Journal of Progressive Research in Mathematics(JPRM)
ISSN: 2395-0218

SCITECH
RESEARCH ORGANISATION|

Volume 2, Issue 1
February 26, 2015|

Journal of Progressive Research in Mathematics
www.scitecresearch.com

Variable Precision Rough Set Approximations in Concept Lattice

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Abstract

The notions of variable precision rough set and concept lattice are can be shared by a basic notion, which is the definability of a set of objects based on a set of properties. The two theories of rough set and concept lattice can be compared, combined and applied to each other based on definability. Based on introducing the definitions of variable precision rough set and concept lattice, this paper shows that any extension of a concept in concept lattice is an equivalence class of variable precision rough set. After that, we present a definition of lower and upper approximations in concept lattice and generate the lower and upper approximations concept of concept lattice. Afterwards, we discuss the properties of the new lower and upper approximations. Finally, an example is given to show the validity of the properties that the lower and upper approximation have.

Keywords: variable precision rough set; concept lattice; lower approximation; upper approximation.

1. INTRODUCTION

Rough set theory, proposed by Pawlaw [1], is an extension of set theory for the study of intelligent systems characterized by inexact, uncertain or vague information, and aims at handling uncertain information. As a generalization, Ziarko [2] provides a new rough set model---variable precision rough set model. The variable precision rough set is proposed to make the rough set decision rules with certain accuracy, namely the judgment under a certain tolerance. Zhang et al. [3] have introduced the variable precision concept lattice. Belohlavek [4] gives the unity of the structure of different variable precision concept lattice. Qiu [5] introduces four kinds of concepts of variable precision.

A concept lattice, another efficient tool for data analysis, is a hierarchical structure defined by a binary relation between objects and attributes in a data set. The researches on concept lattice mainly focus on the mathematical structure and construction algorithm of concept lattice [6,7,8], relationships with rough set [9-14] , and so on [15-17].

Both theories of rough set and concept lattice are methods to model and manipulate uncertainty, imprecise, incomplete and the vague information. Additionally, the common notion of definability links the two theories together. As a result, we can immediately adopt ideas from one to another. In addition, the notion of concept lattice can be introduced into rough set by considering different types of concepts [18-20]. Meantime, rough set approximation operators can be introduced into concept lattice by considering a different type of definability [21-22].

The combination of variable precision rough set and concept lattice has made great progress in recent years (see [23-29]). Yao [21] provides the notions of rough set approximations defined on concept lattices. As a generalization of Yao [21], Mohanty [18] gives another notion about rough set approximation. However, both [18] and [21] do not give variable precision on undefinable objects though they present two different ways on classifications. The important of variable precision rough set model can be seen Ziarko [2]. To research the effect on variable precision on rough set approximations in undefinable objects, we put forward a method using the upper and lower approximations to solve the calculation of the given set of object (undefined object), and find out the equivalent class which is similar to the given set of object.

In this paper, with the variable precision rough set model, we discuss some properties of the variable precision rough set approximations in concept lattices. The content is organized as follows: Section 2 reviews basic definitions of variable precision rough sets and concept lattices. In Section 3, based on β-upper and lower approximations in variable precision rough set, and combining with the characteristics of concept lattice, we present a kind of approximation operators in a concept lattice. After that, we discuss some properties of the new kind of approximation operators. An example relative to the upper and lower approximations is given. The paper is concluded in Section 4.

2. PRELIMINARIES

To make the paper self-contained, this section will review some basic facts in regard of variable precision rough set and concept lattice respectively. For more details, please refer to [2] for variable precision rough set theory, and see [30] and [31] for concept lattice theory. A running example is provided in this section.

2.1 Variable Precision Rough Set

Let U be a finite set, that is, an universe set. Let $X,Y\subseteq U$, and $X,Y\notin\{\varnothing\}$. We say that X is included in Y, or $Y\supseteq X$, if for all $e\in X$ implies $e\in Y$. Clearly, there is no room for even the slightest misclassification according to this definition. The measure $c(X,Y)$ of the relative degree of misclassification of the set X with respect to set Y is defined.

Definition 1[2] We defined $c(X,Y)$ as follows:

$$c(X,Y)=1-\,card(X\cap Y)/\,card(X) \quad \text{if } card(X)>0,$$

$$c(X,Y)=0 \qquad \text{if } card(X)=0,$$

where card(Z) denotes the set cardinality of Z .

