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Causal Theory of Action based utility and consumer surplus

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Abstract

The objective of this paper is to introduce the Causal Theory of Action, (CTA) based utility and consumer surplus, and compare it to the standard utility and consumer surplus that is based on Tversky's normative theory of choice. In addition to utility and consumer surplus, price level setting is also altered based on the (CTA). (CTA) based pricing is introduced.

Keywords:

Causal Theory of Action; utility; consumer surplus; Tversky; normative theory: similarity; risk aversion; beliefs; price levels; Nea work; Hopf algebra of Nea work; pricing techniques.

MSC:

37N40.

1. Introduction

The objective of this paper is to analyze two important topics in economics, utility and consumer surplus from the point of view of the Causal Theory of Action (CTA). Utility and consumer surplus are obtained from a demand curve. A demand curve shows the relationship between price and quantity consumed by an economic agent, (consumer). To expand this statement, let us use Marshalls', [1] definition of utility. A consumer earning a fixed income must choose among a number of goods each with a predetermined price. Consumer derives utility by purchasing quantities of good that maximizes his utility. In this statement Marshall assumes that consumer is a rational person, and remains rational. Therefore, for a consumer who follows rationality as motivation, is always aiming at maximizing his utility. The principal of marginal utility is defined as, utility is maximized when change in utility with respect to the consumption of one extra unit of a good can not be increased. Based on this argument a relationship between price and quantity can be derived as is shown in Figure 1. Utility in Figure 1 is the area $(0p^*a^*q^*)$, given the unit price (p^*) of a good, and the quantity consumed (q^*). It is assumed that at this level of utility, consumer surplus is the area

$(p^* p_{max} a^*)$. Let's look at the demand curve from a different point of view.

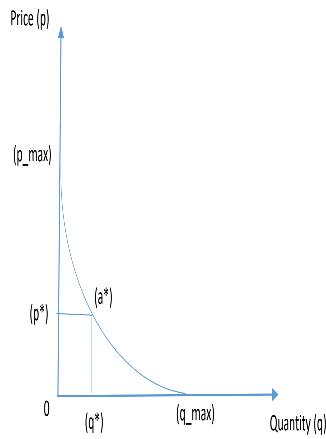

Figure 1. standard demand curve

Assume that consumer is not consistently, and continuously rational. Instead let's assume that the behavior of consumer follows what is known as the Causal Theory of Action (CTA), [2], [3], [4]. CTA explains that behind any action there exists a dynamics of causal occurrences. Looking at the demand curve from this angle, there is only one action that is concretely demonstrated, and that is the action of purchasing (q^*) quantity of a good at a unit price of (p^*), point (a^*), Figure 1. The shape of the demand curve after and before this point is not predetermined. The shape is in fact determined if there exists necessary and sufficient conditions of causal dynamics as information feedback to the consumer. Demand curve based on CTA is given in Figure 2. In Figure 2, at point (a^*), consumer has made the action of purchasing a good. This is a causal action that has occurred because there was necessary and sufficient conditions for the action to happen. Therefore, as an special case,the (CTA) utility is, the same as the utility for the rational consumer. For the causal consumer, it is the consumer surplus that is not fixed. The causal strength of purchasing action is different from the causal strength of consumer surplus. In other words, the gravitational center of purchasing action is stronger than the gravitational center of the surplus perceived by the consumer. Based on whether the causality of consumer surplus perceived is the result of transitional, chain or intentional causality, the consumer surplus curve varies as is shown by the upper segments in Figure 2. The lower segment must necessarily follow the same causal dynamics as the consumer surplus curves. The lower segment indicates the low of diminishing returns at various levels of intensity.

