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SPC using size Biased Maxwell Distribution

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Abstract

This paper elucidates the control limits for size biased Maxwell distribution for different sample size(s) with fixed scale parameter. The performance of the control chart observed through the average run length (ARL). The propose control limits, size biased Maxwell (SBMW), compare with existing Maxwell Distribution with same parameter setting and found majority of times better in performance as compare to existing control chart.

Keywords: Biased Maxwell Distribution; sample size; control charts.

1. Introduction

Statistical Process Control (SPC) is widely used in the production process to control quality products; the control chart is one of the seven major SPC problem-solving tools, so called Magnificent-Seven. Its included Histogram and stem-and-leaf plot, Check sheet, Pareto chart, Cause-and-effect diagram, Defect concentration diagram, Scatter diagram, and Control chart. All of these tools but the Shewhart control chart is the most technically sophisticated. The control charts were introduced by Professor Shewhart Walter in the 1920s and have now become an important tool in quality improvement. Control charts were developed for controlling the mean and variance of the distribution. It uses to show that how the process changes over time. The control chart always has a central line for the central limit, an upper line for the upper control limit, and a lower line for the lower control limit. These control limits are very helpful for reducing defective products and alternately for increasing the profits of industries. The maintenance of quality through control charts brings a good reputation for an industry in the market. Boyapati and Ch(2015) conducted the study on variable control charts based on exponential-gamma distribution. Parkand Wang(2020) investigated research on the x-and s control charts which were used to monitor the mean and variability of variables and it was helped to identify the quality of engineers and investigate the causes of the variation process.

In statistical quality control if data follow normal distribution then commonly used Shewhart control chart and if the data is skewed, then alternative procedure should be adopted to develop the control limits.

The weighted distribution's idea was given by Fisher and Rao. The basic concept of weighted distribution gives us an approach to the problems related to the understanding of the data and the specification of the model. When selecting samples from the developed and original distributions, provides us with a technique to fit models with unknown weight functions. Weighted distributions are suitable when observations are recorded without any experiment, repetition, and random process. This distribution is characterized into two types, which are length biased and size biased distribution. Length biased distribution when the weighted function retains only the length in units. Size-biased distribution occurs when observations from a sample are recorded with probability proportional.

So the different generators have been proposed on one or more than one parameter to develop new distribution. Cheng and Xie (2000) conducted research on control charts for lognormal data. In this paper, the researcher used positively skewed distributed data. Gadde, Fulment, and Joseph at(2019) explored an attribute control chart for a lifetime of the product which follows the dagum distribution.

Mathew and Chesneau (2020) researched marshall–olkin length-biased Maxwell distribution and its applications. Omar, Arafat, Hossain, and Riaz (2021) have assessed the inverse Maxwell distribution statistical process control as an efficient approach for monitoring positively skewed processes.

This paper studies the control chart for the size-biased Maxwell distribution as one of the non-normal skewed distribution. This has immense application in statistical mechanics as well as in life time modeling. In this research paper the one parameter, which is a scale parameter, size-biased.

2. Description of the Distribution

Suppose 'X' is a non-negative random variable with probability density function f (X) . Let w(x) be the non negative weight function, then the probability density function of the weighted random variable Xw can be derived with the help of:

$$f_w(x)=\frac{w(x)f(x)}{E(w(x))} \quad ; \quad x>0 \qquad (2.1)$$

Where w(x) be a non-negative weight function and E [w(x)]=∫w(x) f(x) dx <∞ . For different weighted models, we have different choice of the weight function w(x). When w(x) = xc, the resulting distribution is termed as weighted distribution. In this research, the size biased Maxwell distribution (SBMD) has been obtained. If one will take c=1 in weights xc, then SBMD can be derived as:

$$f_{(x)}=\frac{xf(x)}{E(x)} \qquad (2.2)$$

The PDF of Maxwell distribution is given by

$$f(x;\alpha)=\sqrt{\frac{2}{\pi}}\frac{x^2 e^{\frac{-x^2}{2\alpha^2}}}{\alpha^3} \qquad (2.3)$$