The specified majority requirement the admissible level of classification error β must be within the range $0\le\beta<0.5$.

Be replacing the inclusion relation with a majority inclusion relation in the original definition of lower approximation of a set, the generalized notion of β-lower and upper approximation as follows.

Definition 2[2] For a set $U\supseteq X$, its generalized notion of β-lower approximation or β-positive region is defined by:

$$\underline{R}_{\beta}X=\cup\{E\in R^*|\ c(X,Y)\le\beta\}.$$

The β-upper approximation of the set $U\supseteq X$ is defined as:

$$\overline{R}_{\beta}X=\cup\{E\in R^*|\ c(X,Y)\le 1-\beta\}.$$

Where $R^*=\{E_1, E_2,\ldots, E_n\}$ is an equivalence class on U.

Remark 1 In fact, the β-lower approximation of the set X can be interpreted as the collection of all those elements of U which can be classified into X with the classification error not greater thanβ. Similarly, theβ-upper approximation of the set X is the collection of all those elements of U which can be classified into X with the classification error less than $1-\beta$.

Lemma 1[2] For every $0\le\beta<0.5$ the following relationships are true

(1a) $X\overset{\beta}{\supseteq}\underline{R}_{\beta}X$

(1b) $\overline{R}_{\beta}X\supseteq\underline{R}_{\beta}X$

(2) $\underline{R}_{\beta}\phi=\overline{R}_{\beta}\phi=\phi;\underline{R}_{\beta}U=\overline{R}_{\beta}U=U$

(3) $\overline{R}_{\beta}(X\cup Y)\supseteq\overline{R}_{\beta}X\cup\overline{R}_{\beta}Y$

(4) $\underline{R}_{\beta}X\cap\underline{R}_{\beta}Y\supseteq\underline{R}_{\beta}(X\cap Y)$

(5) $\underline{R}_{\beta}(X\cup Y)\supseteq\underline{R}_{\beta}X\cup\underline{R}_{\beta}Y$

(6) $\overline{R}_{\beta}X\cap\overline{R}_{\beta}Y\supseteq\overline{R}_{\beta}(X\cap Y)$

(7) $\underline{R}_{\beta}(-X)=-\overline{R}_{\beta}X$

(8) $\overline{R}_{\beta}(-X)=-\underline{R}_{\beta}X$.

2.2 Concept Lattice

Concept lattice deals with visual presentation and analysis of data (see [21,31,32]) and focuses on the definability of a set of objects based on a set of attributes, and vice versa.

Definition 3[30,31] A context is a triple (G,M,I) where G and M are sets and $I \subseteq G\times M$. The elements of G and M are called objects and attributes respectively. As usual, instead of writing $(g,m)\in I$, we write gIm and say 'the object g has the attribute m'.

For $A\subseteq G$ and $B\subseteq M$, define

$$A'=\{m\in M \mid (\forall g\in A)\ gIm\};$$

$$B'=\{g\in G \mid (\forall m\in B)\ gIm\}.$$

From Definition 3, the authors point that A' is the set of attributes common to all the objects in A and B' is the set of objects possessing the attributes in B.

Lemma 2[30] Assume that (G,M,I) is a context. Let A, $A_j\subseteq G$ and B, $B_j \subseteq M$, For $j\in J$, there are the following statements:

(1) $A\subseteq A''$, $B\subseteq B''$; (2) $A_1\subseteq A_2 \Rightarrow A_1'\supseteq A_2'$, $B_1\subseteq B_2 \Rightarrow B_1'\supseteq B_2'$;

(3) $A'=A'''$, $B'=B'''$; (4) $A\subseteq B' \Leftrightarrow A'\supseteq B$;

(5) $\left(\bigcup_{j\in J} A_j\right)'=\bigcap_{j\in J} A_j'$, $\left(\bigcup_{j\in J} B_j\right)'=\bigcap_{j\in J} B_j'$.

Definition 4[30,31] For $A \subseteq G$ and $B \subseteq M$, the pair (A,B) is called a concept of (G,M,I) if $A=B'$ and $B= A'$, and A is the extension of the concept, B is the intension of the concept.

