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A New Approach to solve Fuzzy Transportation Problem for Trapezoidal Number

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Abstract

In this paper we are study on fuzzy transportation problem for industries to reduce the transportation cost of commodity from one source to another source. In this paper we are taking transportation cost, demand and supply all are in fuzzy trapezoidal number because the fuzzy number satisfy the condition of vagueness. Here we are using the propose algorithm to obtained the fuzzy optimal solution of fuzzy transportation problem with membership function. The solution procedure is illustrated with numerical example.

Keywords: Fuzzy Transportation Problem; Trapezoidal Number; Fuzzy Optimal Solution; Membership Function.

1. INTRODUCTION

A fuzzy transportation problem is a transportation problem in which the transportation costs, supply and demand quantities are fuzzy quantities. The objective of the fuzzy transportation problem is to determine the shipping schedule that minimizes the total fuzzy transportation cost while satisfying fuzzy supply and demand limits. In the real world is inevitable owing to some unexpected situations. There are cases that the cost coefficients and the supply and demand quantities of a transportation problem may be uncertain due to some uncontrollable factors. To deal quantitatively with imprecise information in making decisions, Bellman and Zadeh [2] and Zadeh [15] introduced the notion of fuzziness. By using this fact much researcher study on fuzzy transportation problem like Nuran [8] solve fuzzy transportation problem at two stages, in first stage they calculated satisfaction level between fuzzy demand and fuzzy supplies and in second stage by considering the unit transportation costs from zero to maximum satisfaction level. Pandian and Natarajan [11] introduced the new algorithm, zero point method to find the fuzzy optimal solution of fuzzy transportation problem. Parra et al [12] proposed a method for solving fuzzy transportation problem and also to find the possibility distribution of the objective value of the transportation problem provided all the inequality constraints are of ≤ types or ≥ types. Chanas et al [3] developed a method for solving transportation problems with fuzzy supplies and demands via the parametric programming technique using the Bellman-Zadeh certerion [2]. Chanas and Kuchta [4] introduced a method for solving a transportation problem with fuzzy cost coefficient by transforming the given problem to a bicriterial transportation problem with crisp objective function which provides only crisp solution to the given transportation problem. Liu and Kao [10] developed a solution procedure for computing the fuzzy objective value of the fuzzy transportation problem, where at least one of the parameters are fuzzy numbers using Zadeh's extension principal. Nagoor Gani and Abdul Razak [11] obtained a fuzzy solution for a two stage cost minimizing fuzzy transportation problem in which supplies and demands are trapezoidal fuzzy numbers using a parametric approach. Ritha and Vinotha [13] presenting two stage cost minimizing fuzzy transportation problem with multi-objective constraints they used fuzzy geometric programming approach to solve multi-objective fuzzy transportation problem. Verma et al [14] apply the fuzzy programming technique with hyperbolic and exponential membership functions to solve multi-objective transportation problem, the solution derived is a compromise solution.

In this paper, we are using proposed algorithm for finding the fuzzy optimal solution for a fuzzy transportation problem where all parameters transportation cost, demand, and supply are in fuzzy trapezoidal fuzzy. Because we know that the fuzzy number satisfies the condition of vagueness i.e. condition of uncertainty of demand of market and the supply of availability.

When we use proposed new algorithm for finding an optimal solution for a fuzzy transportation problem, we have the following advantages:

(i) We o not use linear programming techniques.

(ii) We do not use goal and parametric programming technique.

(iii) The optimal solution is a fuzzy number.

2. BASIC DEFINITION AND FORMULATION:

2.1 Definition: Let A be a classical set and $\mu_A(x)$ be a function from A to [0,1]. A fuzzy set A* with membership function $\mu_A(x)$ is defined by $A^* = \{(x, \mu_A(x)) : x \in A \text{ and } \mu_A(x) \in [0,1]\}$

2.2 Definition: A real fuzzy number A = (a_1, a_2, a_3, a_4) is a fuzzy subset from the real line R with the membership function $\mu_A(x)$ satisfying the following conditions:

(i) $\mu_A(x)$ is a continuous mapping from R to the closed interval [0,1].

