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Al-Tememe Transformation for Solving Some LODE Without using Initial Conditions

Ali Hassan Mohammed
University of Kufa, Faculty of Education for Girls, Department of Mathematics, Iraq
Aabass85@yahoo.com
AlaaSallhHadi
University of Kufa, Faculty of Education for Girls, Department of Mathematics, Iraq
alaasalh.hadi@gmail.com
Hassan NademRasoul
University of Kufa. Faculty of Computer Science and Math. Department of Mathematics, Iraq
Hassan.nadem.rasoul@gmail.com

Abstract
Our aim in this paper is to apply Al-Tememe Transformations to solve linear ordinary differential equations (LODE) with variable coefficients using without any initial conditions.

1. Introduction

We can use Al-Tememe transformation ($\mathcal{T}$.T) to solve (LODE) with variable coefficients without using any initial conditions, this method summarized by taking ($\mathcal{T}$.T) to both sides of the equation with simplistic and taking inverse Al-Tememe transformation ($\mathcal{T}^{-1}.T$) to both sides, so we obtain the solution of (DE) whose solution required.

2. Preliminaries

Definition 1: [1]

Let f is defind function at period (a, b)then the integral transformation for$f$ whose it's symbol $F(p)$ is defined as:

$$F(p) = \int_a^b k(p, x) f(x) dx$$

Where k is a fixed function of two variables, called the kernel of the transformation, and a, b are real numbers or $\mp\infty$, such that the integral above converges.

Definition 2:[3]

The Al-Tememe transformation for the function $f(x)$; $x > 1$ is defined by the following integral:

$$\mathcal{T}[f(x)] = \int_1^\infty x^{-p} f(x) dx = F(p),$$

such that this integral is convergent , p is positive constant.

Property of this transformation 1 :[3]

This transformation is characterized by the linear property ,that is

$$\mathcal{T}[Af(x) + Bg(x)] = A\mathcal{T}[f(x)] + B\mathcal{T}[g(x)]\,,$$

Where $A\,,B$ are constants, the functions $f(x)$, $g(x)$ are defined when ; $x > 1$.

The Al-Tememe transform of some fundamental functions are given in table(1)[3] :

ID	Function ,$f(x)$	$F(p) = \int_1^\infty x^{-p} f(x)dx = \mathcal{T}f(x)$	Regional of convergence
1	$k; k = constant$	$\frac{k}{p-1}$	$p > 1$
2	$x^n \quad , n \in R$	$\frac{1}{p-(n+1)}$	$p > n+1$
3	$\ln x$	$\frac{1}{(p-1)^2}$	$p > 1$
4	$x^n \ln x\,, n \in R$	$\frac{1}{[p-(n+1)]^2}$	$p > n+1$
5	$\sin(a \ln x)$	$\frac{a}{(p-1)^2+a^2}$	$p > 1$
6	$\cos(a \ln x)$	$\frac{p-1}{(p-1)^2+a^2}$	$p > 1$
7	$\sinh(a \ln x)$	$\frac{a}{(p-1)^2-a^2}$	$\lvert p-1\rvert > a$
8	$\cosh(a \ln x)$	$\frac{p-1}{(p-1)^2-a^2}$	$\lvert p-1\rvert > a$

Table 1.

From the Al-Tememe definition and the above table, we get:

Theorem1:

If $\mathcal{T}[\,f(x)] = F(p)$ and $a$is constant, then $\mathcal{T}[x^{-a}f(x)] = F(p+a)$.see [3]

Definition 3: [3]

Let $f(x)$ be a function where ($x > 1$) and $\mathcal{T}[f(x)] = F(p), f(x)$ is said to be an inverse for the Al-Tememe transformation and written as $\mathcal{T}^{-1}[F(p)] = f(x)$, where $\mathcal{T}^{-1}$returns the transformation to the original function.

