-\omega|\asymp\left|\omega-\underset{\sim}{\omega}\right|$ that $|\tilde{t}-t|\asymp\left|\tilde{t}-\underset{\sim}{t}\right|$. Now, if we take into account that the relation $\left|\tilde{t}-\underset{\sim}{t}\right|\asymp\tilde{\delta}\left(t,\frac{1}{n}\right)$ is valid for the considered class of curves, the lemma in the case $s\ge 0$ is proved.

In the case $s<0$ the proof is conducted by means of similar arguments after introduction of the auxiliary function

$$s(z)=\frac{P_n(z)}{(\tilde{z}-z)^s\varphi^{n+|s|}(z)}.$$

The lemma is proved.

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