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(τi , τj) * - Q* g closed sets in Bitopological spaces

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Abstract

The aim of this paper is to introduced the new type of closed sets called (τ_i , τ_j)* - Q* g closed set. We introduce and study a new class of spaces namely (τ_i , τ_j)* - Q*g $T_{1/2}$ space and (τ_i , τ_j)* - Q* g $T_{3/4}$ space. Also we find some basic properties and applications of (τ_i , τ_j)* - Q* g closed sets.

Keywords: (τ_i , τ_j)* - Q* g open; (τ_i , τ_j)* - Q* g closed; (τ_i , τ_j)* - Q*g $T_{1/2}$ space and (τ_i , τ_j)* - Q*g $T_{3/4}$ space.

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1. Introduction

Closed sets are fundamental objects in a topological space. For example one can define the topology on a set by using either the axioms for the closed sets or the Kuratowski closure axioms. In 1971, Levine [7] introduced the concept of generalized closed sets in topological spaces. Also he introduced the notion of semi open sets in topological spaces. Bhattacharyya and Lahiri introduced a class of sets called semi generalized closed sets by means of semi open sets of Levine and obtained various topological properties.

The notion of Q^* - closed sets in a topological space was introduced by Murugalingam and Lalitha [9] in 2010 . In the year 2012, P. Padma introduced the concept of (τ_1 , τ_2)* - Q* closed sets in bitopological spaces . Also she introduced the notion of (τ_1 , τ_2)* - Q* continuous maps in bitopological spaces.

The aim of this paper is to introduced the new type of closed sets called Q* g closed. We introduce and study a new class of spaces namely (τ_i , τ_j)* - Q*g $T_{1/2}$ space and (τ_i , τ_j)* - Q*g $T_{3/4}$ space. Also we find some basic properties and applications of (τ_i , τ_j)* - Q* g closed sets.

2. Preliminaries

Throughout this paper X and Y always represent nonempty bitopological spaces (X , τ_1 , τ_2) and (Y , σ_1 , σ_2). For a subset A of X , τ_i - cl (A) , τ_i - Q^*cl (A) (resp.τ_i - int (A) .τ_i - Q^*int (A)) represents closure of A and Q^*closure of A (resp. interior of A , Q^* - interior of A) with respect to the topology τ_i . We shall now require the following known definitions.

Definition 2.1 - A subset S of X is called $\tau_1\tau_2$ - open if $S \in \tau_1 \cup \tau_2$ and the complement of $\tau_1\tau_2$ - open set is $\tau_1\tau_2$ - closed.

Example 2.1 - Let X = { a , b , c } , τ_1 = { ϕ , X , { a } , {a , b} } and τ_2 = { ϕ , X , { b}}.

Then τ_1 - open sets on X are ϕ , X , { a } , { a, b } and τ_2 - open sets on X are ϕ , X , { b }. Therefore , $\tau_1\tau_2$ - open sets on X are ϕ , X , { a } , { b } , { a , b } and $\tau_1\tau_2$ - closed sets are X , ϕ , { b , c } , {c , a } , { c }.

Definition 2.3 - A subset A of a bitopological space (X , τ_1 , τ_2) is called (τ_1 , τ_2) * - semi generalized closed (briefly (τ_1 , τ_2) * - sg closed) set if and only if $\tau_1\tau_2$ - scl (S) $\subseteq$ F whenever S $\subseteq$ F and F is $\tau_1\tau_2$ - semi open set . The complement of (τ_1 , τ_2) * - semi generalized closed set is $\tau_1\tau_2$ - semi generalized open.

Definition 2.4 - A subset A of a bitopological space (X , τ_1 , τ_2) is called (τ_1 , τ_2) * - generalized closed (briefly (τ_1 , τ_2) * - g closed) set if and only if $\tau_1\tau_2$ - cl (S) $\subseteq$ F whenever S $\subseteq$ F and F is $\tau_1\tau_2$ - open set . The complement of (τ_1 , τ_2) * - generalized closed set is (τ_1 , τ_2) * - generalized open.

Definition 2.5 - A subset A of a topological space (X , τ) is called a Q* ***generalized closed*** set (briefly Q* ***g - closed***) if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is Q* open in (X , τ) . The complement of a Q* g - closed set is called a Q* ***g - open*** set.

3. Q* g closed

In this section we introduce a new type of closed set called Q* g closed.

Definition 3.1 - A subset A of a bitopological space (X , τ_1 , τ_2) is called a (τ_i , τ_j)* - Q* ***generalized closed*** set (briefly (τ_i , τ_j)* - Q* ***g closed***) if τ_i τ_j - cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is τ_i τ_j - Q* open in (X , τ_1 , τ_2) , where i , j = 1 , 2 and i $\neq$ j . The complement of a (τ_i , τ_j)* - Q* g - closed set is called a (τ_i , τ_j)* Q* ***g - open*** set .

