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A Note on One Sided and Two Sided PO-Ternary Ideals in PO-Ternary Semiring

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Abstract

In this paper the term, left(lateral, right and two sided) PO-ternary ideal, maximal left(lateral, right and two sided) PO-ternary ideal, left (lateral, right and two sided) PO-ternary ideal of T generated by a set A, principal left (lateral, right and two sided) PO-ternary ideal generated by an element a left (lateral, right and two sided) simple PO-ternary semiring are introduced. It is proved that (1) the non-empty intersection of any two left (lateral, right and two sided) PO-ternary ideals of a PO-ternary semiring T is a left (lateral, right and two sided) PO-ternary ideal of T. (2) non-empty intersection of any family of left (lateral, right and two sided) PO-ternary ideals of a PO-ternary semiring T is a left(lateral, right and two sided) PO-ternary ideal of T. (3) the union of any left PO-ternary ideals of a PO-ternary semiring T is a left PO-ternary ideal of T. (4) the union of any family of left(lateral, right and two sided) PO-ternary ideals of a PO-ternary semiring T is a left(lateral, right and two sided) PO-ternary ideal of T. (5) The left (lateral, right and two sided) PO-ternary ideal of a PO-ternary semiring T generated by a non-empty subset A is the intersection of all left(lateral, right and two sided) PO-ternary ideals of T containing A. (6) If T is a PO-ternary semiring and $a \in T$ then L(a) = $(T^eT^ea + na]$ = $(T^eT^ea \cup na]$ (M(a) = $(T^eaT^e + T^eT^eaT^eT^e + na]$ = $(T^eaT^e \cup T^eT^eaT^eT^e \cup na]$, R(a) = $(aT^eT^e + na]$ = $(aT^eT^e \cup na]$ and T(a) =$(T^eT^ea + aT^eT^e + T^eT^eaT^eT^e + na]$ = $(T^eT^ea \cup aT^eT^e \cup T^eT^eaT^eT^e \cup na]$). (7) A PO-ternary semiring T is a left(lateral, right) simple PO-ternary semiring if and only if (TTa] = T ((TaT ∪ TTaTT] = T, (aTT] = T) for all $a \in T$.

Keywords: PO-ternary Semiring; left PO-ternary ideal; lateral PO-ternary ideal; right PO-ternary ideal; two sided PO-ternary ideal; left simple; lateral simple; right simple.

1. Introduction

Algebraic structures play a prominent role in mathematics with wide ranging applications in many disciplines such as theoretical physics, computer sciences, control engineering, information sciences, coding theory, topological spaces and the like. This provides sufficient motivation to researchers to review various concepts and results.

The theory of ternary algebraic systems was studied by LEHMER [9] in 1932, but earlier such structures were investigated and studied by PRUFER in 1924, BAER in 1929.

Generalizing the notion of ternary ring introduced by Lister [10], Dutta and Kar [6] introduced the notion of ternary semiring. Ternary semiring arises naturally as follows, consider the ring of integers Z which plays a vital role in the theory of ring. The subset $Z+$ of all positive integers of Z is an additive semigroup which is closed under the ring product,i.e. $Z+$ is a semiring. Madhusudhana Rao and Srinivasa Rao[10] studied about ternary semirings. Sivapasad, Madhusudhana Rao and Srinivasa Rao [11] introduced the concept of PO-ternary Semiring.

2. Preliminaries

In this section, the required preliminaries (concepts, examples and results) are presented.

Definition 2.1 : A nonempty set T together with a binary operation called addition and a ternary multiplication denoted by [ ] is said to be a ***ternary semiring*** if T is an additive commutative semigroup satisfying the following conditions :

i) $[[abc]de] = [a[bcd]e] = [ab[cde]]$,

ii) $[(a + b)cd] = [acd] + [bcd]$,

iii) $[a(b + c)d] = [abd] + [acd]$,

iv) $[ab(c + d)] = [abc] + [abd]$ for all $a; b; c; d; e \in$ T.