And the mean of the Maxwell is

$$E(x)=2\alpha\sqrt{\frac{2}{\pi}} \qquad (2.4)$$

Now, substitute these two equations (2.3) and (2.4) in (2.2) and after some simple algebraic calculations the following expressions has been derived, the PDF, CDF, mean, variance and Standard deviation of SBMD in the following equations (2.5), (2.6),(2.7b),(2.8c),(2.9b):

$$f(x)=\frac{x^3 e^{\frac{-x^2}{2a^2}}}{2a^4} \qquad (2.5)$$

;x>0,a>0

$$F(x)=\frac{2a^4-(a^2x^2+2a^4)e^{\frac{-x^2}{2a^2}}}{2a^4} \qquad (2.6)$$

The mean and standard deviation of SBMD

$$E(x)=\int_0^\infty x \frac{x^3 e^{\frac{-x^2}{2a^2}}}{2a^4}\, dx \qquad (2.7a)$$

$$E(x)=\frac{3a\sqrt{\pi}}{2^{\frac{3}{2}}} \qquad (2.7b)$$

$$E(X^2)=4a^2 \qquad (2.8a)$$

$$v(x)=4a^2-\left(\frac{3a\sqrt{\pi}}{2^{\frac{3}{2}}}\right)^2 \qquad (2.8b)$$

$$v(x)=(0.46572)a^2 \qquad (2.8c)$$

And, standard deviation is given

$$S.D(x)=\sqrt{(0.46572)a^2} \qquad (2.9a)$$

$$S.D(x)=0.6824368(a) \qquad (2.9b)$$

3. Control Limits for size Biased Maxwell distribution

This type of control chart has several values and control limit sets. Therefore, when the process is under control, almost all points are within the upper control limit (UCL) and lower control limit(LCL) [Duncan (1986) and Montgomery (2012)]. Therefore, from above equations the control limits are given by

$$\text{Upper control limit(UCL)} = \frac{3a\sqrt{\pi}}{2^{\frac{3}{2}}}+k\sqrt{(0.46572)a^2}$$

$$\text{Center limit(CL)} = \frac{3a\sqrt{\pi}}{2^{\frac{3}{2}}}$$

Lower control limit (LCL) = $\frac{3a\sqrt{\pi}}{2^{\frac{3}{2}}} - k\sqrt{(0.46572)a^2}$

Now with the extensive simulation study conducted on "R" the following Table3.1 and Table 3.2 have been developed

Table 3.1: The Control limits for SBMWD at a = 0.05

"a"	K at 290	Samplesize	ucl	cl	lcl
0.05	2.1535	2	0.1677697	0.09440	0.02103025
	1.7542	3	0.1541656	0.09440	0.03463441
	1.3445	5	0.1402071	0.09440	0.04859288
"a"	K at300	Samplesize	ucl	cl	lcl
0.05	2.1805	2	0.16868960	0.09440	0.026110360
	1.7529	3	0.1541213	0.09440	0.0346787
	1.3582	5	0.1406739	0.09440	0.04812613

It can be observed from Table3.1, that for the fixed value of scale parameter 'a'= 0.05, the control limits have been contracted with the increase in sample size(s).

Table 3.2: ARL for SBMWD and MWD at n=2,3,5 and scale parameter "a" = 0.05 when ARL0=290

n=2	SBMD with k= 2.1535	Maxwell with k = 2.2304	Shift
	287.814	306.007	1
	29.5065	34.282	1.2
	6.7985	8.737	1.4
	2.9375	4.057	1.6
	1.915	2.499	1.8
	1.627	1.8605	2.0
	1.3145	1.616	2.2
n=3	SBMD withk=1.7542	Maxwell with k =1.7782	Shift
	285.7	287.5005	1
	20.9535	25.717	1.2
	5.108	5.8595	1.4
	2.057	2.743	1.6
	1.4595	1.8125	1.8

	1.303	1.4715	2.0
	1.1565	1.265	2.2
n =5	SBMD with k=1.3445	Maxwell with k =1.3682	Shift
	280.857	300.203	1
	13.085	18.9185	1.2
	3.069	3.5785	1.4
	1.417	1.8285	1.6
	1.1575	1.343	1.8
	1.0805	1.136	2.0
	1.0225	1.0685	2.2

The control chart is monitoring the shift in the scale parameter, as the SBMD having just one scale parameter only, by sample mean. The shift has been taken in scale parameter as "a*f" (from 1 to2.2) of shift 0.2. It can be observed that increase in the shift the average run length is decreasingwhichisaworthysign irrespectiveof distributionand samplesizes.

4. Conclusion

In this paper the control limits for size biased Maxwell distribution for different samplesizes, with scale parameter is 0.05 have been calculated. It can be observed from Table 3.1, that for the fixed value of scale parameter 'a'= 0.05, the control limits have been contracted with the increase in samplesize(s). The efficiency of control charts can be observed with the help of average run length, it refers how much point it has to take to detect a "k" sigma shift. In this research paper the shift has been take ninescale parameter as "a*f"(from1to2.2) of 0.2. It can be observed from Table

3.2 That with increase in the shift the average run length for SBMWD decreases or atleast same as compare with Maxwell distribution. So, SBMWD can be considered more efficient than Maxwell Distribution a sit detecting the shift quickly\early as compare to Maxwell in majority of times and better in performance than Maxwell Distribution.

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