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Qualitative Short-Time (ST) Dynamical Systems Analysis of Changes in Smoke Patterns with Applications to other Dynamical Systems in Nature

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Abstract

The smoking of a cigarette is an obvious metaphor for the observation of qualitative short-time (ST) dynamical patterns in life as a function of heat, diffusion, and the eventual death of a system of molecules, which includes particles that make up common ingredients in a cigarette, including nicotine, tobacco, and the associated artificial chemicals used to deliver these materials into the blood stream. I use the basic materials associated with smoking a cigarette as a framework for exploring qualitative patterns observed in ST dynamical systems. The actual process of smoking a cigarette was used to test the following hypotheses: (1) Do the patterns of change over time of smoking a cigarette from start to finish demonstrate ST dynamical patterns that can be analyzed with simple time series (TS) analysis tools?(2) Does the qualitative ST dynamical behavior generated from smoking a cigarette follow a predictable pattern? Next, I place my qualitative observations within a quantitative framework. Lastly, I use results obtained from hypotheses (1) and (2) to propose how change over time in qualitative ST dynamical behavior of the simple act of smoking a cigarette can be applied to other experiments, especially experiments examining the qualitative and quantitative ST dynamics of patterns observed in naturally occurringsystems.

Key Words: Thermodynamics; Non-linear dynamics; Time-series analysis; Short-term dynamical behavior; Diffusion.

Introduction

Any study on the effects of a cigarette on qualitative ST dynamical behavior must begin with a brief overview of the role that thermodynamics plays in determining that behavior. Throughout the remainder of this paper, I use STbecause of the fact that the qualitative dynamical behavior of cigarette smoke occurs and disappears rapidly (often, in fractions of a second).

Clearly, the role of temperature has a profound impact on the ST dynamical behavior exhibited by smoking a cigarette. The zero or zeroth law of thermodynamics [4, 5, 10, 11] states the relationship that exists between change in the temperature of open and closed (adiabatic) systems and its effects on changes in energy through time [5]. The zeroth law of thermodynamics is best described by the following equation

$$\beta = \frac{1}{KT} \tag{1}$$

In equation (1), K = Boltzmann's constant, which is a measure of changes in the state of a system at low energy (low temperature (T) to a state of high energy (high T)), where the collision of molecules necessarily makes dynamical behavior more possible [5, 12]. Thus, T is a measure of the absolute temperature in degrees Kelvin, which simply measures temperature with respect to energy, and is separate from the other measures of temperature, namelyFahrenheit and Celsius [5, 10, 11]. In equation (1), T is the thermodynamical temperature, and it is a measure of the absolute temperature with T = absolute zero corresponding to the lowest possible temperature, and T = 373 K corresponding to the highest possible temperature [5]. Thus, β is a measure of the movement of molecules from a low energy state (low T) to a high energy state (high T)[12]. In terms of a cigarette, as it becomes hotter (as T increases), the smoke (molecules in motion) that is produced becomes more energetic and dynamical patterns are more likely to be observed. Thus, the likelihood of observing a qualitative ST dynamical event increases over the duration of smoking a cigarette.

The Diffusion of Smoke and Dynamical Behavior

The diffusion equation is a partial differential equation, which describes density dynamics in material undergoing diffusion. It is also used to describe processes exhibiting diffusive-like behavior[6].

This is covered in the standard convection-diffusion type of equation)[6]:

$$\frac{\partial \phi(\mathbf{r},t)}{\partial t} = \nabla \cdot \big[D(\phi,\mathbf{r})\ \nabla\phi(\mathbf{r},t)\big], \tag{2}$$

where $\phi(\mathbf{r},\, t)$ is the density of the diffusing material at location $\mathbf{r}$ and time t and $D(\phi,\, \mathbf{r})$ is the collective diffusion coefficient for density ϕ at location $\mathbf{r}$; and ∇ represents the vector differential operatordel. If the diffusion coefficient depends on the density then the equation is nonlinear, otherwise it is linear [6].

The diffusion equation can be trivially derived from the continuity equation, which states that a change in density in any part of the system is due to inflow and outflow of material into and out of that part of the system. Effectively, no material is created or destroyed:

$$\frac{\partial \phi}{\partial t} + \nabla \cdot \mathbf{j} = 0, \tag{3}$$

where **j** is the flux of the diffusing material. The diffusion equation can be obtained easily from this when combined with the phenomenological Fick's first law [19], which states that the flux of the diffusing material in any part of the system is proportional to the local density gradient:

$$\mathbf{j} = -D(\phi,\mathbf{r})\,\nabla\phi(\mathbf{r},t). \tag{4}$$

If drift must be taken into account, the Smoluchowski equation provides an appropriate generalization [7].