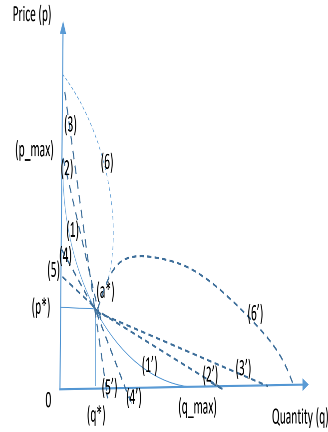

Figure 2. Causal Action variable utility and consumer surplus

The important aspect of the CTA approach is that marginal consumer surplus is variable while marginal utility stays fixed at (a^*). Marginal utility of good (X), $(M_u(X))$ is the change in utility given change in quantity (q), $(M_u(X) = \frac{\partial(u(X))}{\partial(q(X))})$, where $(u(X))$ is the utility, and $(q(X))$ is the quantity of good (X). Marginal consumer surplus, $(M_{C_i}(X))$ is change in consumer surplus given change in quantity $(q(X))$, $(M_{C_i}(X) = \frac{\partial(C_i^n(X))}{\partial(q(X))})$, where $(C_i^n(X))$ is consumer surplus, and (i) represents various consumer surplus curves, and (n) is the number of causal elements relating to consumer surplus. Marginal diminishing returns, $(M_{R_i}(X))$ is the loss in perceived satisfaction with extra units of consumption of good (X). $(M_{R_i}(X))$ is change in the return $(R_i^m(X))$ due to change in quantity consumed $(q(X))$, $(M_{R_i}(X) = \frac{\partial(R_i^m(X))}{\partial(q(X))})$, where (m) is the number of causal elements relating to diminishing returns, $(m \subseteq n)$.

It is relevant to compare standard indifference curves with CTA indifference curves. The standard indifference curves is based on Tversky's hypothesis, [5]. Consumer weighs in the risks and gains of acquiring good (X), versus good (Y). The indifference curves reflect the level of risk acceptability by an individual consumer. Therefore, consumers' indifference curves (utility curves) are defined in terms of the level of his risk aversion, as is shown in Figure 3.

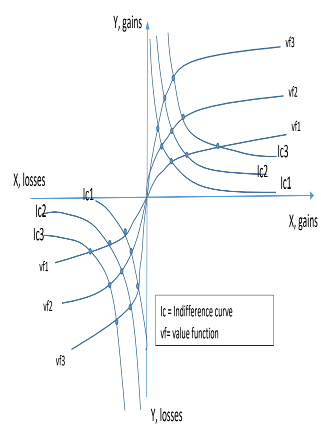

Figure 3. Tversky based indifference curves

The CTA indifference curves are functions of micro-macro movements that constitute consumer actions. It is the number of micro-macro actions that justify a purchase of a product for a consumer. The causality is established through the number and the pattern of micro-macro movements. The pattern of micro-macro movements is the combination of micro-macro movements. The CTA indifference curves are shown in Figure 4. The CTA indifference curves show consumer preference among two goods (X), and (Y). This preference is based on micro-macro movements are the result of a vector of micro-macro movements that constitute a causality. Figure 4, demonstrates CTA based indifference curves. The CTA indifference curve represents consumer preference based on the number and the pattern of micro-macro movements, (w_+^+). It is assumed that more of good (X) is purchased, if it this purchase requires a larger number of micro-macro movements. The inflection point is the point where the same number and pattern of micro-macro movements are required for both goods (X), and (Y). This is considered to be a unique point. It is assumed that at the inflection point the consumer surplus is the same for both goods (X), and (Y). The upper and lower segments of the indifference curve are dependent on the level of causal explanations, and its components. Causal components are consumers' aim, goal, purpose or intention, and finally desire to consume. It is the gravitational force of each component that determines the shape of the upper and lower segments of the indifference curve. The gravitational force is defined as the strength of a causal element in terms of provoking action on the part of an economic agent.