Property 2:[3]

If $\mathcal{T}^{-1}[F_1(p)\,] = f_1(x)$, $\mathcal{T}^{-1}[F_2(p)\,] = f_2(x)$,…, $\mathcal{T}^{-1}[F_n(p)\,] = f_n(x)$ and $a_1, a_2, \dots, a_n$are constants then,

$$\mathcal{T}^{-1}[a_1F_1(p) + a_2F_2(p) + \cdots + a_nF_n(p)] = a_1f_1(x) + a_2f_2(x) + \cdots + a_nf_n(x),$$

Definition 4: [4]

The equation,

$$a_0x^n\frac{d^ny}{dx^n} + a_1x^{n-1}\frac{d^{n-1}y}{dx^{n-1}} + \cdots + a_{n-1}x\frac{dy}{dx} + a_ny = f(x),$$

Where $a_1, a_2, \dots, a_n$ are constants and $f(x)$ is a function of x , is called ***Euler's equation***.

Theorem 2:[3]

If the function $f(x)$ is defined for $x > 1$ and its derivatives $f^{(1)}(x), f^{(2)}(x), \dots, f^{(n)}(x)$ are exist then:

$$\mathcal{T}\left[x^n f^{(n)}(x)\right] = -f^{(n-1)}(1) - (p-n)f^{(n-2)}(1) - \cdots - (p-n)\big(p-(n-1)\big)\dots((p-2)f(1) + (p-n)!\,F(p)$$

We will use Theorem(2) to prove that

$$\mathcal{T}(\ln x)^n = \frac{n!}{(p-1)^{n+1}}; \quad n \in \mathbb{N}$$

$$\text{if} \quad n = 1 \Longrightarrow \mathcal{T}(\ln x) = \frac{1}{(p-1)^2} \qquad (\text{Table1}) \qquad \dots(1)$$

If $n = 2 \quad \Longrightarrow y = (\ln x)^2 \qquad \Longrightarrow y(1) = 0$

$$y' = 2\ln x \,.\frac{1}{x} = \frac{2}{x}\ln x \qquad \Longrightarrow \quad xy' = 2\ln x$$

$$\mathcal{T}(xy') = 2\mathcal{T}(\ln x) = 2 \cdot \frac{1}{(p-1)^2} = \frac{2}{(p-1)^2}$$

$$\because \mathcal{T}(xy') = -y(1) + (p-1)\mathcal{T}(y) \quad \Longrightarrow \mathcal{T}(xy') = (p-1)\mathcal{T}(y)$$

$$\therefore \ (p-1)\mathcal{T}(y) = \frac{2}{(p-1)^2} \quad \Longrightarrow \ \mathcal{T}(y) = \frac{2}{(p-1)^3} = \frac{2!}{(p-1)^3} \dots(2)$$

If $n = 3 \quad \Longrightarrow y = (\ln x)^3 \qquad \Longrightarrow y(1) = 0$

$$y' = 3\,(\ln x)^2 \cdot \frac{1}{x} = \frac{3}{x}(\ln x)^2 \qquad \Longrightarrow \quad xy' = 3\,(\ln x)^2$$

$$\mathcal{T}(xy') = 3\,\mathcal{T}(\ln x)^2 = 3.\frac{2}{(p-1)^3} = \frac{6}{(p-1)^3}$$

$$\because \quad \mathcal{T}(xy') = (p-1)\mathcal{T}(y)$$

$$(p-1)\mathcal{T}(y) = \frac{6}{(p-1)^3} \Longrightarrow \mathcal{T}(y) = \frac{6}{(p-1)^4} = \frac{3!}{(p-1)^4} \dots \qquad (3)$$

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Also, $\quad y = (\ln x)^n \qquad \Longrightarrow y(1) = 0$

$$y' = n\,(\ln x)^{n-1} \cdot \frac{1}{x} \qquad \Longrightarrow \ xy' = n\ (\ln x)^{n-1}$$

$$\mathcal{T}(xy') = n\,\mathcal{T}(\ln x)^{n-1} = n \cdot \frac{(n-1)!}{(p-1)^n} = \frac{n!}{(p-1)^n}$$

$$\because \mathcal{T}(xy') = (p-1)\mathcal{T}(y)$$

$$\therefore (p-1)\mathcal{T}(y) = \frac{n!}{(p-1)^n} \qquad \Longrightarrow \mathcal{T}(y) = \frac{n!}{(p-1)^{n+1}} \qquad \dots(n)$$

$$\boxed{\therefore \mathcal{T}(\ln x)^n = \frac{n!}{(p-1)^{n+1}}; \quad n \in \mathbb{N}}$$