Note 2.2 : For the convenience we write $x_1x_2x_3$ instead of $[x_1x_2x_3]$

Note 2.3 : Let T be a ternary semiring. If A, B and C are three subsets of T , we shall denote the set ABC = $\{\Sigma abc : a \in A, b \in B, c \in C\}$.

Note 2.4 : Let T be a ternary semiring. If A, B are two subsets of T , we shall denote the set A + B = $\{a+b : a \in A, b \in B\}$ and 2A = $\{ a + a : a \in A\}$.

Note 2.5 : Any semiring can be reduced to a ternary semiring.

Example 2.6 : Let T be an semigroup of all $m \times n$ matrices over the set of all non negative rational numbers. Then T is a ternary semiring with matrix multiplication as the ternary operation.

Definition 2.7: A ternary semiring T is said to be a ***partially ordered ternary semiring*** or simply ***PO Ternary Semiring*** or ***Ordered Ternary Semiring*** provided T is partially ordered set such that $a \leq b$ then

(1) $a + c \leq b + c$ and $c + a \leq c + b$,

(2) $acd \leq bcd$, $cad \leq cbd$ and $cda \leq cdb$ for all $a, b, c, d \in$ T.

Throughout T will denote as PO-ternary semiring unless otherwise stated.

Note 2.8 : Some times we write $a \geq b$ for $a \leq b$. That is "$\geq$" is the dual relation of "$\leq$".

Example 2.9: Consider the set T = {0,1,2,3,….} with $m + n = \max(m, n)$ or $\min(m, n)$, $mn = m + n$. where the addition in the ternary multiplication is the usual addition, for all m, n in S and the order being the usual order relation . Then (T, +, [ ], $\leq$) is a PO-ternary semiring.

Definition 2.10 : A PO-ternary semiring T is said to be ***commutative PO-ternary semiring*** provided $abc = bca = cab = bac = cba = acb$ for all $a,b,c \in$ T.

Definition 2.11 : A PO-ternary semiring T is said to be ***quasi commutative PO-ternary semiring*** provided T is a quasi commutative ternary semiring.

Note 2.12 : A PO-ternary semiring T is ***quasi commutative*** provided for each $a, b, c \in$ T, there exists an odd natural number n such that $abc = b^n ac = bca = c^n ba = cab = a^n cb$.

Definition 2.13 : PO- ternary semiring T is said to be ***normal PO- ternary semiring*** provided T is normal ternary semiring.

3. Main Results

Definition 3.1 : A nonempty subset A of a PO-ternary semiring T is said to be ***left (lateral, right and two sided) PO-ternary ideal*** of T if

(1) $a, b \in$ A implies $a + b \in$ A.

(2) $b, c \in$ T, $a \in$ A implies $bca \in$ A ($bac \in$ A, $abc \in$ A and $bca \in$ A, $abc \in$ A) .

(3) $t \in T, a \in A$, $t \leq a \Rightarrow t \in$ A.

Note 3.2 : A nonempty subset A of a PO-ternary semiring T is a left (lateral, right and two sided) PO-ternary ideal of T if and only if A is additive subsemigroup of T, TTA $\subseteq$ A (TAT $\subseteq$ A, ATT $\subseteq$ A and left as well as right) and (A] $\subseteq$ A.

Note 3.3: Let T be a PO-ternary semiring. Then the set (TTa] = {$t \in$ T / $t \leq \sum_{i=1}^{n} x_i y_i a$ for some x_i, $y_i \in$ T and $n \in$ N}.

Example 3.4 : In the PO-ternary semiring Z^0 , nZ^0 is a left PO-ternary ideal for any $n \in$ N.

Theorem 3.5: Let T be a PO-ternary semiring. Then (TT*a*] is a left PO-ternary ideal of T for all *a* ∈ T.