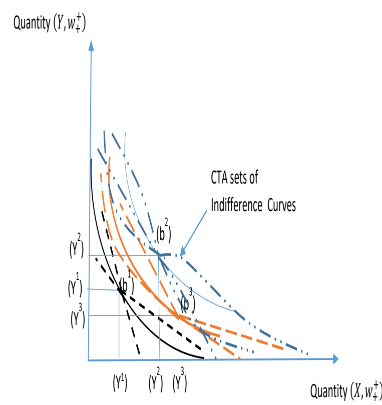

Figure 4. Causal Action induced set of indifference curves

In conventional economics, each single utility function has a single indifference curve for each pair of (X,Y) goods. In the CTA context,for each CTA utility set, there exists a set of indifference curves that depends on micro-macro movements (w_+^+) required for each pair of (X,Y) goods. The advantage of Causal Action (CTA)utility set curves is that it provides a more realistic view of individual consumption behavior and provides alternative estimation of consumer appreciation of a product. It gives a means of forecasting the longevity of a product, in terms of consumer appreciation. Exploring causality dynamics of action behind consumers' decisions and perceptions and provides a better understanding of an individuals' behavior towards price. It provides an alternative to the theory of rational consumer with monetary (gain-loss) utility function, and thus opens a new venue for exploring the theory of consumer demand. Mainly it deviates from the standard economic theory based on price to an economic theory based on Causal Action (CTA). In classical economics price is considered to be an independent variable outside of the influence of consumers, [6]. Generally price is set as a function of marginal cost of production. Demand is an adaptation to a fixed price. In the (CTA) economics, price becomes a function of causal behavior of consumer. Therefore, it is a function of causal dynamics. Details on alternate price description are explored in the paper.

In sections (2), and (3), the objective is to show and analyze the Causal Theory of Action, (CTA) based utility and consumer surplus. Utility and consumer surplus based on (CTA) are functions of economic agent's mental processes that causes action (macro, and micro displacements, purchasing action, and all micro physical movements that this purchasing involves). It is analyzed that the sole aspect of action can considerably change the way utility and consumer surplus are perceived. The (CTA) approach is then compared to the classical utility and consumer surplus that is based on Tversky's definition of mental process of decision based on similarity, risk aversion, and beliefs. (CTA) produces multiple consumer surpluses that lead to multiple utilities that are micro-macro movements causality. This means that an economic agent is a dynamic entity with evolving mental processes that cause actions related to the act of purchasing a product. In Tversky's theory of utility, it is utility that is fixed due to very specific mental analysis of an economic agent, (similarity comparison, risk perception, and either rational or irrational beliefs). In Tverskys' version, consumer surplus is an afterthought that clarifies or explains this fixed utility. The CTA based consumer surplus, is an integral part of the utility function that is formulated as the derivative of the utility function. In section (4), price determination is analyzed based on the CTA utility and consumer surplus. The CTA pricing and price evolution is compared to Tversky pricing and price evolution. Section (5) discusses the compatibility of mirco and macro actions, through the construction, and proofs of the existence of coalgebra using the Hopf algebra tools. Section (6), discusses price evolution in the (CTA) model as compared to the Tversky model.

2 CTA Consumer surplus

In the (CTA) approach, the process of consumer purchasing of (q_X) quantity of product (X) at a given price (p_X) depends on a thought process shown in Figure 5.