Remark 2 The authors [30] and [31] indicate that a subset A of G is the extension of some concept if and only if $A''=A$ in which case the unique concept of which A is an extension of (A,A'').

Definition 5[30] (1) The set of all concepts from a context (G,M,I) called a concept lattice and is denoted by:

$$\boldsymbol{B}=\boldsymbol{B}(G,M,I)=\{(A,B) \mid A\in G, B\in M \text{ and } A=B',\ B=A'\}.$$

Then we can define:

$$\boldsymbol{B}_G=\{A \mid A\in G,\ (A,B)\in \boldsymbol{B}(G,M,I)\},\ \boldsymbol{B}_M=\{B \mid B\in M,\ (A,B)\in \boldsymbol{B}(G,M,I)\}.$$

(2) For concepts (A_1,B_1) and (A_2,B_2) in $\boldsymbol{B}(G,M,I)$, we write $(A_1,B_1)\le(A_2,B_2)$, if $A_1\subseteq A_2$. Also $A_1\subseteq A_2$ implies that $A_1'\supseteq A_2'$, and the reverse implication is also valid, because $A_1=A_1''$ and $A_2''=A_2$. We therefore have

$$(A_1,B_1)\le(A_2,B_2) \Leftrightarrow A_1\subseteq A_2 \Leftrightarrow B_1\supseteq B_2.$$

Remark 3 The author [30] state that the relation $\le$ is an order on $\boldsymbol{B}(G,M,I)$. We still call $\langle \boldsymbol{B}(G,M,I),\le\rangle$ a concept lattice.

Lemma 3[30] Let (G,M,I) be a context. Then $\langle \boldsymbol{B}(G,M,I),\le\rangle$ is a complete lattice in which join and meet are given by:

$$\bigvee_{j\in J}(A_j,B_j)=\left(\left(\bigcup_{j\in J}A_j\right)'',\bigcap_{j\in J}B_j\right),$$

$$\bigwedge_{j\in J}(A_j,B_j)=\left(\left(\bigcap_{j\in J}A_j\right),\left(\bigcup_{j\in J}B_j\right)''\right).$$

Where J is an index set and for every $j\in J$, (A_j,B_j) is a concept.

Example 1 The ideas of concept lattice can be illustrated by [21, Example 1]. Let (G,M,I) as Table 1, where the meaning of each attribute is given as follows: a: needs water to live; b: lives in water; c: lives on land; d: needs chlorophyll to produce food; e: two seed leaves; f: one seed leaf; g: can mood around; h: has limbs; i: suckles its off spring. [21, Example 1] points that the concept lattice of (G,M,I) is Figure 1.

Table 1: A context

	a	b	c	d	e	f	g	h	i
1.Leech	×	×					×		
2.Bream	×	×					×	×	
3.Frog	×	×	×				×	×	
4.Dog	×		×				×	×	×
5.Spike-weed	×	×		×		×			
6.Reed	×	×	×	×		×			
7.Bean	×		×	×	×				
8.Maize	×		×	×		×			

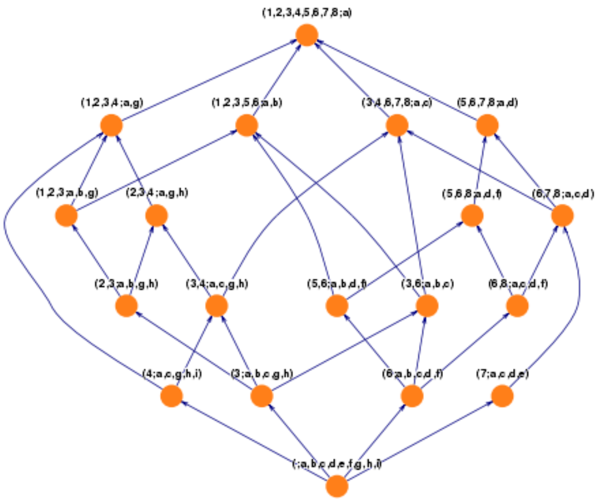

Fig.1: concept lattice for the context of Table 1

2.3 Equivalence Class and the Corresponding Relation of Concept Lattice

Lemma 4 [32] Let (G,M,I) be a context, $P \subseteq M$ and $P \neq \phi$. The following statements holds:

(1) Let $(A,B) \in \boldsymbol{B}(G,M,I)$. Then $A = [x]_B$ is correct for any $x \in A$.