Proof **:** Suppose that s, $t \in$ (TTa], then there exist x_i, y_i, x_j, $y_j \in$ T such that $s \leq \sum_{i=1}^{n} x_i y_i a$ and $t \leq \sum_{j=1}^{n} x_j y_j a$.

Since T is a PO-ternary semiring and TTa is a left PO-ternary ideal of T.

We have $s + t \leq \sum_{i=1}^{n} x_i y_i a + \sum_{j=1}^{n} x_j y_j a \in$ TTa and hence $s + t \in$ (TTa].

Therefore (TTa] is the additive subsemigroup of T.

Let $t \in$ (TTa], r, $s \in$ T. $t \in$ (TTa] $\Rightarrow t \leq \sum_{i=1}^{n} x_i y_i a$ where x_i, $y_i \in$ T and $n \in$ N.

Now $rst \leq rs(\sum_{i=1}^{n} x_i y_i a) = (rs \sum_{i=1}^{n} x_i y_i)a \in$ (TTa]

Therefore $t \in$ (TTa], r, $s \in$ T $\Rightarrow rst \in$ (TTa] and hence (TTa] is a PO- left ternary ideal of T.

Now $s \in$ (TTa] and $t \in$ T such that $t \leq s$.

$s \in$ (TTa] then there exist x_i, $y_i \in$ T such that $s \leq \sum_{i=1}^{n} x_i y_i a$.

Now $t \leq s$, $s \leq \sum_{i=1}^{n} x_i y_i a \Rightarrow t \leq \sum_{i=1}^{n} x_i y_i a$.

Therefore $s \in$ (TTa] and $t \in$ T such that $t \leq s \Rightarrow t \in$ (TTa] and hence (TTa] is a left PO-ternary ideal of T.

Theorem 3.6: Let T be a PO-ternary semiring. Then (T*a*T] is a lateral PO-ternary ideal of T for all *a* ∈ T.

Theorem 3.7: Let T be a PO-ternary semiring. Then (TT*a*TT] is a lateral PO-ternary ideal of T for all *a* ∈ T.

Theorem 3.8: Let T be a PO-ternary semiring. Then (T*a*T∪TT*a*TT] is a lateral PO-ternary ideal of T for all *a* ∈ T.

Theorem 3.9: Let T be a PO-ternary semiring. Then (*a*TT] is a right PO-ternary ideal of T for all *a* ∈ T.

Theorem 3.10 : The nonempty intersection of any two left (lateral, right and two sided) PO-ternary ideals of a PO-ternary semiring T is a left PO-ternary ideal of T.

Proof : Let A , B be two left PO-ternary ideals of T. Let a, $b \in$ A$\bigcap$B and c, $d \in$T

a, $b \in$ A $\bigcap$B $\Rightarrow$ a, $b \in$ A and a, $b \in$ B. a, $b \in$ A ; c, $d \in$ T , A is a left PO-ternary ideal of T

$\Rightarrow$ $a + b \in$ A, $cda \in$ A and $c \leq a \Rightarrow c \in$ A.

a, $b \in$ B ; c, $d \in$ T , B is a left ideal of T $\Rightarrow$ $a + b \in$ B and $cda \in$ B and $c \leq a \Rightarrow c \in$ B.

$a + b \in$ A and $cda \in$ A, $a + b \in$ B and $cda \in$ B $\Rightarrow$ $a + b \in$ A $\bigcap$ B and $cda \in$ A $\bigcap$ B.

Now $c \leq a \Rightarrow c \in$ A ∩ B. Therefore A$\bigcap$B is a left PO-ternary ideal of T.

Similarly we can prove the remaining parts.

Theorem 3.11 : The nonempty intersection of any family of left (right, lateral and two sided) PO-ternary ideals of a PO-ternary semiring T is a left PO-ternary ideal of T.

proof : Let $\{A_\alpha\}_{\alpha\in\Delta}$ be a family of left PO-ternary ideals of T and let A = $\bigcap\limits_{\alpha\in\Delta} A_\alpha$

Let $a, b \in$ A ; $c, d \in$ T. Now $a, b \in$ A, $a, b \in \bigcap\limits_{\alpha\in\Delta} A_\alpha \Rightarrow a, b \in A_\alpha$ for each $\alpha \in \Delta$.