Example 3.1 : Let X = { a , b , c } , τ_1 = { ϕ , X } τ_2 = { ϕ , X , { a } } then (τ_i , τ_j)* - Q* g closed sets are ϕ , X , { b } , { c } , { a , b } , { b , c } , { c , a }and (τ_i , τ_j)* - Q* g open sets are ϕ , X , { a , c } , { a , b } , { c } , { a } , { b }.

Theorem 3.1 : Every (τ_i , τ_j)* - Q* g closed set is (τ_i , τ_j)* - semi closed.

Remark 3.1 : The converse of the above theorem need not be true . The following example supports our claim.

Example 3.2 : Let X = { a , b , c } , τ_1 = { ϕ , X , { a } , { b } , {a , b} } and τ_2 = { ϕ , X } then (τ_i , τ_j)* - Q* g closed sets are ϕ , X , { c } , { b , c } , { c , a } . Here { a , b } is (τ_i , τ_j)* - semi closed but not (τ_i , τ_j)* - Q* g closed.

Theorem 3.2 : Every (τ_i , τ_j)* - Q* open set is (τ_i , τ_j)* - Q* g open.

Remark 3.2 : The converse of the above theorem is not true as shown in the following example.

Example 3.3 : In example 3.2 , { a } is (τ_i , τ_j)* - Q* g - open set but not (τ_i , τ_j)* - Q* open.

Theorem 3.3 : Every (τ_i , τ_j)* - Q* closed set is (τ_i , τ_j)* - Q* g closed.

Proof : Let A be an (τ_i , τ_j)* - Q* closed set . Then X – A is (τ_i , τ_j)* - Q* open . We have to show that A is (τ_i , τ_j)* - Q* g closed . Since every (τ_i , τ_j)* - Q* open set is (τ_i , τ_j)* - Q* g open , we have X – A is

(τ_i , τ_j)* - Q* g open . Thus , A is (τ_i , τ_j)* - Q* g closed.

Remark 3.3 : The converse of the above theorem is not true as shown in the following example.

Example 3.4 : In example , { b , c } is (τ_i , τ_j)* - Q* g closed set but not (τ_i , τ_j)* - Q* closed.

Theorem 3.4 : Every (τ_i , τ_j)* - Q* g closed set is (τ_i , τ_j)* - g closed.

Proof : Let A be an (τ_i , τ_j)* - Q* g closed set and U be any (τ_i , τ_j)* - Q* open set containing A in X . We have to show that A is (τ_i , τ_j)* - g closed . Since every (τ_i , τ_j)* - Q* open set is (τ_i , τ_j)* - open and A is (τ_i , τ_j)* - Q* g closed we have τ_i τ_j - cl (A) $\subseteq$ U whenever A $\subseteq$ U and U is (τ_i , τ_j)* - open in X . Hence X is (τ_i , τ_j)* - g closed.

Remark 3.4 : The converse of the above theorem is true as shown in the following example.

Example 3.5 : Let X = { a , b , c } , τ_1 = { ϕ , X } , τ_2 = { ϕ , X , { c } , { a , c } } then (τ_i , τ_j)* - g closed sets and (τ_i , τ_j)* - Q* g closed sets are ϕ , X , { b } , { a , b } , { b , c }.

Theorem 3.5 : Every (τ_i , τ_j)* - Q* g closed set is (τ_i , τ_j)* - αg** closed.

Proof : Let A be an (τ_i , τ_j)* - Q* g closed set such that A $\subseteq$ U , where U is (τ_i , τ_j)* - Q* open . We have to show that A is (τ_i , τ_j)* - αg** closed . Since every (τ_i , τ_j)* - Q* open set is (τ_i , τ_j)* - open , (τ_i , τ_j)* - closed set is (τ_i , τ_j)* - α closed and A is (τ_i , τ_j)* - Q* g closed we have τ_i τ_j - α cl (A) $\subseteq$ U whenever A $\subseteq$ U and U is (τ_i , τ_j)* - g** open in X . Hence A is (τ_i , τ_j)* - αg** closed.

Remark 3.5 : The converse of the above theorem is true .

Example 3.6 : In example 3.1 , (τ_i , τ_j)* - αg** closed and (τ_i , τ_j)* - Q* g closed sets are ϕ , X , { b } , { c } , { a , b } , { b , c } , { c , a }.

Theorem 3.6 : Every (τ_i , τ_j)* - open set is (τ_i , τ_j)* - Q* g open.

Proof : Let S be an (τ_i , τ_j)* - open set in X . Then X – S is (τ_i , τ_j)* - closed.