Also we will use Theorem(2) to find $\mathcal{T}[x^m(\ln x)^n]$; $\quad n, m \in \mathbb{N}$

The first case: If $n = 1$

$$\mathcal{T}[x^m \ln x] = \frac{1!}{[p-(m+1)]^2}; m \in \mathbb{N} \text{ Table1}$$

The second case: If $n = 2$ To find $\mathcal{T}[x^m (\ln x)^2]$

If $m = 1 \quad \Longrightarrow \mathcal{T}[x(\ln x)^2]$

Consider, $\quad y = x(\ln x)^2 \Longrightarrow y(1) = 0$

$$y' = x \cdot 2(\ln x) \cdot \frac{1}{x} + (\ln x)^2 \quad \Longrightarrow \quad xy' = 2x(\ln x) + x(\ln x)^2$$

$$\mathcal{T}(xy') = 2\mathcal{T}[x(\ln x)] + \mathcal{T}(y) = 2 \cdot \frac{1}{(p-2)^2} + \mathcal{T}(y)$$

$$\because \mathcal{T}(xy') = (p-1)\mathcal{T}(y) \Rightarrow (p-1)\mathcal{T}(y) = \frac{2}{(p-2)^2} + \mathcal{T}(y)$$

$$\Longrightarrow (p-2)\mathcal{T}(y) = \frac{2}{(p-2)^2} \quad \Longrightarrow \mathcal{T}[x\,(\ln x)^2] = \frac{2}{(p-2)^3} \ \ldots \qquad (4)$$

If $\quad m = 2 \quad \Longrightarrow \mathcal{T}[x^2 (\ln x)^2]$

Consider, $\ y = x^2(\ln x)^2 \Longrightarrow \ y(1) = 0$

$$y' = x^2 \cdot 2(\ln x) \cdot \frac{1}{x} + 2x(\ln x)^2 \ \Longrightarrow \ xy' = 2x^2(\ln x) + 2x^2(\ln x)^2$$

$$\mathcal{T}(xy') = 2\mathcal{T}[x^2(\ln x)] + 2\mathcal{T}(y) = 2 \cdot \frac{1}{(p-3)^2} + 2\mathcal{T}(y)$$

$$\because \mathcal{T}(xy') = (p-1)\mathcal{T}(y) \ \Longrightarrow \ (p-1)\mathcal{T}(y) = \frac{2}{(p-3)^2} + 2\mathcal{T}(y)$$

$$\Longrightarrow (p-3)\mathcal{T}(y) = \frac{2}{(p-3)^2} \quad \Longrightarrow \mathcal{T}[x^2(\ln x)^2] = \frac{2}{(p-3)^3} \qquad \ldots (5)$$

If $\ m = 3 \Longrightarrow \mathcal{T}[x^3 (\ln x)^2]$

Consider, $\quad y = x^3(\ln x)^2 \Longrightarrow \ y(1) = 0$

$$y' = x^3 \cdot 2(\ln x) \cdot \frac{1}{x} + 3x^2(\ln x)^2 \Longrightarrow xy' = 2x^3(\ln x) + 3x^3(\ln x)^2$$

$$\mathcal{T}(xy') = 2\mathcal{T}[x^3(\ln x)] + 3\mathcal{T}(y) = 2 \cdot \frac{1}{(p-4)^2} + 3\mathcal{T}(y)$$

$$(p-1)\mathcal{T}(y) = 2 \cdot \frac{1}{(p-4)^2} + 3\mathcal{T}(y) \Longrightarrow \mathcal{T}[x^3(\ln x)^2] = \frac{2}{(p-4)^3} \ \ldots \qquad (6)$$