From equations (2-4), we can infer that some of the types of dynamical behavior that can be produced by a diffusion process could include the diffusion of smoke from a cigarette. From equation (2), if the diffusioncoefficientdepends on the density then the equation is nonlinear, a prerequisite for the types of dynamical behavior that one should observe. In smoke patterns generated by a single cigarette. Such behavior includes stable and unstable limit cycles, and the most prevalentbehavior generated by equation (2-4), are random walks [2]. These are the types of behavior that are observed for qualitativeST dynamical systems like smoke generated from a cigarette [8]. As I will demonstrate later in the paper, this type of behavior is robust, and occurred frequently and continuously over the duration of this study of the smoking of a single cigarette over time lengths of one minute up to twelve minutes.

Materials and Methods

The subject of the study was a 39-year-old male who had been smoking a minimum of 20 cigarettes (1 pack) up to 60 cigarettes (3 packs) per day for approximately 3 months. The cigarette brand smoked during the study wasnon-filter Natural American Spirit 100%, withadditive-free natural tobacco and organic menthol [9]. The ingredients listed on a pack include organic tobacco and organic menthol. Additional chemicals acting to transport the menthol and nicotine into the subjects blood stream are not listed on the package, nor are they listed on the American Spirit website [9].

Smoking occurred in the subject’s car with the windows closed and the heater running (on cold mornings and days) and with the driver and passenger windows completely open on warm days (with or without the heater running). Smoking times varied from 2 to 3 hour intervals up to 8 to 10 hour intervals. Ash produced by cigarettes was dispensed in an ash tray found in the front of the car. In-between smoking events, the subject left the car and returned to their apartment to use the restroom or get some food, which was either consumed in the apartment or in the car.

Methods for obtaining information about qualitative ST change in the dynamical behavior of cigarette smoke patterns was recorded for each smoking event (n = 50 events = 50 cigarettes), and the environmental condition in which the cigarettes were smoked was recorded for each smoking event (i.e. the smoking of one cigarette).

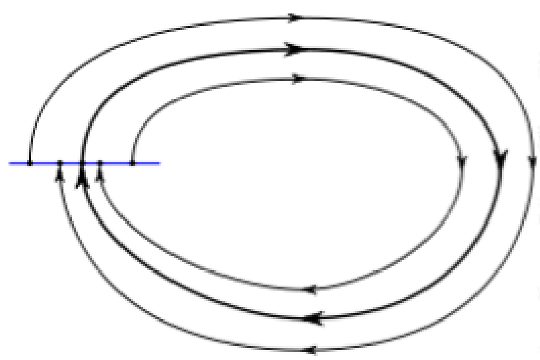
Measurements of the following particulars occurred over the duration of the three month study: (1) The length of time it took to smoke a single cigarette given the environmental conditions inside the car, and (2) The observed qualitative ST dynamical behavior generated by smoke emanating from a single cigarettevs. time.

The length of time it took to a smoke a single cigarette was recorded from measurements produced by (1) and the associated qualitative ST dynamical behavior for each data point was recorded.

Obviously, data was not collected for every smoking event over the 3-month duration of the study due to impracticability. As a consequence, n = 50 data points (50 smoking events) were collected and stored in an Excel spreadsheet.

Results

The duration of one smoking event (i.e. the smoking of 1 cigarette) varied from 1 minute up to 12 minutes. Illustrations of the observed dynamical behavior produced by one smoking event are given in Figure 1.



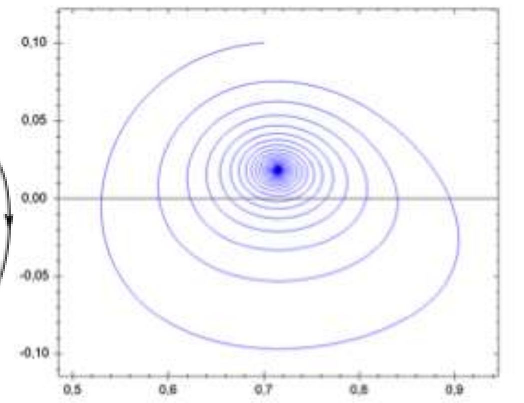


Figure 1(a) **Figure 1(b)**

Figure 1 (a,b). Illustrations of the observed qualitativeST dynamics exhibited by the smoking of cigarettes over a 3 month period. Figure 1(a) illustrates the dynamics of a stable limit cycle. Figure 1(b) illustrates the dynamics of an unstable limit cycle (axis labels are arbitrary)

One can observe the types of ST dynamical behavior that was exhibited for n = 50 events (50 smoking events) that took place for a duration of 1 minute to 12 minutes (figure 2) .