Claim : S is a (τ_1 , τ_2) * - Q* g open set . i.e) to prove X – S is a (τ_1 , τ_2) * - Q* g closed set . i.e) to prove τ_1 τ_2 - cl (X – S) $\subseteq$ F whenever X – S $\subseteq$ F , F is (τ_i , τ_j)* - open . Let X – S $\subseteq$ F and F is (τ_i , τ_j)* - open . Since X – S is (τ_i , τ_j)* - closed , we have τ_1 τ_2 - cl (X – S) = X – S $\subseteq$ F.

$\Rightarrow$ X – S is a (τ_1 , τ_2) * - Q* g closed set.

Thus , S is a (τ_1 , τ_2) * - Q* g open set.

Theorem 3.7 : Every (τ_i , τ_j)* - closed set is (τ_i , τ_j)* - Q* g closed.

Proof : Let A be an (τ_i , τ_j)* - closed set. Then X – A is (τ_i , τ_j)* - open . We have to show that A is (τ_i , τ_j)* - Q* g closed . Since every open set is (τ_i , τ_j)* - Q* g open , we have X – A is (τ_i , τ_j)* - Q* g open. Thus, A is (τ_i , τ_j)* - Q* g closed.

Remark 3.6 : The converse of the above theorem is not true as shown in the following example.

Example 3.7 : In example 3.1 , { a , b } is (τ_i , τ_j)* - Q* g closed but not (τ_i , τ_j)* - closed in X.

Note 3.1 : The family of all (τ_i , τ_j)* - Q*g closed subsets of a bitopological space X is denoted by (τ_i , τ_j)* - Q* g.

Proposition 3.1 : If A , B $\in$ (τ_i , τ_j)* - Q* g then A $\cup$ B $\in$ (τ_i , τ_j)* - Q* g.

Proof : Let A and B be (τ_i , τ_j)* - Q* g closed sets in X.

Claim : A $\cup$ B be a (τ_i , τ_j)* - Q* g closed sets in X . i.e) to prove τ_i τ_j - cl (A $\cup$ B) $\subseteq$ U whenever A $\cup$ B $\subseteq$ U and U is (τ_i , τ_j)* - Q* open in X . Since, A and B be (τ_i , τ_j)* - Q* g closed sets in X we have τ_i τ_j - cl (A) $\subseteq$ U whenever A $\subseteq$ U and U is (τ_i , τ_j)* - Q* open in X and τ_i τ_j - cl (B) $\subseteq$ U whenever B $\subseteq$ U and U is (τ_i , τ_j)* - Q* open in X. Since X be a bitopological space , we have finite union of closed sets are closed.

$\Rightarrow$ τ_i τ_j - cl (A $\cup$ B) $\subseteq$ U whenever (A $\cup$ B) $\subseteq$ U and U is (τ_i , τ_j)* - Q* open in X.

$\Rightarrow$ A $\cup$ B is (τ_i , τ_j)* - Q* g closed sets in X .

$\Rightarrow$ A $\cup$ B $\in$ (τ_i , τ_j)* - Q*g.

Remark 3.7 : The intersection of 2 (τ_i , τ_j)* - Q* g closed sets are (τ_i , τ_j)* - Q* g closed . The following example supports our claim.

Example 3.8 : In example 3.2 , { b , c } and { c } are (τ_i , τ_j)* - Q* g closed set . { b , c } $\cap$ { c } = { c } is (τ_i , τ_j)* - Q* g closed.

Remark 3.8 : The union of 2 (τ_i , τ_j)* - Q* g closed sets are (τ_i , τ_j)* - Q* g closed . The following example supports our claim.

Example 3.10 : In example 3.1 , { c , a } and { c } are (τ_i , τ_j)* - Q* g closed set . { c , a } $\cup$ { c } = { a , c } is (τ_i , τ_j)* - Q* g closed.

Theorem 3.8 : The intersection of a (τ_i , τ_j)* - Q* g closed and a (τ_i , τ_j)* - g closed set is always (τ_i , τ_j)* - Q* g closed.

Proof : Let A be a (τ_i , τ_j)* - Q* g closed and let F be (τ_i , τ_j)* - g closed . If U is an (τ_i , τ_j)* - Q* open set with A $\cap$ F $\subseteq$ U then A $\subseteq$ U $\cup$ F^c and so τ_i τ_j - cl (A) $\subseteq$ U $\cup$ F^c . Now τ_i τ_j - cl (A $\cap$ F) $\subseteq$ τ_i τ_j - cl (A) $\cap$ F $\subseteq$ U . Hence A $\cap$ F is (τ_i , τ_j)* - Q* g closed.

Example 3.11 : In example 3.2 , { b } is (τ_i , τ_j)* - Q* g closed and { a , b } is (τ_i , τ_j)* - g closed . { b } $\cap$ { a , b } = { b } is (τ_i , τ_j)* - Q* g closed.

Proposition 3.2 : If A is (τ_i , τ_j)* - Q* g open and (τ_i , τ_j)* - Q* g closed subset of X then A is an (τ_i , τ_j)* - Q* closed subset of X.