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$$\mathcal{T}[x^m (\ln x)^2]; m \in \mathbb{N} \ \Rightarrow y = x^m(\ln x)^2 \Longrightarrow \ y(1) = 0$$

$$y' = x^m \cdot 2(\ln x) \cdot \frac{1}{x} + mx^{m-1}(\ln x)^2 \Longrightarrow xy' = 2x^m(\ln x) + mx^m(\ln x)^2$$

$$\mathcal{T}(xy') = 2\mathcal{T}[x^m(\ln x)] + m\mathcal{T}(y) = 2\cdot\frac{1}{[p-(m+1)]^2} + m\mathcal{T}(y)$$

$$(p-1)\mathcal{T}(y) = 2\cdot\frac{1}{[p-(m+1)]^2} + m\mathcal{T}(y)$$

$$\Rightarrow \mathcal{T}[x^m(\ln x)^2] = \frac{2!}{[p-(m+1)]^3}; m\in\mathbb{N} \quad \dots(m)$$

Note: These cases are also true for $m \in \mathbb{Q}$

The third case: To find $\mathcal{T}[x^m(\ln x)^3]$

If $m = 1 \Longrightarrow \mathcal{T}[x(\ln x)^3]$

Consider, $y = x(\ln x)^3 \Longrightarrow y(1) = 0$

$$y' = x\cdot 3(\ln x)^2\cdot\frac{1}{x} + (\ln x)^3 \Longrightarrow xy' = 3x(\ln x)^2 + x(\ln x)^3$$

$$\mathcal{T}(xy') = 3\mathcal{T}[x(\ln x)^2] + \mathcal{T}(y) = 3\cdot\frac{2}{(p-2)^3} + \mathcal{T}(y)$$

$$\Longrightarrow (p-1)\mathcal{T}(y) = \frac{3!}{(p-2)^3} + \mathcal{T}(y)$$

$$\Longrightarrow (p-2)\mathcal{T}(y) = \frac{3!}{(p-2)^3} \quad \Longrightarrow \mathcal{T}[x\,(\ln x)^2] = \frac{3!}{(p-2)^4} \dots(7)$$

If $m = 2 \Longrightarrow \mathcal{T}[x^2(\ln x)^3]$

Consider, $y = x^2(\ln x)^3 \Longrightarrow y(1) = 0$

$$y' = x^2\cdot 3(\ln x)^2\cdot\frac{1}{x} + 2x(\ln x)^3 \Longrightarrow xy' = 3x^2(\ln x)^2 + 2x^2(\ln x)^3$$

$$\mathcal{T}(xy') = 3\mathcal{T}[x^2(\ln x)^2] + 2\mathcal{T}(y) = 3\cdot\frac{2}{(p-3)^3} + 2\mathcal{T}(y)$$

$$(p-1)\mathcal{T}(y) = 3\cdot\frac{2}{(p-3)^3} + 2\mathcal{T}(y) \quad \Longrightarrow \mathcal{T}(x^2(\ln x)^3) = \frac{3!}{(p-3)^4} \quad \dots(8)$$

If $m = 3 \Longrightarrow \mathcal{T}[x^3(\ln x)^3]$

Consider, $y = x^3(\ln x)^3 \Longrightarrow y(1) = 0$

$$y' = x^3\cdot 3(\ln x)^2\cdot\frac{1}{x} + 3x^2(\ln x)^3$$

$$xy' = 3x^3(\ln x)^2 + 3x^3(\ln x)^3 \Longrightarrow \mathcal{T}(xy') = 3\mathcal{T}[x^3(\ln x)^2] + 3\mathcal{T}(y)$$

$$= 3\cdot\frac{2}{(p-4)^3} + 3\mathcal{T}(y)$$

$$(p-1)\mathcal{T}(y) = 3\cdot\frac{2}{(p-4)^3} + 3\mathcal{T}(y) \quad \Longrightarrow \mathcal{T}[x^3(\ln x)^3] = \frac{3!}{(p-4)^4} \dots(9)$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$\mathcal{T}[x^m(\ln x)^3]$