(ii) $\mu_A(x)$ = 0 for every $a\epsilon(-\infty, a_1]$

(iii) $\mu_A(x)$ is strictly increasing and continuous on $[a_1, a_2]$

(iv) $\mu_A(x)$ = 1 for every $a\epsilon[a_2, a_3]$

(v) $\mu_A(x)$ is strictly decreasing and continuous on $[a_3, a_4]$

(vi) $\mu_A(x)$ = 0 for every $a\epsilon[a_4, +\infty]$

2.3 Definition: A fuzzy number A is a trapezoidal fuzzy number denoted by **A** = (a_1, a_2, a_3, a_4) where a_1, a_2, a_3 and a_4 are real numbers and its membership function $\mu_A(x)$ is given below:

$$\mu_A(x) = \begin{cases} 0 & \text{for } x \le a_1 \\ \frac{(x - a_1)}{(a_2 - a_1)} & \text{for } a_1 \le x \le a_2 \\ 1 & \text{for } a_2 \le x \le a_3 \\ \frac{(a_4 - x)}{(a_4 - a_3)} & \text{for } a_3 \le x \le a_4 \\ 0 & \text{for } x \ge a_4 \end{cases}$$

We need the following definition of the basic arithmetic operators on fuzzy trapezoidal numbers based on the function principal which can be found in [3,5].

2.4 Definition: If $A = (a_1, a_2, a_3, a_4)$ is trapezoidal fuzzy number, then the defuzzified value or the ordinary crisp of A, a given below

$$a = \frac{(a_1 + 2a_2 + 2a_3 + a_4)}{6}$$

We need the following definitions of ordering on the set of fuzzy numbers based on the magnitude of a fuzzy number which can be found in [1]

2.5 Definition:

- **Fuzzy Feasible solution:** Any set of fuzzy non negative allocations $x_{ij} = [-2\delta, -1\delta, 1\delta, 2\delta]$ where δsmall positive number, which satisfies the row is and column sum is a fuzzy feasible solution.
- **Fuzzy basic feasible solution:** A feasible solution is a fuzzy basic feasible solution if the number of non negative allocation is at most (m + n -1) where m is the number of rows and n is the number of columns in transportation table.
- **Fuzzy non degenerate basic feasible solution:** Any fuzzy feasible solution to a transportation problem containing m origins and n destinations is said to be fuzzy non degenerate, if it contains exactly (m + n -1) occupied cells.
- **Fuzzy degenerate basic feasible solution:** If a fuzzy basic solution contains less than (m + n -1) non negative allocations, it is said to be degenerate.

2.6 Definition: If A = (a_1, a_2, a_3, a_4) and B = (b_1, b_2, b_3, b_4) two trapezoidal fuzzy numbers then the arithmetic operations on A and B as follows:

- **Addition :**

$$(a_1, a_2, a_3, a_4) \oplus (b_1, b_2, b_3, b_4) = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$$

- **Subtraction:**

$$(a_1, a_2, a_3, a_4) \ominus (b_1, b_2, b_3, b_4) = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$$

- **Multiplication:** $(a_1, a_2, a_3, a_4) \otimes (b_1, b_2, b_3, b_4) = (t_1, t_2, t_3, t_4)$

Where
$$t_1 = minimum\{a_1b_1, a_1b_4, a_4b_1, a_4b_4\}$$
$$t_2 = minimum\{a_2b_2, a_2b_3, a_3b_2, a_3b_3\}$$
$$t_3 = maximum\{a_2b_2, a_2b_3, a_3b_2, a_3b_3\}$$
$$t_4 = maximum\{a_1b_1, a_1b_4, a_4b_1, a_4b_4\}$$

We need the following definition of the defuzzified value of fuzzy number based on graded mean integration method.

2.6 Fuzzy Transportation Problem:

Let a transportation problem with m fuzzy origins and n fuzzy destinations. Let $C_{ij} = \left(c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}, c_{ij}^{(4)}\right)$ be the transportation cost of one unit of the product from fuzzy origins ith to fuzzy destination jth. Let $a_i = \left(a_i^{(1)}, a_i^{(2)}, a_i^{(3)}, a_i^{(4)}\right)$ be the quantity of commodity available at fuzzy origin i, $b_j = \left(b_j^{(1)}, b_j^{(2)}, b_j^{(3)}, b_j^{(4)}\right)$ be the quantity of commodity needed at fuzzy destination j, $X_{ij} = \left(x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}\right)$ is quantity transported from ith fuzzy origins to jth fuzzy destination. The linear programming model representing the fuzzy transportation is given by