Proof : Since A is (τ_i , τ_j)* - Q* g open and (τ_i , τ_j)* - Q* g closed , τ_i τ_j - cl(A) $\subseteq$ U . Hence A is (τ_i , τ_j)* - g closed.

Remark 3.9 : The following example supports our claim.

Example 3.12 : In example 3.1 , { a , c } is (τ_i , τ_j)* - Q* g open and (τ_i , τ_j)* - Q* g closed . Then { a , c } is (τ_i , τ_j)* - g closed.

Theorem 3.9 : If A is (τ_i , τ_j)* - closed set in X then A is (τ_i , τ_j)* - Q* g closed if and only if τ_i τ_j - c l (A) – A is (τ_i , τ_j)* - g closed.

Proof : Necessity : Let A be a (τ_i , τ_j)* - closed set.

Claim : τ_i τ_j - Cl (A) – A is (τ_i , τ_j)* - g - closed . ie) To prove τ_i τ_j - cl (A) – A = ϕ . Since A is (τ_i , τ_j)* - closed , we have τ_i τ_j - cl (A) = A . ie) τ_i τ_j - cl (A) – A = ϕ . Hence τ_i τ_j - cl (A) – A is (τ_i , τ_j)* - g - closed.

Sufficiency : Let τ_i τ_j - cl (A) – A be a (τ_i , τ_j)* - g - closed set.

Claim : A is (τ_i , τ_j)* - closed . ie) To prove τ_i τ_j - cl (A) = A . Since , τ_i τ_j - cl (A) – A is (τ_i , τ_j)* - g closed.

$\Rightarrow$ τ_i τ_j - cl (A) – A = ϕ . Also A is (τ_i , τ_j)* - Q* g closed . Therefore , A is (τ_i , τ_j)* - closed.

Proposition 3.3 : If A is (τ_i , τ_j)* - closed subset of X then A is (τ_i , τ_j)* - Q* g closed.

Proof : Let A be (τ_i , τ_j)* - closed in X.

Claim : A is (τ_i , τ_j)* - Q* g closed. ie., to prove τ_i τ_j - cl (A) $\subseteq$ U whenever A $\subseteq$ U and U is (τ_i , τ_j)* - Q* open in X. Let A $\subseteq$ U and U is (τ_i , τ_j)* - Q* open in X. Since A is (τ_i , τ_j)* - closed in X , we have τ_i τ_j - cl (A) = A

Since A $\subseteq$ U, we have τ_i τ_j - cl (A) = A $\subseteq$ U . Therefore , A is (τ_i , τ_j)* - Q* g closed.

Remark 3.10 : But the converse of the above proposition is not true in general. It is shown in the example 3.1 . The subset { b } of a bitopological space X is (τ_i , τ_j)* - Q* g closed but it is not (τ_i , τ_j)* - closed in X.

Proposition 3.4 : If A is a (τ_i , τ_j)* - Q* g closed set of X such that A $\subseteq$ B $\subseteq$ τ_i τ_j - cl (A) then B is also an (τ_i , τ_j)* - Q* g closed set of Y.

Proof : Let A be an (τ_i , τ_j)* - Q* g closed set of X such that A $\subseteq$ B $\subseteq$ τ_i τ_j - cl (A).

Claim : B is (τ_i , τ_j)* - Q* g closed set of X . ie) to prove τ_i τ_j - cl (B) $\subseteq$ U whenever B $\subseteq$ U & U is (τ_i , τ_j)* - Q* open in X . Let B $\subseteq$ U & U is (τ_i , τ_j)* - Q* open in Y . Since A is (τ_i , τ_j)* - Q* g closed in X , we have τ_i τ_j - cl (A) $\subseteq$ U whenever A $\subseteq$ U & U is (τ_i , τ_j)* - Q* open in X. Since A is (τ_i , τ_j)* - closed in X , we have τ_i τ_j - cl (A) = A . Since A $\subseteq$ B $\subseteq$ τ_i τ_j - cl (A) , we have B is (τ_i , τ_j)* - closed in X & τ_i τ_j - cl (B) = B . Since B

$\subseteq$ U we have B is (τ_i , τ_j)* - Q* g closed.

Remark 3.11 : The following example show that (τ_i , τ_j)* - Q* g closeness is independent from (τ_i , τ_j)* - $\hat{g}$ closeness , (τ_i , τ_j)* - sg closeness , (τ_i , τ_j)* - gα closeness and (τ_i , τ_j)* - α closeness.

Example 3.11 : In example 3.1 , (τ_i , τ_j)* - { a , b } is Q* g closed but neither (τ_i , τ_j)* - $\hat{g}$ closed , (τ_i , τ_j)* - sg closed and the set { a , c } is (τ_i , τ_j)* - Q* g closed but neither (τ_i , τ_j)* - gα closed and (τ_i , τ_j)* - α closed.