Consider $\Longrightarrow y = x^m(\ln x)^3 \Longrightarrow y(1) = 0$

$$y' = x^m\cdot 3(\ln x)^2\cdot\frac{1}{x} + mx^{m-1}(\ln x)^3 \Longrightarrow xy' = 3x^m(\ln x)^2 + mx^m(\ln x)^3$$

$$\mathcal{T}(xy') = 3\mathcal{T}[x^m(\ln x)^2] + m\mathcal{T}(y) = 3\cdot\frac{2}{[p-(m+1)]^3} + m\mathcal{T}(y)$$

$$(p-1)T(y) = 3\cdot\frac{2}{[p-(m+1)]^3} + mT(y)$$

$$\vdots \quad \vdots \quad \vdots \quad \vdots$$

Gradually we find now

Consider, $y = x^m(\ln x)^n \Longrightarrow y(1) = 0$

$$y' = x^m \cdot n(\ln x)^{n-1}\cdot\frac{1}{x} + mx^{m-1}(\ln x)^n \Longrightarrow xy' = nx^m(\ln x)^{n-1} + mx^m(\ln x)^n$$

$$\mathcal{T}(xy') = n\mathcal{T}[x^m(\ln x)^{n-1}] + m\mathcal{T}(y) = n\cdot\frac{(n-1)!}{[p-(m+1)]^n} + m\mathcal{T}(y)$$

$$(p-1)\mathcal{T}(y) = n\cdot\frac{(n-1)!}{[p-(m+1)]^n} + m\mathcal{T}(y)$$

$$\mathcal{T}[x^m(\ln x)^n] = \frac{n!}{[p-(m+1)]^{n+1}} \quad ; m\in\mathbb{N}\ , n\in\mathbb{N}$$

How to use Al-Tememe Transformation for solving (LODE) without using any initial conditions

We can generalized the idea of the researcher Mohammed [2] to find solution of an ODEs that have constant coefficients without using any initial conditions by using Laplace transform

Suppose we have a linear ordinary differential equation (LODE) of order (n) with variables coefficients it's from :

$$a_0x^ny^{(n)} + a_1x^{n-1}y^{(n-1)} + \cdots + a_{n-1}xy' + a_ny = f(x) \qquad \dots(10)$$

Without using any initial conditions, that is mean $y(1), \dots, y^{(n-1)}(1), y^{(n)}(1)$ are unknown and the Al-Tememe transformation of $f(x)$ is known. To solve equation (10) we take ($\mathcal{T}$.T) to both sides we get:

$$\mathcal{T}(y) = \frac{h(p)}{[L(p).K(p)]} \qquad \dots(11)$$

Where as $L(p)$ is a polynomial of (p) represents the denominator of ($\mathcal{T}$.T) of the function $f(x)$ its degree (n) , and $K(p)$ is also a polynomial of (p) its degree $m$so the degree of $[L(p).k(p)]$equal $(n+m)$,and $h(p)$ is also a polynomial of $p$and its degree less than$(n+m)$, and not necessary to know the terms of $h(p)$we only denoted it by this symbol. Now by taking $\mathcal{T}^{-1}$ to both sides of equation (11), we get the following solution:

$$y = A_1g_1(x) + A_2g_2(x) + \cdots + A_ng_n(x) + B_1k_1(x) + B_2k_2(x) + \cdots + B_mk_m(x) \qquad \dots(12)$$

$$= \sum_{i=1}^{n} A_ig_i(x) + \sum_{j=1}^{m} B_jk_j(x)$$

Whereas $A_1, A_2, \dots, A_n$and $B_1, B, \dots, B_m$ are constants, $g_1, g_2, \dots, g_n$ and $k_1, k_2, \dots, k_m$ are functions of x.

The number of the constants $B_i$and the number of the functions $k_i, i = 1,2, \dots, m$ are equal to the degree of $k(p)$which is supposed to be (m) .

Note: Since the order of equation(10) is (n), therefore its general solution contains (n) constants, but the solution in (12) contains $(n + m)$ constants and to solve this problem we can eliminate some of these constants $(B_1, B, \dots, B_m)$ whose values obtaining by substituting the solution (12) in equation (10), so we get a solution contains (n) constants (as unknown) as the required solution. By this method we get the general solution of equation (10) without using any initial conditions by using Al-Tememe transformation.