Minimize = $Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \left(c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}, c_{ij}^{(4)}\right) \left(x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}\right)$

Subject to the constraint

$\sum_{j=1}^{n} \left(x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}\right) = \left(a_i^{(1)}, a_i^{(2)}, a_i^{(3)}, a_i^{(4)}\right)$ for i = 1,2,....,m

$\sum_{j=1}^{n} \left(x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}\right) = \left(b_j^{(1)}, b_j^{(2)}, b_j^{(3)}, b_j^{(4)}\right)$ for j = 1,2,....,n

$\left(x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}\right) \geq 0$

The given fuzzy transportation problem is said to be balanced if

$\sum_{i=1}^{m} \left(a_i^{(1)}, a_i^{(2)}, a_i^{(3)}, a_i^{(4)}\right) = \sum_{j=1}^{n} \left(b_j^{(1)}, b_j^{(2)}, b_j^{(3)}, b_j^{(4)}\right)$

i.e if the total fuzzy capacity is equal to the total fuzzy demand.

3. NEW PROPOSED ALGORITHM

Step1: Check the given problem is balanced.

Step 2: We check the number of rows and columns are equal or not.

If number of rows are not equal to number of columns and vice versa. The dummy row or dummy column must be added with zero cost elements with zero demand/supply, so our matrix becomes a square matrix.

Step3: Find the smallest element of each row of the given matrix and subtract this smallest element from all element of that row. Therefore, there will be at least one zero in each row of this new matrix. In this new matrix find the smallest element of each column of the given matrix and subtract this smallest element from all element of that column.

Thus each row and column have at least one zero in new reduced matrix.

Step4: In reduced matrix, find a row with exactly single zero. Make assignment to this single zero by square box and mark cross overall other zeros in the corresponding column, proceed in this way until all the row have been examined. Same process we have to use for column.

Step 5: If the number of assignment is equal to number of column or row, the problem in idle condition.

If the number of assignment is not equal to number of column or row, the solution is not in idle condition, and then we go to next step

Step 6: We draw the minimum number of horizontal and vertical line crossing the all zeros.

Step7: Develop the new revised cost matrix as follows:

(i) Find the smallest element of reduced matrix not covered by the any minimum number of line.

(ii) Subtract this element from all uncovered elements and add this smallest element to all the elements lying at the intersection of horizontal and vertical lines.

Step 8: Go to step 5 and repeat the procedure until an idle condition is obtained.

Step 9: Firstly we have select those cell for allocation in row/column of idle transportation table which have only assignment, after that we have reaming cells for allocation in transportation table.

Step 10: Now we check the solution is optimal or not. If solution is not optimal then we are using MODI method to get the optimal solution.

4. NUMERICAL EXAMPLE

To solve the following fuzzy transportation problem starting with initial fuzzy feasible solution obtained by using proposed algorithm:

Table 1

	D1	**D2**	**D3**	**D4**	**Demand**
S1	[1,2,3,4]	[1,3,6,8]	[-1,0,1,2]	[3,5,6,8]	[0,2,4,6]
S2	[4,8,12,16]	[6,7,11,12]	[2,4,6,8]	[1,3,5,7]	[2,5,9,13]
S3	[1,5,9,13]	[0,4,8,12]	[0,6,8,14]	[4,7,9,12]	[2,4,6,7]
Supply	[1,3,5,7]	[0,2,4,6]	[1,3,5,7]	[1,3,5,7]	

By using definition 2.4, Fuzzy Transportation Problem is balanced i.e Sum of supply = Sum of demand

[3, 11, 17, 27] = [4, 11, 19, 27]

Our problem is balanced but number of row and number of column are not equal, so by using step 2 we get the following table 2

Table 2

	D1	**D2**	**D3**	**D4**	**Demand**
S1	[1,2,3,4]	[1,3,6,8]	[-1,0,1,2]	[3,5,6,8]	[0,2,4,6]
S2	[4,8,12,16]	[6,7,11,12]	[2,4,6,8]	[1,3,5,7]	[2,5,9,13]
S3	[1,5,9,13]	[0,4,8,12]	[0,6,8,14]	[4,7,9,12]	[2,4,6,7]
S4	0	0	0	0	0
Supply	[1,3,5,7]	[0,2,4,6]	[1,3,5,7]	[1,3,5,7]	