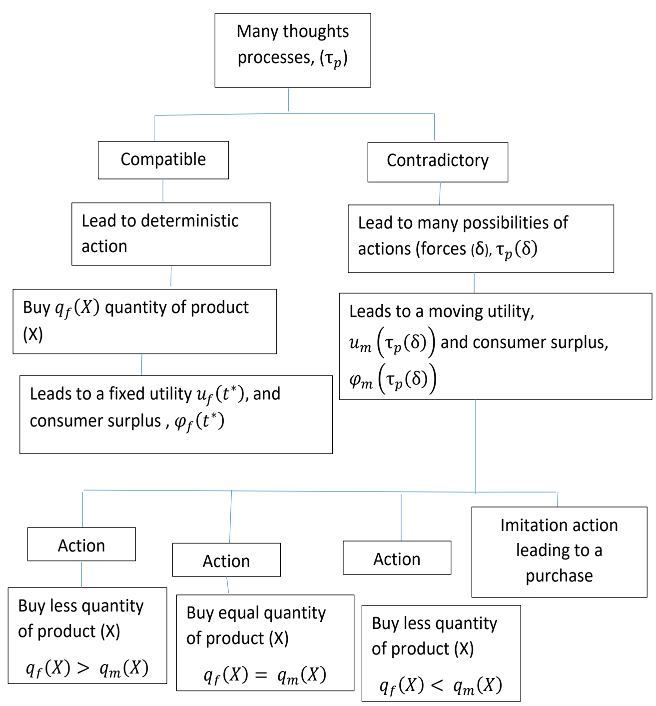

Figure 5. Consumer thought process under the CTA approach

At the instance, when a consumer makes the act of purchasing, it can be assumed that many thought processes, (τ_p) are going through his mind. (p) refers to the plurality of thought processes. Two possibilities exist. One possibility is that these thought processes are compatible. Compatible thought processes are the ones that are similar in principals. Choosing one compatible thought does not negate another compatible thought that is built on the same set of principals. In this case, compatible thought processes lead to actions that culminate to one final outcome which is the act of purchasing. This action is definitive and leads to a fix consumer surplus, $(\varphi_f(t^*))$, and utility, $(u_f(t^*))$ as is shown in Figure 5. (t^*) represents the instant of purchase, and (f) stands for fixed, the action itself is not consequential to purchasing product (X),rather it is assumed that the purchase is the outcome of the thought processes, (τ_p). Both fixed utility,and fixed consumer surplus are constant entities meaning that the demand curve keeps the same curvature, and trend. Fixed consumer surplus, $(\varphi_f(t^*))$, and utility, $(u_f(t^*))$ are the standard utility and consumer surplus. The second possibility is that the thought processes of the consumer are contradictory. Contradictory thought processes are those thought processes that are not in the chain of intermediary thoughts that lead to a final thought, or are not induced by some set of initial thoughts. There is no correlation between contradictory thoughts. Contradictory thoughts are independent thoughts and can be in contrast to each other. This leads to many possibilities of actions, all leading to causing a moving consumer surplus, $(\varphi_m(\tau_p(\delta)))$, and a moving utility, $(u_m(\tau_p(\delta)))$. (δ) represents the force of action in determining an outcome, which in this case is the purchase of product (X), and $((\tau_p(\delta)))$ represents contradictory thought processes that create a dynamic force that leads to actions ending in a purchase. The quantity purchased in the case of moving consumer surplus, and moving utility, is either less than the quantity purchased in the case of fixed consumer surplus, and fixed utility, $(q_m(X) < q_f(X))$ or equal $(q_m(X) = q_f(X))$, or greater than $(q_m(X) > q_f(X))$. Graphical comparison of quantities purchased between fixed and moving utility and consumer surplus when $(q_m(X) = q_f(X))$ is shown in Figure 6a.

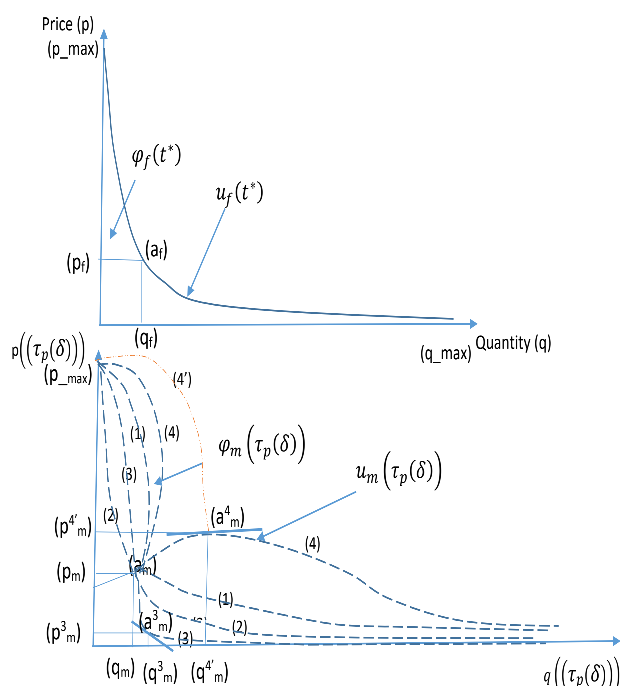

Figure 6a. Fixed and moving utility and consumer surplus when $(q_m(X) = q_f(X))$