$a, b \in A_\alpha$, $c, d \in$ T, A_α is a left PO-ternary ideal of T $\Rightarrow a + b \in A_\alpha$ and $cda \in A_\alpha$

$a + b \in A_\alpha$ and $cda \in A_\alpha$ for all $\alpha \in \Delta \Rightarrow a + b \in \bigcap\limits_{\alpha\in\Delta} A_\alpha$ and $cda \in \bigcap\limits_{\alpha\in\Delta} A_\alpha \Rightarrow a + b \in$ A and $cda \in$ A.

Therefore A is a left ternary ideal of T. Now suppose that $t \in$ T, $a \in$ A and $t \leq a$. $a \in$ A $\Rightarrow a \in \bigcap\limits_{\alpha\in\Delta} A_\alpha$ $\Rightarrow a \in A_\alpha$ for each $\alpha \in \Delta$. $t \in$ T, $a \in A_\alpha$ and $t \leq a$, A_α is a left PO-ternary ideal of T $\Rightarrow t \in A_\alpha$ for each $\alpha \in \Delta \Rightarrow t \in \bigcap\limits_{\alpha\in\Delta} A_\alpha \Rightarrow t \in$ A. Therefore $t \in$ T, $a \in$ A and $t \leq a \Rightarrow t \in$ A and hence A is a left PO-ternary ideal of a PO-ternary semiring T. Similarly we can prove the remaining parts.

Theorem 3.12 : The union of any two left (lateral, right and two sided) PO-ternary ideals of a PO-ternary semiring T is a left(lateral, right, two sided) PO-ternary ideal of T.

Proof : Let A_1, A_2 be two left PO-ternary ideals of a PO-ternary semiring T.

Let A = $A_1 \cup A_2$. Clearly A is a nonempty subset of T.

Let $a, b \in$ A ; $c, d \in$ T. Now $a, b \in$ A $\Rightarrow a, b \in A_1 \cup A_2 \Rightarrow a, b \in A_1$ or $a, b \in A_2$.

Suppose $a, b \in A_1$. So $a, b \in A_1$; $c, d \in$ T; A_1 is a left PO-ternary ideal of T

$\Rightarrow a + b \in A_1$ and $cda \in A_1 \subseteq A_1 \cup A_2$ = A $\Rightarrow a + b \in$ A and $cda \in$ A.

Suppose $a, b \in A_2$. So $a, b \in A_2$; $c, d \in$ T; A_2 is a left PO-ternary ideal of T

$\Rightarrow a + b \in A_2$ and $cda \in A_2 \subseteq A_1 \cup A_2$ = A $\Rightarrow a + b \in$ A and $cda \in$ A.

Therefore $a, b \in$ A ; $c, d \in$ T $\Rightarrow a + b \in$ A and $cda \in$ A and hence A is a left ternary ideal of T.

Suppose $t \in$ T, $a \in$ A and $t \leq a$. $a \in$ A $\Rightarrow a \in A_1 \cup A_2 \Rightarrow a \in A_1$ and $a \in A_2$.

$t \in$ T, $a \in A_1$, $t \leq a$ and A_1 is a left PO-ternary ideal of T $\Rightarrow t \in A_1$

Now $t \in$ T, $a \in A_2$, $t \leq a$ and A_2 is a left PO-ternary ideal of T $\Rightarrow t \in A_2$

Therefore $t \in A_1 \cup A_2$ = A. Hence $t \in$ T, $a \in$ A, $t \leq a \Rightarrow t \in$ A.

Therefore A is left PO-ternary ideal of T. Similarly we can prove the remaining parts.