(2) $([x]_P, [x]_{P'}) \in \boldsymbol{B}(G,M,I)$ is correct for any $x \in G$.

Lemma 4 shows that any extension of a concept in concept lattice must be an equivalence class of rough set. Conversely, any of the equivalence class in a rough set is the extension of a concept in concept lattice. Based on these corresponding relationships, the upper and lower approximations will make those undefinable sets of the objects approximate be definable sets of objects which are the extension of a concept lattice.

APPROXIMATIONS IN CONCEPT LATTICE

A concept lattice consists of a definable set of objects and a definable set of attributes, which uniquely determine each other. The concept lattice is the family of all such definable concepts. An arbitrary set of objects may not be the extension of a concept. Therefore, the set can be viewed as an undefinable set of objects. Following variable precision rough set theory, such a set of objects can be approximated by definable sets of objects. In this section, a method of approximations is discussed by using the subsystem based on formulation of variable precision rough set theory.

2.4 Definitions of Lower and Upper Approximations

In order to classify the undefinble object of concept lattice, find out the extension of concept lattice which is similar to the undefinble object, and combine the lower and upper approximations of the variable precision rough set, this section will put forward the variable precision rough set approximation in concept lattice, and some properties on rough set are discussed.

Definition 6 Let $\boldsymbol{B}(G,M,I)$ be a concept lattice, $X \subseteq G$ and $0 \le \beta < 0.5$. Then the lower approximation of X is defined by:

$$\underline{LR}_\beta X = \vee\{E \subseteq G \mid c(E,X) \le \beta, \exists Y \in M, (E,Y) \in \boldsymbol{B}\},$$

And the upper approximation of X is defined by:

$$\overline{LR}_\beta X = \vee\{E \subseteq G \mid c(E,X) < 1-\beta, \exists Y \in M, (E,Y) \in \boldsymbol{B}\}.$$

We called $(\underline{LR}_\beta X, (\underline{LR}_\beta X)')$ the lower approximation concept of $\boldsymbol{B}(G,M,I)$, and $(\overline{LR}_\beta X, (\overline{LR}_\beta X)')$ called the upper approximation concept of $\boldsymbol{B}(G,M,I)$..

The set $X \subseteq G$ is rough with respect to the operator LR_β if and only if $\underline{LR}_\beta X \ne \overline{LR}_\beta X$, otherwise X is exact with respect to the operator LR_β.

Remark 4 (1) From the definition of $\underline{LR}_\beta X$, we know that the concept $(\underline{LR}_\beta X, (\underline{LR}_\beta X)')$ is the supremum of those concepts whose the relative degree of misclassification of the extensions with respect to X is less than or equal to β, and $(\overline{LR}_\beta X, (\overline{LR}_\beta X)')$ is the supremum of those concepts whose the relative degree of misclassification of the extensions with respect to X is less than 1-β.

(2) For a concept (A,B), the complementary of A may not necessarily be the extension of a concept. $\boldsymbol{B}(G,M,I)$. May not be a complemented lattice. The approximation operators $\underline{LR}_\beta X$ and $\overline{LR}_\beta X$ are not necessarily dual operators.

As confirm know that an intersection of extensions is an extension of a concept. However, the union of extensions may not be the extension of a concept. This follows that $\underline{LR}_\beta X$ may contain X. Hence, $c(E,X)>\beta$ may be set up. We may easily attain that the new approximation operators do not satisfy properties (1a), (7) and (8) in Lemma 1.

To continue our discussion, we need to find some properties for the lower and upper approximation operators.