Example 1: To solve the differential equation:

$$xy' + 2y = cos(ln\, x) \qquad \dots(13)$$

By using ($\mathcal{T}$.T) without any initial conditions we take ($\mathcal{T}$.T) to both sides of it we can write:

$$\mathcal{T}(y) = \frac{h(p)}{(p+1)[(p-1)^2+1]} \dots \qquad (14)$$

Whereas $h(p)$ has a degree less than three, $(p + 1)$ is the coefficients of $\mathcal{T}(y)$ and $\frac{1}{(p-1)^2+1}$is the denominator of $cos(ln\, x)$.

By taking $\mathcal{T}^{-1}$ to both sides of equation (14) we get the following solution :

$$y = \mathcal{T}^{-1}\left[\frac{A}{(p+1)} + \frac{Bp + C}{(p-1)^2 + 1}\right]$$

So, $$y = Ax^{-2} + Bcos(ln\, x) + Dsin(lnx) \qquad \dots(15)$$

Such that $D = B + C$

The given equation is of order one, so the general solution of it must contain only one constant while equation (15) contains three constants, therefore, we should eliminate the contants B and D, for this, we get y$'$ from equation (15) as follows:

$$y' = -2Ax^{-3} - Bx^{-1}sin(ln\, x) + Dx^{-1}cos(lnx) \qquad \dots(16)$$

And substitute y, y'in equation (16) to find the values of B and D, so:

$$2B + D = 1 \qquad \dots(17)$$

$$-B + 2D = 0 \qquad \dots(18)$$

By solving equations (17) and (18), we get:

$B = {}^2/_5$, $D = {}^1/_5$

Therefore the general solution of the given ODE is given by :

$$y = Ax^{-2} + {}^2/_5\, cos(ln\, x) + {}^1/_5\, sin(lnx)$$

This solution contains only one constant (A) and this is equal to the order of equation (13)

Example 2: To solve the differential equation:

$$x^2y'' + xy' = sinh(2ln\, x)$$

By using ($\mathcal{T}$.T) without any initial condition. We take ($\mathcal{T}$.T) to both sides of it we can write:

$$\mathcal{T}(y) = \frac{h(p)}{(p-1)^2[(p-1)^2-4]} \qquad \dots(19)$$

Whereas h(p) has a degree less than four.

By taking $\mathcal{T}^{-1}$ of both sides of equation (19) we get:

$$y = \mathcal{T}^{-1}\left[\frac{A}{(p-1)} + \frac{B}{(p-1)^2} + \frac{Cp+D}{(p-1)^2-4}\right]$$

$$\Longrightarrow y = A + B\ln x + C\cosh(2\ln x) + E\sinh(2\ln x)\,; \;\; E = C + D \qquad \ldots(20)$$

The given equation is of order two, so the general solution must contain only two constants while equation (20) contains four constants, therefore, we should eliminate the contants C and E, for this, we get y', y'' from equation (20) as follows:

$$y' = Bx^{-1} + 2Cx^{-1}\sinh(2\ln x) + 2Ex^{-1}\cosh(2\ln x)$$

$$y'' = -Bx^{-2} + (4C - 2E)x^{-2}\cosh(2\ln x) + (4E-2C)x^{-2}\sinh(2\ln x)$$

And after we substitute y and y'' in (D.E) to find the values of E, C we get:

$$E = {}^{1}/_{4} \;\;, C = 0$$

$$\Longrightarrow y = A + B\ln x + {}^{1}/_{4}\sinh(2\ln x)$$

Example 3: To solve the differential equation:

$$x^3y''' - x^2y'' - 2xy' + 6y = 2x^{-4}$$

Take ($\mathcal{T}$) to both sides of it and we can write:

$$\mathcal{T}(y) = \frac{h(p)}{p(p-4)(p-3)(p+3)}$$

Where as h(p) has a degree less than four.