Theorem 3.13 : The union of any family of left (lateral, right and two sided) PO-ternary ideals of a PO-ternary semiring T is a left (lateral, right, two sided) PO-ternary ideal of T.

Proof : Let $\{A_\alpha\}_{\alpha\in\Delta}$ be a family of left PO-ternary ideals of a PO-ternary semiring T.

Let A = $\bigcup\limits_{\alpha\in\Delta} A_\alpha$. Clearly A is a non-empty subset of T. Let $a, b \in$ A ; $c, d \in$ T. $a, b \in$ A

$\Rightarrow a, b \in \bigcup_{\alpha\in\Delta} A_\alpha \Rightarrow a, b \in A_\alpha$ for some $\alpha \in \Delta$. $a, b \in A_\alpha$, $c, d \in$ T, A_α is a left PO-ternary ideal of T

$\Rightarrow a + b \in A_\alpha$ and $cda \in A_\alpha \subseteq \bigcup_{\alpha\in\Delta} A_\alpha$ = A. Now suppose that $t \in$ T, $a \in$ A and $t \leq a$. $a \in$ A

$\Rightarrow a \in \bigcup_{\alpha\in\Delta} A_\alpha \Rightarrow a \in A_\alpha$ for some $\alpha \in \Delta$; $\Rightarrow a \in A_\alpha$ $t \in$ T, $a \in$ A and $t \leq a$, A_α is a left PO-ternary ideals of T $\Rightarrow t \in A_\alpha$ for some $\alpha \in \Delta \Rightarrow t \in$ A. Therefore $c, d \in$ T $\Rightarrow a + b \in$ A and $cda \in$ A and $t \in$ T, $a \in$ A and $t \leq a$ $\Rightarrow t \in$ A. Therefore A is a left PO-ternary ideal of T. Similarly we can prove the remaining parts.

We now introduce a maximal left (lateral, right and two sided) PO-ternary ideal and left (lateral, right and two sided) PO-ternary ideal generated by a subset of a PO-ternary semiring.

Definition 3.14 : A left (lateral, right and two sided) PO-ternary ideal A of a PO-ternary semiring T is said to be a ***maximal left* (*lateral, right and two sided*) *PO-ternary ideal*** or simply ***maximal left* (*lateral, right and two sided*) *PO-ideal*** provided A is a proper left (lateral, right and two sided) PO-ternary ideal of T and is not properly contained in any proper left (lateral, right and two sided) PO-ternary ideal of T.

Definition 3.15 : Let T be a PO-ternary semiring and A be a non-empty subset of T. The smallest left(lateral, right, two sided) PO-ternary ideal of T containing A is called ***left*(*lateral, right, two sided*) *PO-ternary ideal of T generated by A***.

Theorem 3.16 : The left(lateral, right, two sided) PO-ternary ideal of a PO-ternary semiring T generated by a non-empty subset A is the intersection of all left (lateral, right, two sided) PO-ideals of T containing A.

Proof : Let Δ be the set of all left PO-ternary ideals of T containing A . Since T itself is a left PO-ternary ideal of T containing A, $T \in \Delta$. So $\Delta \neq \varnothing$. Let $S^* = \bigcap_{S\in\Delta} S$. Since A $\subseteq$S for all S $\in \Delta$, A $\subseteq$ S^*. By theorem 3,11, S^* is a left PO-ternary ideal of T. Let K be a left PO-ternary ideal of T containing A, K is a left PO-ternary ideal of T. Clearly A ⊆ K. Therefore K $\in \Delta \Rightarrow S^* \subseteq$ K and hence S^* is the left PO-ternary ideal of T generated by A. Similarly we can show the remaining parts.

Theorem 3.17: Let A be a left (lateral, right, two sided) PO-ternary ideal of T. Then (A] is a left (lateral, right, two sided) partially ordered ternary ideal of T generated by A.