By using step 3 to 5 we get the following table 3

Table 3

	D1	**D2**	**D3**	**D4**	**Demand**
S1	[-1,1,3,5]	[-1,2,6,9]	**0**	[1,4,6,9]	[0,2,4,6]
S2	[-3,3,9,15]	[-1,2,8,11]	[-5,-1,3,7]	**0**	[2,5,9,13]
S3	[-11,-3,5,13]	**0**	[-12,-2,4,14]	[-8,-1,5,12]	[2,4,6,7]
S4	**0**	0	0	0	0
Supply	[1,3,5,7]	[0,2,4,6]	[1,3,5,7]	[1,3,5,7]	

By using step 9 we get the following table 4

Table 4

	D1	D2	D3	D4	Demand
S1	[1,2,3,4]	[1,3,6,8]	**[0,2,4,6]** [-1,0,1,2]	[3,5,6,8]	[0,2,4,6]
S2	**[-12,-3,7,17]** [4,8,12,16]	[6,7,11,12]	**[-5,-1,3,7]** [2,4,6,8]	**[1,3,5,7]** [1,3,5,7]	[2,5,9,13]
S3	**[-4,0,4,7]** [1,5,9,13]	**[0,2,4,6]** [0,4,8,12]	[0,6,8,14]	[4,7,9,12]	[2,4,6,8]
Supply	[1,3,5,7]	[0,2,4,6]	[1,3,5,7]	[1,3,5,7]	

After applying proposed new algorithm we get the number of occupied cell m + n -1 = 6 and get the following allotment which supply from one destination to another destination,

x_{13} = [0,2,4,6] x_{21} = [-12,-3,7,17] x_{23} = [-5,-1,3,7]

x_{24} = [1,3,5,7] x_{31} = [-4,0,4,7] x_{32} = [0,2,4,6]

This is a non degenerate fuzzy basic feasible solution. Here we get fuzzy feasible value [-289, -9, 199, 552] of our fuzzy transportation problem and we get the crisp value of the fuzzy transportation problem z is 107.167.

Since the above solution is not optimal so we are using MODI method to get the optimum solution and get the following occupied cell

x_{11} = [-12,-3,7,17] x_{13} = [-17,-5,7,18] x_{23} = [-17,-4,10,24]

x_{24} = [1,3,5,7] x_{31} = [-4,0,4,7] x_{32} = [0,2,4,6]

Transportation Cost = [1,2,3,4]* [-12,-3,7,17] + [-1,0,1,2]* [-17,-5,7,18]+[2,4,6,8]* [-17,-4,10,24] + [1,3,5,7]*[1,3,5,7] + [1,5,9,13]* [-4,0,4,7] + [0,4,8,12]* [0,2,4,6]

= [-48, -9, 21, 68] + [-34, -5, 7, 36] + [-136, -24, 60, 192] + [1, 9, 25, 49] + [-52, 0, 36, 91] + [0, 8, 32, 72]

= [-261, -15, 171, 484]

Here we get fuzzy optimal value [-261, -15, 171, 484] of our fuzzy transportation problem and we get the crisp value of the fuzzy transportation problem z is 89.167.

Now by the definition2.3, we are finding the fuzzy membership function for the fuzzy transportation cost as follows

$$\mu_{c_{11}} = \begin{cases} \frac{(x-1)}{1} & 1 \le x \le 2 \\ 1 & 2 \le x \le 3 \\ \frac{(4-x)}{1} & 3 \le x \le 4 \\ 0 & atherwise \end{cases}$$

To compute the interval of confidence for each level α the trapezoidal shapes will be described by function of α in the following manner.