Theorem 1 For the set of objects $X, Y \in G$, we have obtain:

(1b) $\overline{LR}_\beta X \supseteq \underline{LR}_\beta X$

(2) $\underline{LR}_\beta \phi = \overline{LR}_\beta \phi = \phi; \underline{LR}_\beta U = \overline{LR}_\beta U = U$

(3) $\overline{LR}_\beta (X \cup Y) \supseteq \overline{LR}_\beta X \cup \overline{LR}_\beta Y$

(4) $\underline{LR}_\beta X \cap \underline{LR}_\beta Y \supseteq \underline{LR}_\beta (X \cap Y)$

(5) $\underline{LR}_\beta (X \cup Y) \supseteq \underline{LR}_\beta X \cup \underline{LR}_\beta Y$

(6) $\overline{LR}_\beta X \cap \overline{LR}_\beta Y \supseteq \overline{LR}_\beta (X \cap Y)$

Proof. (1b) By Definition 6 we can conclude the inclusion $\overline{LR}_\beta X \supseteq \underline{LR}_\beta X$.

(2) Since $c(E,\phi)=1$ for any given extensions in $\boldsymbol{B}(G,M,I)$, we get $\underline{LR}_\beta \phi = \phi$ and $\overline{LR}_\beta \phi = \phi$. Similarly, since $c(E,U)=0$, we can obtain $\underline{LR}_\beta U = U$ and $\overline{LR}_\beta U = U$.

(3) By Definition 1 we affirm that if X, $Y \subseteq G$, then $c(E, X \cup Y) \le c(E,X)$ and $c(E,X \cup Y) \le c(E,Y)$. By Definition 6, we confirm $\overline{LR}_\beta (X \cup Y) \supseteq \overline{LR}_\beta X$ and $\overline{LR}_\beta (X \cup Y) \supseteq \overline{LR}_\beta$ Y.

Thus $\overline{LR}_\beta (X \cup Y) \supseteq \overline{LR}_\beta X \cup \overline{LR}_\beta Y$ is correct.

(4) This inclusion is also consequence of the relationship $c(E, X \cap Y) \ge c(E,X)$ and $c(E, X \cap Y) \ge c(E,Y)$. By

Definition 6, we can conclude

$$\underline{LR}_{\beta}X \supseteq \underline{LR}_{\beta}(X \cap Y) \text{ and } \underline{LR}_{\beta}Y \supseteq \underline{LR}_{\beta}(X \cap Y).$$

Then, we have $\underline{LR}_{\beta}X \cap \underline{LR}_{\beta}Y \supseteq \underline{LR}_{\beta}(X \cap Y)$.

(5) This property follows again from the fact given in item (3).

(6) Analogously to the discussion as item (4).

2.5 Some Exploration of Approximation in Concept Lattice

This section mainly from three aspects to explore the variable precision rough set approximation in concept lattice.

1) The effect on the lower and upper approximations for the change of β;
2) The lower and upper approximations concept lattice;
3) The relation between the lower approximation $\underline{LR}_0 X$ and X''.

The above 1)---3) will be described as Subsections 3.2.1---3.2.3 respectively. To illustrate the theory validity, we give an example.

2.5.1 Measure of Approximation

We give an definition of measure of approximation in order to observe the effect on the lower and upper approximation for the change of β.

Definition 7 We define the measure of approximation as follows:

$$\alpha(LR,\beta,X) = card\left(\underline{LR}_{\beta}X\right)/card\left(\overline{LR}_{\beta}X\right),$$

where $0 \le \beta < 0.5$.

Remark 5 The β-accuracy represents the imprecision of the approximate characterization of the set X relative to assumed classification error β. It is interesting to note that with the increase of β the cardinality of the upper approximation will tend downward and the size of the lower approximation will tend upward which leads to the conclusion that is consistent with intuition that the relative accuracy may increase at the expense of a higher classification error.

2.5.2 Change of β

Theorem 2 Let $X \subseteq G$ and $0 \le \beta_1 < \beta_2 < 0.5$. Then

$$\underline{LR}_{\beta_1}X \subseteq \underline{LR}_{\beta_2}X, \qquad (3.2.1)$$

$$\overline{LR}_{\beta_1}X \supseteq \overline{LR}_{\beta_2}X. \qquad (3.2.2)$$

Proof. To prove the expression (3.2.1).

From this question $c(E,X) \le \beta_1 < \beta_2 < 0.5$ holds, from Definition 6, we obtain $\underline{LR}_{\beta_1}X \subseteq \underline{LR}_{\beta_2}X$;

To prove the expression (3.2.2).

From this question, we can have $1-\beta_1 > 1-\beta_2$, and $c(E,X) < 1-\beta_2 < 1-\beta_1$ holds. According to Definition 6, we confirm $\overline{LR}_{\beta_1}X \supseteq \overline{LR}_{\beta_2}X$.