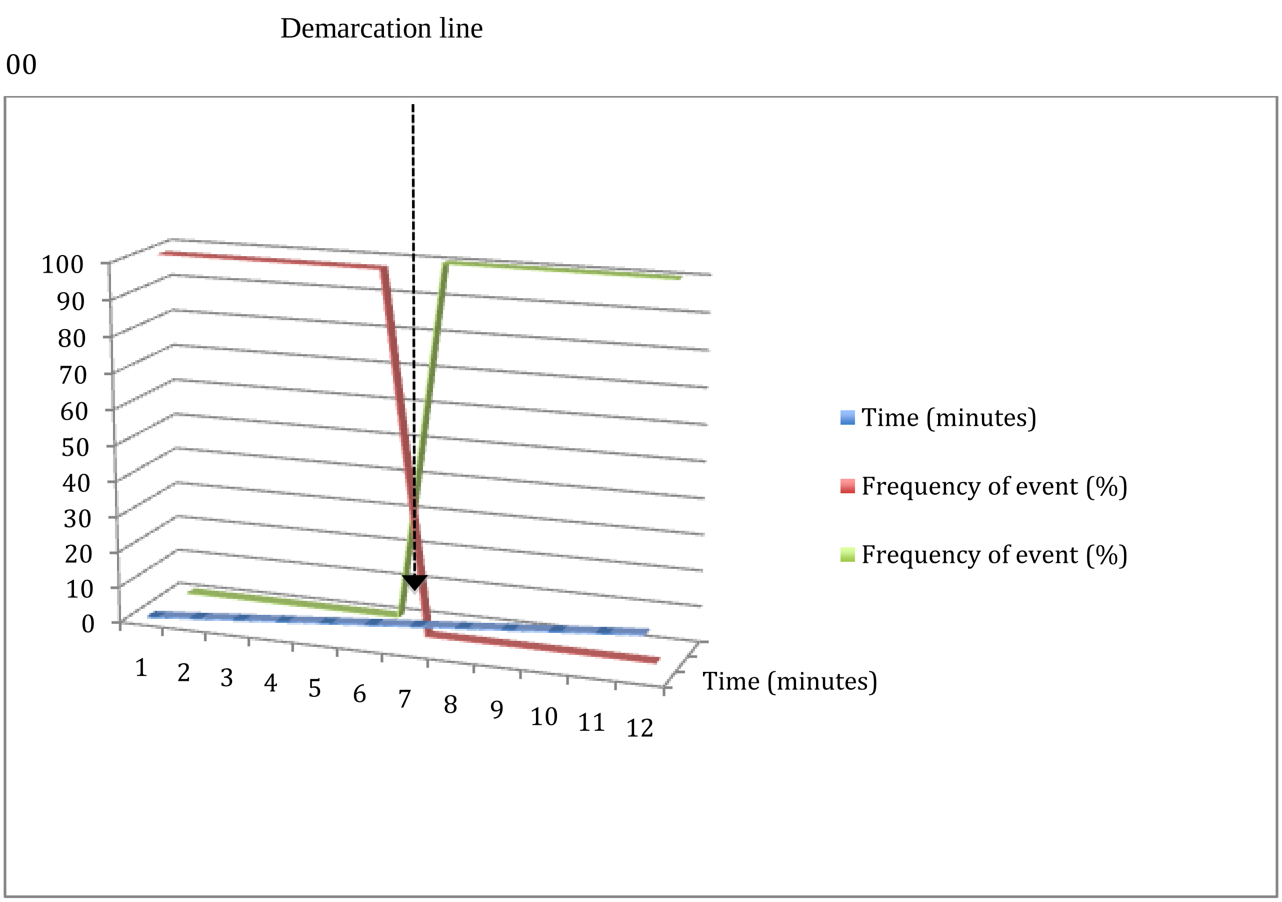


Figure 2. Observed dynamical behavior generated from the smoking of n = 50 cigarettes. The frequency of each event (%) is either 0 or 100. The blue line indicates the time of the event (t = 1…12 minutes). Each event represents n = 4 measurements, with exception of t = 6 minutes, where n = 6 measurements were taken. the red line indicates the frequency of stable ST dynamics, while the green line indicates the frequency of unstable ST events. The dashed line with an arrow indicates the demarcation point (time when the system switched from stable ST dynamics to unstable ST dynamics) as a function of time. Therefore, the dashed vertical line indicates the half-life of the cigarette (i.e., when half of it had been smoked or approximately 6 minutes) and hence, the demarcation point represents the time when the system's qualitative ST dynamical behavior changed from stable to unstable limit cycles.