Proof: Clearly (A] is nonempty since A ⊆ (A]. Suppose that $t \in$ T and $a, b \in$ (A]. Then there exist $x, y \in$ A such that $a \leq x$ and $b \leq y$ by the definition of (A]. Since T is an ordered ternary semiring and A is a left ternary ideal of T, we have $rsa \leq rsx \in$ A and $a + b \leq x + y \in$ A. It follows that $rsa \in$ (A] and $a+b \in$ (A]. As for ((A]] ⊆ (A], it is clear. Hence (A] is a left ordered ternary ideal of T containing A. Moreover, if L is an arbitrary left ordered ideal of T containing A, then (A] ⊆ (L] ⊆ L. Thus (A] is the least left ordered ideal of T containing A. That is to say, (A] is a left ordered ideal of T generated by A, as required. Similarly we can prove the remaining parts.

We now introduce a principal left(lateral, right, two sided) PO-ternary ideal of a PO-ternary semiring and characterize principal left(lateral, right, two sided) PO-ternary ideal.

Definition 3.18 : A left(lateral, right, two sided) PO-ternary ideal A of a PO-ternary semiring T is said to be the ***principal left*(*lateral, right, two sided*) *PO-ternary ideal generated by a*** if A is a left(lateral, right, two sided) PO-ternary ideal generated by $\{a\}$ for some $a \in$ T. It is denoted by L (a) or $< a >_l$ ($< a >_m$, $< a >_r$ and $< a >_t$).

Theorem 3.19 : If T is a PO-ternary semiring and $a \in$ T then

L(a) = (A] where A= $\left\{\sum_{i=1}^{n} r_i t_i a + na : r_i, t_i \in T,\ n \in z_0^{\ +}\right\}$**, and Σ denotes a finite sum and $z_0^{\ +}$ is the set of all positive integer with zero.**

Proof : Given that A = $\left\{\sum_{i=1}^{n} r_i t_i a + na : r_i, t_i \in T,\ n \in z_0^{\ +}\right\}$. Let $a, b \in$ A.

$a, b \in$ A . Then $a = \sum r_i t_i a + na$ and $b = \sum r_j t_j a + na$ for r_i, t_i, r_j, $t_j \in$ T, $n \in z_0^+$.

Now $a + b = \sum r_i t_i a + na + \sum r_j t_j a + na \Rightarrow a + b$ is a finite sum.

Therefore $a + b \in$ A and hence A is a additive subsemigroup of T.

For t_1, $t_2 \in$ T and $a \in$ A.

Then $t_1t_2a = t_1t_2(\sum r_i t_i a + na) = \sum r_i t_i (t_1 t_2 a) + n(t_1 t_2 a) \in$ A

Therefore $t_1t_2a \in$ A and hence A is a left ternary ideal of T. By theorem 3.17, we have (A] is a left ordered ternary ideal of T containing a. Thus L(a) ⊆ (A]. On the other hand, L(a) is also a left ordered ideal of T containing a, so we have A ⊆ L(a). Thus (A] ⊆ L(a) since (A] is a left ordered ternary ideal of T generated by A. Therefore L(a) = (A], as required.

Note 3.20 : if T is ternary semiring and $a \in$ T then L(a) = $(T^eT^ea + na]$ = $(T^eT^ea \bigcup na]$.

Theorem 3.21 : If T is a PO-ternary semiring and $a \in$ T then

M(a) = (A], where A = $\left\{ \sum_{i=1}^{n} r_i a t_i + \sum_{j=1}^{n} u_j v_j a p_j q_j + na : r_i, t_i, u_j v_j p_j q_j \in T,\ n \in z_0^+ \right\}$, and Σ denotes a finite sum and z_0^+ is the set of all positive integer with zero.

Proof: Similar to theorem 3.19.

NOTE 3.22 : if T is PO-ternary semiring and $a \in$ T then M(a) = $(T^eaT^e + T^eT^eaT^eT^e + na]$ = $(T^eaT^e \bigcup T^eT^eaT^eT^e \bigcup na]$.