Remark 6 For a given set X of objects, we can classify X as we like mathematical ways. However, in general, it allows a certain error classification to be existed. When β changes, the accuracy of the classification also changes. According to the different requirements for accuracy, β can be selected appropriately.

The following will obtain the results about the lower and upper approximation concept lattices with the changing of β.

Theorem 3 (1) Let $0 \le \beta_1 < \beta_2 < \beta_3 < \ldots < \beta_n < 0.5$. Then $\underline{LR}_{\beta_1}X \subseteq \underline{LR}_{\beta_2}X \subseteq \underline{LR}_{\beta_3}X \subseteq \ldots \subseteq \underline{LR}_{\beta_n}X$.

(2) Let $P = \left\{\underline{LR}_{\beta_1}X, \underline{LR}_{\beta_2}X, \ldots, \underline{LR}_{\beta_n}X\right\}$. Then P is a chain.

(3) Let $P(G,M,I)=\left\{\left(\underline{LR}_{\beta_i}X,\left(\underline{LR}_{\beta_i}X\right)'\right)|1\le i\le n,\underline{LR}_{\beta_i}X=\left(\underline{LR}_{\beta_i}X\right)''\right\}$. Then $\langle P(G,M,I),\le\rangle$ is a lower approximation concept lattice, and $P(G,M,I)\subseteq \boldsymbol{B}(G,M,I)$ holds.

Proof. To prove item (1), let $0\le\beta_1<\beta_2<\beta_3<\ldots<\beta_n<0.5$. Using Theorem 2, we obtain

$$\underline{LR}_{\beta_1}X\subseteq\underline{LR}_{\beta_2}X\subseteq\underline{LR}_{\beta_3}X\subseteq\ldots\subseteq\underline{LR}_{\beta_n}X,$$

To prove item (2), let $P=\{\underline{LR}_{\beta_1}X,\underline{LR}_{\beta_2}X,\ldots,\underline{LR}_{\beta_n}X\}$, then we know P is an ordered set, for any element of P are comparable, thus P is a chain.

To prove item (3), from the Definition 6, $\underline{LR}_{\beta_i}X$ is the extension of the concept $\left(\underline{LR}_{\beta_i}X,\left(\underline{LR}_{\beta_i}X\right)'\right)$.

Let $P(G,M,I)=\left\{\left(\underline{LR}_{\beta_i}X,\left(\underline{LR}_{\beta_i}X\right)'\right)|1\le i\le n,\underline{LR}_{\beta_i}X=\left(\underline{LR}_{\beta_i}X\right)''\right\}$.

Then by Definition 5(2), we obtain that $\langle P(G,M,I),\le\rangle$ is a lower approximation concept lattice. From Definition 6, we confirm $\left(\underline{LR}_{\beta_i}X,\left(\underline{LR}_{\beta_i}X\right)'\right)\in\boldsymbol{B}(G,M,I)$. Thus, we claim $P(G,M,I)\subseteq\boldsymbol{B}(G,M,I)$.

Dually, Q is also a chain, when $Q=\{\overline{LR}_{\beta_1}X,\overline{LR}_{\beta_2}X,\ldots,\overline{LR}_{\beta_n}X\}$, $\langle Q(G,M,I),\le\rangle$ is a upper approximation concept lattice, when

$$Q(G,M,I)=\left\{\left(\overline{LR}_{\beta_i}X,\left(\overline{LR}_{\beta_i}X\right)'\right)|1\le i\le n,\overline{LR}_{\beta_i}X=\left(\overline{LR}_{\beta_i}X\right)''\right\},$$

and $Q(G,M,I)\subseteq\boldsymbol{B}(G,M,I)$.

2.5.3 β=0

Theorem 4: Let $\boldsymbol{B}(G,M,I)$ is a concept lattice. Then $\underline{LR}_0X\subseteq X''$ holds for any $X\subseteq G$.

Proof. By Definition 6 and β=0, we attain

$$\underline{LR}_0X=\vee\{E\subseteq G\,|\,c(E,X)=0,\exists Y\in M,(E,Y)\in\boldsymbol{B}\}.$$

Since $c(E,X)=0$ follows $1-\frac{card(E\cap X)}{card(E)}=0$. Moreover, we obtain $\frac{card(E\cap X)}{card(E)}=1$.This means $card(E\cap X)=cardE$.