Theorem 3.10 : Every (τ_i , τ_j)* - Q* g closed set is (τ_i , τ_j)* - sg closed.

Proof : Let A be an (τ_i , τ_j)* - Q* g closed.

Claim : A is (τ_i , τ_j)* - sg closed . Since every (τ_i , τ_j)* - closed set is (τ_i , τ_j)* - semi closed and A is (τ_i , τ_j)* - Q* g closed we have τ_i τ_j - scl (A) $\subseteq$ U whenever A $\subseteq$ U & U is (τ_i , τ_j)* - open in X . Hence A is (τ_i , τ_j)* - sg closed.

Remark 3.12 : The following example shows that converse of the above theorem need not be true.

Example 3.12 : In example 3.2 , { a } is (τ_i , τ_j)* - sg closed but not (τ_i , τ_j)* - Q* g closed.

Theorem 3.11 : A subset S of X is (τ_i , τ_j)* - Q* g closed if and only if (τ_i , τ_j)* - cl (S) – S contains no non - empty (τ_i , τ_j)* - closed set.

Proof:

Necessity : Suppose that S is (τ_i , τ_j)* - Q*g closed.

Claim : (τ_i , τ_j)* - cl (S) – S contains no non - empty (τ_i , τ_j)* - closed set . Assume the contrary . Let F be a (τ_i , τ_j)* - semi closed set such that F $\subseteq$ (τ_i , τ_j)* - cl (S) – S.

Since F $\subseteq$ (τ_i , τ_j)* - cl (S) – S , we have F $\subseteq$ (τ_i , τ_j)* - cl (S) $\cap$ (X – S).

$\Rightarrow$ F $\subseteq$ (τ_i , τ_j)* - cl (S) and F $\subseteq$ X – S .... (3 . 1).

Since F is (τ_i , τ_j)* - closed , we have X – F is (τ_i , τ_j)* - open in X . Since F $\subseteq$ X – S , we have S $\subseteq$ X – F . Therefore , S $\subseteq$ X – F and X – F is (τ_i , τ_j)* - open.

By the definition of (τ_i , τ_j)* - Q* g closed set , it follows that

(τ_i , τ_j)* - cl (S) $\subseteq$ X – F.

$\Rightarrow$ F $\subseteq$ X – (τ_i , τ_j)* - cl (S) .... (3 . 2).

From (3 . 1) and (3 . 2) , we have

F $\subseteq$ [(τ_i , τ_j)* - cl (S)] $\cap$ [X – (τ_i , τ_j)* - cl (S)] = ϕ.

Therefore , $(\tau_i , \tau_j)^*$ - cl (S) – S contains no non - empty $(\tau_i , \tau_j)^*$ - closed set.

Sufficiency : Suppose that $(\tau_i , \tau_j)^*$ - cl (S) – S contains no non empty $(\tau_i , \tau_j)^*$ - closed set.

Claim : S is $(\tau_i , \tau_j)^*$ - Q* g closed . i.e) to prove $(\tau_i , \tau_j)^*$ - cl (S) $\subseteq$ G whenever S $\subseteq$ G and G is $(\tau_i , \tau_j)^*$ - open in X . Let S $\subseteq$ G and G is $(\tau_i , \tau_j)^*$ - open in X . If $(\tau_i , \tau_j)^*$ - cl (S) $\nsubseteq$ G , we have $(\tau_i , \tau_j)^*$ - cl (S) $\subseteq$ X – G and ((($\tau_i , \tau_j)^*$ - cl (S)) $\cap$ (X – G) $\neq \phi$.

Since S $\subseteq$ G , we have X – G $\subseteq$ X – S.

$\Rightarrow (\tau_i , \tau_j)^*$ - cl (S) $\cap$ (X – G) $\subseteq (\tau_i , \tau_j)^*$ - cl (S) $\cap$ (X– S) .

$\Rightarrow (\tau_i , \tau_j)^*$ - cl (S) $\cap$ (X – G) $\subseteq (\tau_i , \tau_j)^*$ - cl (S) – S.

Since G is $(\tau_i , \tau_j)^*$ - open , we have X – G is a $(\tau_i , \tau_j)^*$ - closed set . Therefore , $(\tau_i , \tau_j)^*$ - cl (S) $\cap$ (X – G) is a non - empty $(\tau_i , \tau_j)^*$ - closed set . This is a contradiction to our assumption that $(\tau_i , \tau_j)^*$ - cl (S) – S contains no non - empty $(\tau_i , \tau_j)^*$ - closed set . Therefore , S is $(\tau_i , \tau_j)^*$ - Q* g closed.

Corollary 3.1 : Let S be $(\tau_1 , \tau_2)^*$ - Q* g closed set in X . Then S is $(\tau_i , \tau_j)^*$ - closed if and only if $(\tau_i , \tau_j)^*$ - cl (S) – S is $(\tau_1 , \tau_2)^*$ - closed.