Theorem 3.23 : If T is a ternary semiring and $a \in$ T then

R(a) = (A], where A = $\left\{ \sum_{i=1}^{n} a r_i t_i + na : r_i, t_i \in T,\ n \in z_0^+ \right\}$, Σ denotes a finite sum and z_0^+ is the set of all positive integer with zero.

Proof: Similar to theorem 3.19.

Note 3.24: If T is a ternary semiring and $a \in$ T then R (a) = $(aT^eT^e + na]$ = $(aT^eT^e \cup na]$.

Theorem 3.25: If T is a PO-ternary semiring and $a \in$ T then T(a) = (A], where A = $\left\{ \sum_{i=1}^{n} r_i s_i a + \sum_{j=1}^{n} a t_j u_j + \sum_{k=1}^{n} l_k m_k a p_k q_k + na : r_i, s_i, t_j, u_j, l_k m_k, p_k, q_k \in T \text{ and } n \in Z_0^+ \right\}$ and Σ denotes a finite sum and z_0^+ is the set of all positive integer with zero.

Note 3.26 : if T is ternary semiring and $a \in$ T then

T(a) = $(T^eT^ea + aT^eT^e + T^eT^eaT^eT^e + na]$ = $(T^eT^ea \bigcup aT^eT^e \bigcup T^eT^eaT^eT^e \bigcup na]$.

We now introduce a left(lateral, right) simple PO-ternary semiring and characterize left(lateral, right) simple PO-ternary semiring.

Definition 3.27 : A PO-ternary semiring T is said to be ***left*(lateral, right) *simple*** PO-***ternary semiring*** if T is its only left(lateral, right) PO-ternary ideal.

Theorem 3.28 : A PO-ternary semiring T is a left simple PO-ternary semiring if and only if (TTa]= T for all $a \in$ T.

Proof : Suppose that T is a left simple PO-ternary semiring and $a \in$ T. By theorem 3.5, (TTa] is a left PO-ternary ideal of T. Since T is a left simple PO-ternary semiring, (TTa] = T. Therefore (TTa] = T for all $a \in$ T.

Conversely suppose that (TTa] = T for all $a \in$ T.

Let L be a left PO-ternary ideal of T. Let $l \in$ L. Then $l \in$ T. By assumption (TTl] = T.

Let $t \in$ T. Then $t \in$ (TTl] $\Rightarrow t \leq \sum_{i=1}^{n} u_i v_i l$ for some u_i, $v_i \in$ T.

$l \in$ L; u_i, $v_i \in$ T and L is a left PO-ternary ideal of T $\Rightarrow \sum_{i=1}^{n} u_i v_i l \in$ L $\Rightarrow t \in$ L.

Therefore T $\subseteq$ L. Clearly L $\subseteq$ T and hence L = T.

Therefore T is the only left PO-ternary ideal of T. Hence T is left simple PO-ternary semiring.

Theorem 3.29 : A PO-ternary semiring T is a lateral simple PO-ternary semiring if and only if (T*a*T ∪ TT*a*TT] = T for all $a \in$ T.

Theorem 3.30 : A PO-ternary semiring T is a right simple PO-ternary semiring if and only if (*a*TT] = T for all $a \in$ T.

Definition 3.31 : A PO-ternary semiring T is said to be a ***left duo PO-ternary semiring*** provided every left PO-ternary ideal of T is a two sided PO-ternary ideal of T.

Definition 3.32 : A PO-ternary semiring T is said to be a ***right duo PO-ternary semiring*** provided every right ideal of T is a two sided PO-ternary ideal of T.

Definition 3.33 : A PO-ternary semiring T is said to be a ***duo PO-ternary semiring*** provided it is both a left duo PO-ternary semiring and a right duo PO-ternary semiring.

Theorem 3.34 : A PO-ternary semiring T is a duo PO-ternary semiring if and only if (xT^{e}T^e] = (T^{e}T^ex] for all $x \in$ T.