A Mathematical Framework

The continuous differential equations that can be used to generate the dynamic behavior illustrated in figure 1 and observed in figure 2 are given by the analog equations

$$\frac{dx}{dt} = xg(x) - yp(x)$$
$$\frac{dy}{dt} = y(-\gamma + q(x)$$
$$(5a)$$

$$\frac{dx}{dt} = xg(x) - yp(x)$$
$$\frac{dy}{dt} = y[-r + cp(x)] - zq(y)$$
$$\frac{dz}{dt} = z[-s + mq(y)] \qquad (5b)$$

In equation 5a [24, 25], x can be identified as a predator for example, and its prey by y [26]. In equation 5a, g(x) is the predators "growth rate" and p(x) are the "competition coefficients" or death rates of x. Furthermore, γ is a term that measures the negative effects of intraspecific interactions amongst prey and q is the prey's growth rate. Qualitative and quantitative analysis of equation 5a shows that it generates stable limit cycles, whereas the same analysis of equation 6a shows that it generates unstable limit cycles [20], given certain initial values of the model's parameters.

Discussion and Conclusions

Although the data presented in this paper is qualitative (i.e., a function of observation only), I do not believe this falsifies the results that were obtained. Clearly, a more thorough quantitative analysis in future studies that start with the qualitative framework for studying dynamics of real-world phenomena is required to justify the qualitative nature of the data presented in this study.

However, all is not lost, as I believe that the falsifiability and predictivepower of this study could change if it wereplaced within a strong quantitative framework. For example, if one assumes that the diffusion of smoke from a single cigarette is a dynamical process, and that the diffusion of smoke is a function of the zero or first law of thermodynamics, one may also assume that as temperature in a the cigaretteincreases over time, energy increases, and the system changes from one of stability, to one of instability (higher molecular collisions through increased motion). In this study, the demarcation point where the system changes from one of stability to instability appears to occur after the cigarette has reached its half-life (see figure 2). This demarcation point between stability and instability (see figure 2), may be a function of the half-life of the cigarette, when half of it has been smoked, at which point one assumes there is a temperature shift from hot to hotter that results in a shift in the observed qualitative ST dynamical behavior of the system for stable to unstable.

Another aspect of the quantitative framework within which this study was performed is the use of the zero or first law of thermodynamics (equation 1), diffusion models (equations 2-4) and equations 5a-5b. Using this quantitative framework may help us understand that simple processes are best described, and have the most predictive power within a qualitative and quantitative framework that shows change in the system from one of stability, to one of unstable dynamics.

Applications

Embedding qualitative observations withina quantitative framework is one path that could lead future research that studies dynamical change in many naturally occurring systems to have high predictive power or operationality. Examples in the literature using this approach are course, however, it is the use of a simple experiment, such as observing the diffusion of smoke from a cigarette that could be used as a proxy by research from such fields as physics (small-scale), biology (small scale cellular processes and biochemical reactions) up to the scale of large ecosystems. Some examples from the literature are based on using the ST qualitative dynamics of a system include, but are not limited to:

(1) The investigation of Jump Markov processes, in which the differential equations used to predict behavior generated by the dynamics of the system have no memory of past events [23].

(2) The cell cycle, where several differential equation models have been developed including a generic eukaryotic cell cycle model which can represent a specific eukaryote. However, the qualitative dynamics generated from simulations of the model show sensitivity to initial conditions as a function of slight differences in the parameters in the model. Often, these models are simple analogs of equations 5(a) and 5(b) [15, 20], where their predictions match those of models designed to predict the existence of a limit cycle as a result of predator-prey interactions, in which both species reach an equilibrium point and are not susceptible to invasion by another species. When the parameters of the model are changed, such that predator-prey interactions lead to unstable limit cycles (i.e. the absence of an equilibrium point), the system is susceptible to invasion by another species [20].

(3) The application of mathematical models similar to those described by equations (2)-(4) and 5(a,b) to predict the qualitative dynamics of swarming behavior in insects [15]

(4) The application of mathematical models similar to those described by equation (1), (2)-(4), and 5(a, b) to predict the qualitative ST dynamics of the spatial distribution of organisms in large ecosystems [22].

All of these examples are in a sense, scaling-up from small-scale observation of qualitiative ST dynamical behavior and the development of simple mathematical models to describe this behavior.

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