Proof :

Necessity : Let S be a $(\tau_1 , \tau_2)^*$ - Q* g closed set in X . Suppose that S is $(\tau_1 , \tau_2)^*$ - closed.

Claim : $(\tau_i , \tau_j)^*$ - cl (S) – S is $(\tau_i , \tau_j)^*$ - closed . Since S is $(\tau_i , \tau_j)^*$ - closed , we have $(\tau_i , \tau_j)^*$ - cl (S) = S . $\Rightarrow (\tau_i , \tau_j)^*$ - cl (S) - S = ϕ . Therefore , $(\tau_i , \tau_j)^*$ - cl (S) – S is $(\tau_i , \tau_j)^*$ - closed.

Sufficiency : Suppose that $(\tau_i , \tau_j)^*$ - cl (S) – S is $(\tau_i , \tau_j)^*$ - closed.

Claim : S is $(\tau_1 , \tau_2)^*$ - closed . Since S be $(\tau_1 , \tau_2)^*$ - Q*g closed set in X and $(\tau_i , \tau_j)^*$ - cl (S) – S be $(\tau_i , \tau_j)^*$ - closed . Then $(\tau_i , \tau_j)^*$ - cl (S) – S be does not contain any non empty $(\tau_i , \tau_j)^*$ - closed subset.

$\Rightarrow (\tau_i , \tau_j)^*$ - cl (S) – S = ϕ.

Thus , $(\tau_i , \tau_j)^*$ - cl (S) = S . Therefore , S is $(\tau_i , \tau_j)^*$ - closed in X.

Proposition 3.5 : Let A $\subseteq$ Y $\subseteq$ X and suppose that A is $(\tau_i , \tau_j)^*$ - Q* g closed in X. Then A is $(\tau_i , \tau_j)^*$ - g closed relative to Y.

Proof : Let A $\subseteq$ Y $\subseteq$ X . Suppose that A is $(\tau_i , \tau_j)^*$ - Q* g closed in X.

Claim : A is $(\tau_i , \tau_j)^*$ - g closed relative to Y.

i.e) to prove $\tau_i \tau_j$ - cl (A) $\subseteq$ U whenever A $\subseteq$ U and U is $\tau_i \tau_j$ - open in Y. Let A $\subseteq$ U and U is $\tau_i \tau_j$ - open in Y.

Since , A is (τ_i , τ_j)* - Q* g closed in X we have $\tau_i \tau_j$ - cl (A) $\subseteq$ U whenever A $\subseteq$ U and U is $\tau_i \tau_j$ - open in X.

Since A $\subseteq$ Y $\subseteq$ X , we have A is $\tau_i \tau_j$ - closed in Y and $\tau_i \tau_j$ - cl (A) = A. Since A $\subseteq$ U we have A is (τ_i , τ_j)* - g closed in Y.

Theorem 3.12 : In (X , τ_1 , τ_2) , (τ_i , τ_j)* - gO (X) = (τ_i , τ_j)* - gC (X) if and only if every subset of X is (τ_i , τ_j)* - Q* g closed.

Proof :

Sufficiency : Suppose that (τ_i , τ_j)* - gO (X) = (τ_i , τ_j)* - gC (X) . Let A be a subset of X.

Claim : A is (τ_i , τ_j)* - Q* g closed.

i.e) to prove $\tau_i \tau_j$ - cl (A) $\subseteq$ F whenever A $\subseteq$ F and F is (τ_i , τ_j)* - Q* open . Since every Q* open set is open and let A $\subseteq$ F and F is (τ_i , τ_j)* - open . Since (τ_i , τ_j)* - Q*O (X) = (τ_i , τ_j)* - Q*C (X) , we have F is (τ_i , τ_j)* - closed in X . Since A $\subseteq$ F , we have $\tau_i \tau_j$ - cl (A) $\subseteq$ $\tau_i \tau_j$ - cl (F) = F . Thus A is (τ_i , τ_j)* - Q*g closed.

Necessity : Suppose that A is (τ_i , τ_j)* - Q*g closed.

Claim : (τ_i , τ_j)* - gO (X) = (τ_i , τ_j)* - gC (X) . Let F is (τ_i , τ_j)* - g open in X . Since A is (τ_i , τ_j)* - Q* g closed , F is (τ_1 , τ_2) * - Q* g closed .

$\Rightarrow \tau_i \tau_j$ - cl (F) $\subseteq$ F.