***Proof* :** Suppose that T is a duo PO-ternary semiring and $x \in$ T.

Let $t \in$ (xT^{e}T^e]. Then $t \leq \sum_{i=1}^{n} x u_i v_i$ for some u_i , $v_i \in$ T^e.

Since T^{e}T^ex is a left PO-ternary ideal of T, (T^{e}T^ex] is a PO-ternary ideal of T.

So $x \in$ (T^{e}T^ex], u_i , $v_i \in$ T, (T^{e}T^ex] is a PO-ternary ideal of T $\Rightarrow \sum_{i=1}^{n} x u_i v_i \in$ (T^{e}T^ex]

$\Rightarrow t \in$ (T^{e}T^ex]. Therefore (xT^{e}T^e] $\subseteq$ (T^{e}T^ex]. Similarly we can prove that (T^{e}T^ex] $\subseteq$ (xT^{e}T^e].

Therefore(xT^{e}T^e] = (T^{e}T^ex] for all $x \in$ T.

Conversely suppose that (xT^{e}T^e] = (T^{e}T^ex] for all $x \in$ T. Let A be a left PO-ternary ideal of T. Let $x \in$ A, u_i , $v_i \in$ T. Then $\sum_{i=1}^{n} x u_i v_i \in$ (xT^{e}T^e] = (T^{e}T^ex] $\Rightarrow \sum_{i=1}^{n} x u_i v_i \leq \sum_{i=1}^{n} s_i t_i x$ for some s_i, $t_i \in$ T^e. Let $x \in$ A, s_i, $t_i \in$ T, A is a left PO-twenary ideal of T $\Rightarrow \sum_{i=1}^{n} s_i t_i x \in$ A $\Rightarrow \sum_{i=1}^{n} x u_i v_i \in$ A.

Therefore A is a right PO-ternary ideal of T and hence A is a PO-ternary ideal of T.

Therefore T is left duo PO-ternary semiring. Similarly we can prove that T is a right duo PO-ternary semiring. Hence T is duo PO-ternary semiring.

Theorem 3.35 : Every commutative PO-ternary semiring is a duo PO-ternary semiring.

***Proof* :** Suppose that T is a commutative PO-ternary semiring. Therefore xT^{e}T^e = T^{e}T^ex for all $x \in$ T implies that (xT^{e}T^e] = (T^{e}T^ex]. By theorem 3.34, T is a duo ternary semiring.

Theorem 3.36 : Every normal PO-ternary semiring is a duo ternary semiring.

***Proof* :** Suppose that T is normal PO-ternary semiring. Then xyT = Txy for all x, $y \in$ T

$\Rightarrow$ (xTT] = (TTx] for all $x \in$ T $\Rightarrow$ (xT^{e}T^e] = (T^{e}T^ex] for all $x \in$ T.

Therefore by theorem 3.34, T is a duo PO-ternary semiring.

Theorem 3.37 : Every quasi commutative PO-ternary semiring is a duo PO-ternary semiring.

Proof **:** Suppose that T is a quasi commutative PO-ternary semiring. Then for each $a, b, c \in$ T, there exists a odd natural number n such that $abc = b^n ac = bca = c^n ba = cab = a^n cb$. Let A be a left PO-ternary ideal of T. Therefore (TTA] ⊆ (A]. Let $a \in$ A and $s, t \in$ T. Since T is a quasi commutative PO-ternary semiring, there exist a odd natural number n such that $ast = t^n sa \in$ (TTA] ⊆ (A]. Therefore $ast \in$ (A] for all $a \in$ A and $s, t \in$ T and hence (ATT] ⊆ (A]. Thus A is right PO-ternary ideal of T. Therefore T is a left duo PO-ternary semiring. Similarly we can prove that T is a right duo POO-ternary semiring. Therefore every quasi commutative PO-ternary semiring is a duo PO-ternary semiring.

Conclusion : In this paper mainly we studied about left, lateral, right and two sided PO-ternary ideal in PO-ternary semiring.

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