In light of Definition 1, we affirm $E\cap X=E$, this implies $E\subseteq X$, and $\underline{LR}_0X=\vee\{E\subseteq G\,|\,E\subseteq X,\exists Y\in M,(E,Y)\in\boldsymbol{B}\}$ holds.

For $E\in\boldsymbol{B}_G$, $E\in X$, we have $E=E''\subseteq X''$, if $E_1,E_2\in\boldsymbol{B}_G$, $E_1\subseteq X,E_2\subseteq X$, then $E_1\subseteq X'',E_2\subseteq X''$, that is $E_1\cup E_2\subseteq X''$, $(E_1\cup E_2)'\supseteq X'''=X'$, $(E_1\cup E_2)''\subseteq X''$, through the recurrence method we can obtain $\underline{LR}_0X\subseteq X''$.

Remark 7: $\boldsymbol{B}_G$ is the set of all the extensions of concept lattice.

(1) It is notes that with the decrease of β the lower approximation will tend to $\underline{lapr}(X)$, $\underline{lapr}(X)$ shown in [4]. When β =0, $\underline{LR}_0X=\underline{lapr}(X)$;

(2) When X only contains E, and $E\ne\phi,E\in\boldsymbol{B}_G$, then $\underline{LR}_0X=E$, and the lower approximation concept of the concept lattice is (E,E');

(3) When X contains $E_1,E_2,\ldots,E_i,(i\ge2)$ and $E_1\cup E_2\cup\ldots\cup E_i=E\in\boldsymbol{B}_G$, then $\underline{LR}_0X=E$, and the lower approximation concept is (E,E');

(4) When X contains $E_1,E_2,\ldots,E_i,(i\ge2)$ and $E_1\cup E_2\cup\ldots\cup E_i=E\in\boldsymbol{B}_G$ not establish, then $\underline{LR}_0X=E''$, especially,

when $E_1 \cup E_2 \cup \ldots \cup E_i = X$, then $\underline{LR}_0 X = X''$, and the lower approximation concept is (X'', X');

(5) When $X \in \boldsymbol{B}_G$, then $\underline{LR}_0 X = X$, and the lower approximation concept is (X'', X');

(6) For $\forall X \subseteq G$, we have $\overline{LR}_0 X = U$, and the upper approximation concept is (U, U').

2.6 An Example

In this section we give an example to show the validities of the results in Subsections 3.1 and 3.2.

Example 2: Let (*G*,*M*,*I*) be shown as Table 1. According to Example 1, we obtain ***B***(*G*,*M*,*I*) as Figure 1. Let $X = \{3,5,6\} \subseteq G$ and $\beta = 0.4$. From Figure 1, we can obtain all of the extensions of concept lattice, they are also equivalence classes:

$E_1 = \{1,2,3,4,5,6,7,8\}$, $E_2 = \{1,2,3,4\}$, $E_3 = \{1,2,3,5,6\}$, $E_4 = \{3,4,6,7,8\}$, $E_5 = \{5,6,7,8\}$, $E_6 = \{1,2,3\}$, $E_7 = \{2,3,4\}$
$E_8 = \{5,6,8\}$, $E_9 = \{6,7,8\}$, $E_{10} = \{2,3\}$, $E_{11} = \{3,4\}$, $E_{12} = \{5,6\}$, $E_{13} = \{3,6\}$, $E_{14} = \{6,8\}$, $E_{15} = \{4\}$, $E_{16} = \{3\}$, $E_{17} = \{6\}$,
$E_{18} = \{7\}$, $E_{19} = \{\phi\}$ and $X = \{3,5,6\}$.