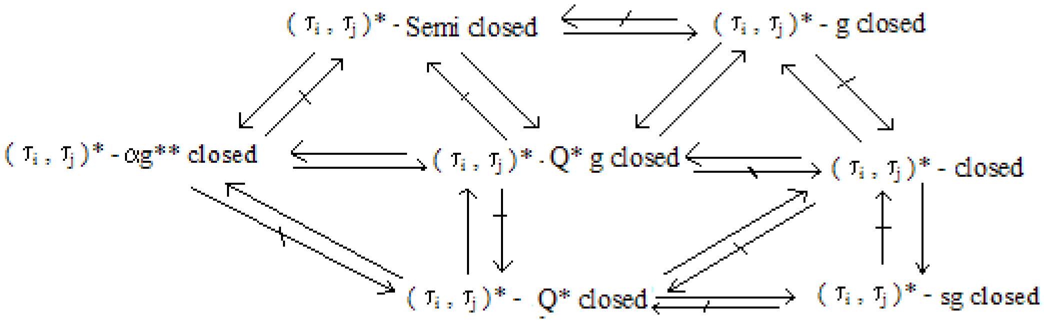
$\Rightarrow \tau_i \tau_j$ - cl (F) = F.

Therefore , F is (τ_i , τ_j)* - closed in X.

Let G is (τ_i , τ_j)* - closed in X . Then X – G is (τ_i , τ_j)* - open . Since X – G is (τ_i , τ_j)* - Q*g closed , we have X – G is (τ_i , τ_j)* - g closed in X . $\Rightarrow$ G is (τ_i , τ_j)* - g open in X.

Therefore , (τ_i , τ_j)* - g O (X) = (τ_i , τ_j)* - gC (X).

Remark 3.12 : The following diagram shows the relationships of Q* g closed sets with known existing sets . A $\rightarrow$ B represents A implies B but not conversely.



Proposition 3.6 : If A is both $(\tau_i , \tau_j)^*$ - g open and $(\tau_i , \tau_j)^*$ - Q*g closed, then A is $(\tau_i , \tau_j)^*$ - g closed.

Proof : Let A be $(\tau_i , \tau_j)^*$ - g open and $(\tau_i , \tau_j)^*$ - Q*g closed.

Claim : A is $(\tau_i , \tau_j)^*$ - g closed.

ie) to prove $\tau_i \tau_j$ - cl (A) = A . Obviously $A \subseteq A$ and A is $(\tau_i , \tau_j)^*$ - g open. Since, A is $(\tau_i , \tau_j)^*$ - Q*g closed, we have $(\tau_i , \tau_j)^*$ - cl (A) $\subseteq$ A . Also A $\subseteq$ $(\tau_i , \tau_j)^*$ - cl (A)

$\Rightarrow (\tau_i , \tau_j)^*$ - cl (A) = A

$\Rightarrow$ A is $(\tau_i , \tau_j)^*$ - closed.

$\Rightarrow$ A is $(\tau_i , \tau_j)^*$ - g closed in X.

4. Applications :

We introduce the following definitions

Definition 4.1 : A space X is called a

i. $(\tau_i , \tau_j)^*$ - Q*g $T_{1/2}$ space if every $(\tau_i , \tau_j)^*$ - closed set is $(\tau_i , \tau_j)^*$ - Q* g closed

ii. $(\tau_i , \tau_j)^*$ - Q* g $T_{3/4}$ space if every $(\tau_i , \tau_j)^*$ - Q* g closed set is $(\tau_i , \tau_j)^*$ - g closed

iii. $(\tau_i , \tau_j)^*$ - Q* g T_S space if every $(\tau_i , \tau_j)^*$ - Q* g closed set is $(\tau_i , \tau_j)^*$ - semi closed

iv. $(\tau_i , \tau_j)^*$ - Q* g T_{sg} space if every $(\tau_i , \tau_j)^*$ - Q* g closed set is $(\tau_i , \tau_j)^*$ - sg closed

v. $(\tau_i , \tau_j)^*$ - Q* g T_C space if every $(\tau_i , \tau_j)^*$ - Q* closed set is $(\tau_i , \tau_j)^*$ - Q* g closed

Proposition 4.1 : A bitopological space X is an $(\tau_i , \tau_j)^*$ - Q*g $T_{1/2}$ - space if and only if { x } is $(\tau_i , \tau_j)^*$ - open or $(\tau_i , \tau_j)^*$ - Q* g closed for each $x \in X$.

Proof : Suppose that X is $(\tau_i , \tau_j)^*$ - Q*g $T_{1/2}$ and for each $x \in X$, {x} is not $(\tau_i , \tau_j)^*$ - Q* g closed . Since X is the only $(\tau_i , \tau_j)^*$ - Q* g open set containing $\{x\}^c$, $\{x\}^c$ is $(\tau_i , \tau_j)^*$ - Q* g closed and thus $(\tau_i , \tau_j)^*$ - Q* closed. Hence { x } is $(\tau_i , \tau_j)^*$ - open.

Conversely, assume that {x} is $(\tau_i , \tau_j)^*$ - open or $(\tau_i , \tau_j)^*$ - g closed for each $x \in X$.