Here $\alpha = \left(x_1^{(\alpha)} - 1\right)$ and $\alpha = \left(4 - x_2^{(\alpha)}\right)$

Therefore $c_{11} = \left[x_1^{(\alpha)}, x_2^{(\alpha)}\right] = [\alpha + 1, 4 - \alpha]$ (i)

and

$$\mu_{x_{11}} = \begin{cases} \frac{(x+12)}{9} & -12 \le x \le -3 \\ 1 & -3 \le x \le 7 \\ \frac{(11-x)}{4} & 7 \le x \le 1 \\ 0 & otherwise \end{cases}$$

Here $\alpha = \left(x_1^{(\alpha)} + 12\right)/9 \; and \; \alpha = \left(11 - x_2^{(\alpha)}\right)/4$

Therefore $x_{11} = \left[x_1^{(\alpha)}, x_2^{(\alpha)}\right] = [9\alpha - 12, 11 - 4\alpha]$ (ii)

Now from equation (i) and equation (ii), we get

$$c_{11} * x_{11} = [(\alpha+1)(9\alpha-12), (4-\alpha)(11-4\alpha)]$$
$$= [9\alpha^2 - 3\alpha + 12, 4\alpha^2 - 27\alpha + 44] \quad \text{(A)}$$

Similarly we find the other values,

$$c_{13} * x_{13} = [(\alpha-1)(12\alpha-17), (17-12\alpha)(18-11\alpha)]$$
$$= [12\alpha^2 - 29\alpha + 17, 132\alpha^2 - 403\alpha + 306] \quad \text{(B)}$$
$$c_{23} * x_{23} = [(2\alpha+2)(13\alpha-17), (8-2\alpha)(24-14\alpha)]$$
$$= [26\alpha^2 - 8\alpha - 34, 28\alpha^2 - 160\alpha + 192] \quad \text{(C)}$$
$$c_{24} * x_{24} = [(2\alpha+1)(2\alpha+1), (7-2\alpha)(7-2\alpha)]$$
$$= [4\alpha^2 + 4\alpha + 1, 4\alpha^2 - 28\alpha + 49] \quad \text{(D)}$$
$$c_{31} * x_{31} = [(4\alpha+1)(4\alpha-4), (13-4\alpha)(7-3\alpha)]$$
$$= [16\alpha^2 - 12\alpha - 4, 12\alpha^2 - 67\alpha + 91] \quad \text{(E)}$$
$$c_{34} * x_{34} = [(4\alpha * 2\alpha), (12-4\alpha)(6-2\alpha)]$$
$$= [8\alpha^2, 8\alpha^2 - 48\alpha + 72] \quad \text{(F)}$$

From equation (A), (B), (C), (D), (E) and (F), we get

Fuzzy minimum cost = $c_{13} * x_{13} + c_{21} * x_{21} + c_{23} * x_{23} + c_{24} * x_{24} + c_{31} * x_{31} + c_{33} * x_{33}$

$$= [75\alpha^2 - 48\alpha - 8, 188\alpha^2 - 733\alpha + 754]$$

The equations to be solved are

$$75\alpha^2 - 48\alpha - 8 - x_1 = 0$$
$$188\alpha^2 - 733\alpha + 754 - x_2 = 0$$

We retain only two roots α in [0, 1]

$$\alpha = \left[48 + \sqrt{(48)^2 - 300(-39 - X_1)}\right] \Big/ 150$$

$$\alpha = \left[733 + \sqrt{(733)^2 - 752(754 - X_1)}\right] \Big/ 376$$

Therefore

$$\mu_{cost\ z}(x) = \begin{cases} \left[48 + \sqrt{(48)^2 - 300(-39 - X_1)}\right] \Big/ 150 & -261 \le x \le -15 \\ 1 & -15 \le x \le 171 \\ \left[733 + \sqrt{(733)^2 - 752(754 - X_1)}\right] \Big/ 376 & 171 \le x \le 484 \\ 0 & otherwise \end{cases}$$

Which are the required fuzzy membership functions of fuzzy transportation minimum cost z [-261, -15, 171, 484]

5. Conclusion

In our study we have provided an optimal solution for fuzzy transportation problems in fuzzy number, but we have seen in some previous study obtained results are in crisp values, and then it might lose some helpful information. The new proposed algorithm provides that the optimal value of the objective function and shipping units are in fuzzy trapezoidal numbers for the fuzzy transportation problem with the unit shipping costs, the supply quantities and the demand quantities all are in fuzzy trapezoidal numbers. This method is very easy to understand and to apply and also, it can serve as an important tool for the decision makers when they are handling various types of logistic problems having fuzzy parameters.

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