The classification errors of the set X computed for all classes are:

$c(E_1, X) = 1 - \dfrac{card(E_1 \cap X)}{card E_1} = 1 - \dfrac{3}{8} = \dfrac{5}{8}$, $c(E_2, X) = \dfrac{3}{4}$, $c(E_3, X) = \dfrac{2}{5}$, $c(E_4, X) = \dfrac{3}{5}$, $c(E_5, X) = \dfrac{1}{2}$, $c(E_6, X) = \dfrac{2}{3}$,
$c(E_7, X) = \dfrac{2}{3}$, $c(E_8, X) = \dfrac{1}{3}$, $c(E_9, X) = \dfrac{2}{3}$, $c(E_{10}, X) = \dfrac{1}{2}$, $c(E_{11}, X) = \dfrac{1}{2}$, $c(E_{12}, X) = 0$, $c(E_{13}, X) = 0$, $c(E_{14}, X) = \dfrac{1}{2}$,
$c(E_{15}, X) = c(E_{18}, X) = 1$, $c(E_{16}, X) = c(E_{17}, X) = c(E_{19}, X) = 0$.

The extension of concept that satisfied $c(E, X) \leq 0.4$ is:

$$\{\phi, \{1,2,3,5,6\}, \{5,6,8\}, \{3,6\}, \{5,6\}, \{3\}, \{6\}\},$$

the corresponding family of concept is:

$\{(\phi, \{a,b,c,d,e,f,g,h,i\}), (\{1,2,3,5,6\}, \{a,b\}), (\{5,6,8\}, \{a,d,f\}), (\{3,6\}, \{a,b,c\}), (\{5,6\}, \{a,b,d,f\}),$
$(\{3\}, \{a,b,c,g,h\}), (\{6\}\{a,b,c,d,f\})\}$,

their join is the concept:

$$\left((\phi \cup \ldots \cup \{6\})'', (\{a,b,c,d,e,f,g,h,i\} \cap \ldots \cap \{a,b,c,d,f\})\right) = (\{1,2,3,4,5,6,7,8\}, \{a\}),$$

the lower approximation is: $\underline{LR}_{0.4} X = \{1,2,3,4,5,6,7,8\}$.

From Definition 6, the extension of concept that satisfied $c(E, X) < 1 - 0.4 = 0.6$ is:

$$\{\phi, 1,2,3,5,6, \{5,6,7,8\}, \{5,6,8\}, \{2,3\}, \{3,4\}, \{3,6\}, \{6,8\}, \{5,6\}, \{3\}, \{6\}\},$$

their join is the concept:

$$\left((\phi \cup \ldots \cup \{6\})'', (\{a,b,c,d,e,f,g,h,i\} \cap \ldots \cap \{a,b,c,d,f\})\right) = (\{1,2,3,4,5,6,7,8\}, \{a\}),$$

the upper approximation is: $\overline{LR}_{0.4} X = \{1,2,3,4,5,6,7,8\}$.

From the result of Example 2, we can exam the validity (1b) in Theorem 1. The other results (2)—(6) in Theorem 1 can be exam as the about similar way.

To exam the validity of Theorem 2, let $\beta = 0.2$, we calculated the lower and upper approximation

$$\underline{LR}_{0.2} X = \{1,2,3,5,6\}, \text{ and } \overline{LR}_{0.2} X = \{1,2,3,4,5,6,7,8\}.$$

We can easily obtain:

$$\underline{LR}_{0.2}X \subseteq \underline{LR}_{0.4}X \text{ and } \overline{LR}_{0.2}X \supseteq \overline{LR}_{0.4}X .$$

To exam the validity of Theorem 4, we can computed $X'' = \{1,2,3,5,6\}$, $\underline{LR}_0 X = \{1,2,3,5,6\}$, there is $\underline{LR}_0 X \subseteq X''$.

3. CONCLUSION

Rough set theory is a hotspot of information science. As a development of rough set, variable precision rough set aims at dealing with uncertain or imprecise information, though it is still restricted under the equivalence relation. There is one-to-one correspondence between extension of a concept in concept lattice and equivalence class of variable precision rough set. Based on variable precision rough set of β-lower and upper approximations, we provide a kind of approximation operators in concept lattice. This operator can classify the undefinable object in the context into the definable objects. Combining variable precision rough set and concept lattice with all discussions, in this paper, we can have a better understanding of data analysis.

In the future, how to combine the advantages of the variable precision rough set and concept lattice effectively, how to research and improvement the knowledge reduction, rule acquisition, uncertainty information processing and huge amounts of data mining are the goal of our study.

Acknowledgments

This paper is granted by NSF of China (61073121) and NSF of Hebei province (A2013201119).

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