Claim : X is an $(\tau_i , \tau_j)^*$ - Q*g $T_{1/2}$ space . i.e) to prove that, every $(\tau_i , \tau_j)^*$ - Q* g closed set is $(\tau_i , \tau_j)^*$ - closed . By assumption, {x } is $(\tau_i , \tau_j)^*$ - open or $(\tau_i , \tau_j)^*$ - Q*g closed for any $x \in X$. $\Rightarrow$ {x} is $(\tau_i , \tau_j)^*$ - open or $(\tau_i , \tau_j)^*$ - Q* g closed for any $x \in \tau_i \tau_j$ - cl (F).

Case (i) Suppose {x} is $(\tau_i , \tau_j)^*$ - open. Since F is $(\tau_i , \tau_j)^*$ - Q* g closed set, we have $\{x\} \cap F \neq \phi$. Therefore , $x \in X$. $\Rightarrow$ F is $(\tau_i , \tau_j)^*$ - closed.

Case (ii) Suppose {x} is (τ_i , τ_j)* - Q* g closed . If x $\notin$ F then {x} $\subseteq$ τ_i τ_j - cl (F) – F which is a contradiction . Therefore , x $\in$ F. $\Rightarrow$ F is (τ_i , τ_j)* - closed. Thus in both cases, we conclude that F is (τ_i , τ_j)* - closed. Hence , X is an (τ_i , τ_j)* - Q*g $T_{1/2}$ space.

Lemma 4.1 : In any space a singleton is (τ_i , τ_j)* - Q* open if and only if it is (τ_i , τ_j)* - open.

Theorem 4.1 : For a bitopological space X , the following conditions are equivalent

i) X is a (τ_i , τ_j)* - Q*g $T_{3/4}$ space

ii) Every singleton { x } is (τ_i , τ_j)* - Q* open or (τ_i , τ_j)* - g closed.

iii) Every singleton { x } is (τ_i , τ_j)* - open or (τ_i , τ_j)* - Q* g closed.

Proof :

i) $\Rightarrow$ ii) : If { x } is not (τ_i , τ_j)* - g closed then X / { x } is not (τ_i , τ_j)* - g open & thus (τ_i , τ_j)* - Q* g closed . By i) , X / { x } is (τ_i , τ_j)* -g closed . ie) { x } is (τ_i , τ_j)* - Q* open.

ii) $\Rightarrow$ i) : Let A $\subseteq$ X is (τ_i , τ_j)* - Q* g closed . Let x $\in$ τ_i τ_j - cl (A) .

We consider the following two cases :

Case i) : Let { x } be (τ_i , τ_j)* - Q* open . Since x belongs to the τ_i τ_j - closure of A then { x } $\cap$ A $\neq$ ϕ . This shows that x $\in$ A .

Case ii) : Let { x } be (τ_i , τ_j)* - g closed . If we assume that x $\notin$ A , then we would have x $\in$ τ_i τ_j - cl (A) / A , which cannot happen according to lemma 4.1 . Hence x $\in$ A .

So in both cases we have τ_i τ_j - cl (A) $\subseteq$ A . Since the reverse inclusion is trivial , then A = τ_i τ_j - cl (A) or equivalently A is (τ_i , τ_j)* - g closed.

ii) $\Rightarrow$ iii) Follows from lemma 4.1 .

Theorem 4.2 : Every (τ_i , τ_j)* - Q*g $T_{1/2}$ space is (τ_i , τ_j)* - Q*g $T_{3/4}$ space.

Remark 4.1 : The converse of the above theorem is true in general . The following example supports our claim .

Example 4.1 : In example 3.1 , X is (τ_i , τ_j)* - Q*g $T_{3/4}$ space and (τ_i , τ_j)* - Q*g $T_{1/2}$ space.

Theorem 4.3 : In a (τ_i , τ_j)* - Q*g $T_{3/4}$ space , every (τ_i , τ_j)* - Q* g closed set is (τ_i , τ_j)* - g closed.

Proof : Let X be a (τ_i , τ_j)* - Q*g $T_{3/4}$ space . Let A be (τ_i , τ_j)* - Q* g closed set of X . We know that every (τ_i , τ_j)* - Q* g closed set is (τ_i , τ_j)* - g closed . Since X is (τ_i , τ_j)* - Q*g $T_{3/4}$ space , A is (τ_i , τ_j)* - g closed.

Theorem 4.4 : If X is (τ_i , τ_j)* - Q*g $T_{1/2}$ space with Y $\subseteq$ X, then Y is (τ_i , τ_j)* - Q*g $T_{1/2}$ space.

Proof : For y $\in$ Y, { y } is (τ_i , τ_j)* - open or (τ_i , τ_j)* - Q* g closed in X. Using proposition 4.1 , { y } is (τ_i , τ_j)* - open or (τ_i , τ_j)* - Q* g closed in Y.

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