Taking $\mathcal{T}^{-1}$ to both sides to the last equation we get :

$$y = \mathcal{T}^{-1}\left[\frac{A}{p} + \frac{B}{(p-4)} + \frac{C}{(p-3)} + \frac{D}{(p+3)}\right]$$

$$y = Ax^{-1} + Bx^3 + Cx^2 + Dx^{-4}$$

The given equation is of order three, so the general solution must contain three constants while equation contains four constants, therefore, we should eliminate the contant D, for this ,we get y', y'', y''' from equation as follows:

$$y' = -Ax^{-2} + 3Bx^2 + 2Cx - 4Dx^{-5}$$

$$y'' = 2Ax^{-3} + 6Bx + 2C + 20Dx^{-6}$$

$$y''' = -6Ax^{-4} + 6B - 120Dx^{-7}$$

And after we substitutey', y'' and y''' in (DE) to find the value of D we get:

$$D = {}^{-1}/_{63}$$

$$\Longrightarrow y = Ax^{-1} + Bx^3 + Cx^2 - {}^{1}/_{63}\,x^{-4}$$

Example 4: To solve the differential equation:

$$x^4y^{(4)} - 6x^2y'' = x^{-1}(\ln x)^2$$

Take ($\mathcal{T}$) to both sides of it we can write:

$$\mathcal{T}(y) = \frac{h(p)}{p^3(p-1)^2(p-2)(p-6)}$$

Where as h(p) has a degree less than seven.

Taking $\mathcal{T}^{-1}$ to both sides of equation we get :

$$y = \mathcal{T}^{-1}\left[\frac{A}{p} + \frac{B}{p^2} + \frac{C}{p^3} + \frac{D}{(p-1)} + \frac{E}{(p-1)^2} + \frac{F}{(p-2)} + \frac{G}{(p-6)}\right]$$

$$y = Ax^{-1} + Bx^{-1}\ln x + Cx^{-1}(\ln x)^2 + D + E\ln x + Fx + Gx^5$$

The given equation is of order four, so the general solution must contain four constants while last equation contains seven constant , therefore, we should eliminate the constants A, B, C, for this ,we get $y', y'', y''', y^{(4)}$ from last equation as follows:

$$y' = (B-A)x^{-2} + (2C-B)x^{-2}\ln x - Cx^{-2}(\ln x)^2 + Ex^{-1} + F + 5Gx^4$$

$$y'' = (2A-3B+2C)x^{-3} + (2B-6C)x^{-3}\ln x + 2Cx^{-3}(\ln x)^2 - Ex^{-2} + 20Gx^3$$

$$y''' = (11B-6A-12C)x^{-4} + (22C-6B)x^{-4}\ln x - 6Cx^{-4}(\ln x)^2 + 2Ex^{-3} + 60Gx^2$$

$$y^{(4)} = (24A+70C-50B)x^{-5} + (24B-100C)x^{-5}\ln x + 24Cx^{-5}(\ln x)^2 - 6Ex^{-4} + 120Gx$$

And after we substitute y, y'', y''' and $y^{(4)}$ in (D.E) to find the values of A,B and C we get:

$A = {}^{169}/_{216}$, $B = {}^{4}/_{9}$, $C = {}^{1}/_{12}$, so

$$y = {}^{169}/_{216}\,x^{-1} + {}^{4}/_{9}\,x^{-1}\ln x + {}^{1}/_{12}\,x^{-1}(\ln x)^2 + D + E\ln x + Fx + Gx^5$$

References:

[1] Gabriel Nagy,“Ordinary Differential Equations” Mathematics Department, Michigan State University, East Lansing, MI, 48824.October 14, 2014.

[2] Mohammed, A.H., “ A Suggested Method of Laplace Transformations Without Using Any Initial Condition” ,Al-Qadisiya university magazine for completely sciences, volume (6), number (2). (2001).

[3] Mohammed, A.H., Athera Nema Kathem, “Solving Euler’s Equation by Using New Transformation”, Karbala university magazine for completely sciences ,volume (6), number (4. (2008).

[4] PhD, Andr’asDomokos, “Differential Equations Theory and Applications”, California State University, Sacramento, Spring, 2015.