Figure 6a, in curve (1) the consumer surplus is the area $(p_{max}a_mp_m)$is significant. Consumer derives satisfaction from consuming as exact amount(q_m) at price (p_m), and is not willing to consume anymore units of product (X). This is shown by checking the curve of diminishing returns of curve (1), where the marginal diminishing return is at point (a_m). In curve (2), ($\varphi_m(\tau_p(\delta))$) is the smallest of the (4) curves. Consumer is willing to consume (q_m) units of product (X), and he derives some satisfaction out of his consumption, but is not willing to consume more units as is shown by the diminishing returns segment of curve (2) which is similar to curve (1). In curve (3), consumer surplus region is larger than in curve (2), consumer derives more satisfaction in consuming (q_m) units of product (X). From the diminishing return segment of curve (3), if consumer consumes less units of product (X),(q_m^3) at a much lower price, (p_m^3), then based on the law of diminishing returns, he will experience dissatisfaction equivalent to the area ($p_m^3 p_m a_m a_m^3$). In curve (4), consumer derives the highest satisfaction from consuming (q_m) units of product (X), but he would be willing to consume more units of the product, (q_m^4), with the consumer surplus the region ($p_{max} a_m^4 p_m^4$).

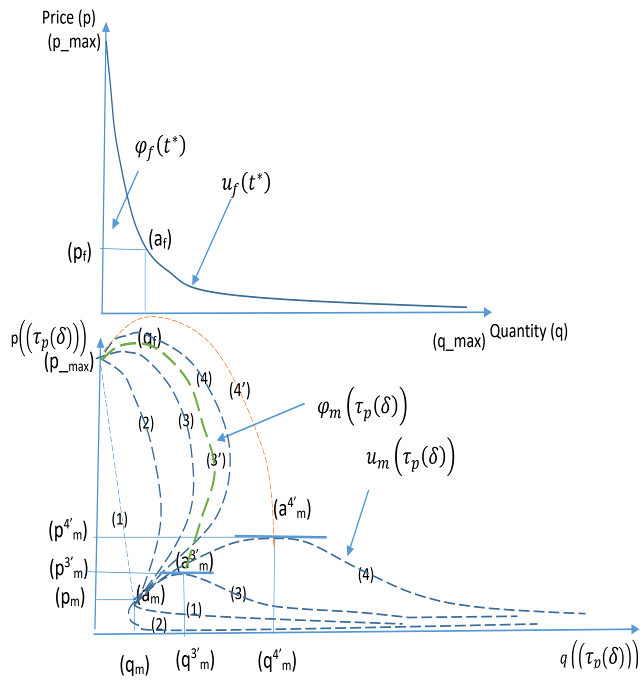

Figure 6b. Fixed and moving utility and consumer surplus when ($q_m(X) < q_f(X)$)

Figure 6b demonstrates the case ($q_m(X) < q_f(X)$), when in the CTA based utility and consumer surplus, consumer buys a smaller quantity of product (X), (q_m) at a lower price,(p_m). This is due to the dynamics of the causality,(($\tau_p(\delta)$)). In curve (1), consumer is at his maximum purchase at (a_m). He does not perceive of product (X) as very gratifying, therefore, his consumer surplus region is small and this is also reflected in his lower segment of the CTA utility curve, his diminishing returns. In curve (4) in contrast, consumer has a small region of consumer surplus ($p_{max} a_m p_m$). Here in contrast to curve (1), consumer perceives small consumer surplus, because he would rather purchase a larger quantity of product (X), so instead of purchasing (q_m) units, he would prefer to purchase ($q_m^{4'}$) units at a price slightly lower, ($p_m^{4'}$), and enjoy consumer surplus, region ($p_{max} a_m^{4'} p_m^{4'}$). (4') indicates that this is a new CTA utility curve. In curve (2) the consumer surplus region is small, since consumer finds the purchasing price, (p_m) too high, and would rather buy more units of product (X) at a lower price. This is shown in the diminishing return segment of the curve. In curve (3) consumer derives much higher satisfaction at consuming (q_m) units of product (X), but he would be interested in buying more units of product (X), ($q_m^{3'}$), at a slightly lower price, ($p_m^{3'}$), and thus would derive a higher consumer surplus shown as region $(p_{max}a_m^{3'}p_m^{3'})$.

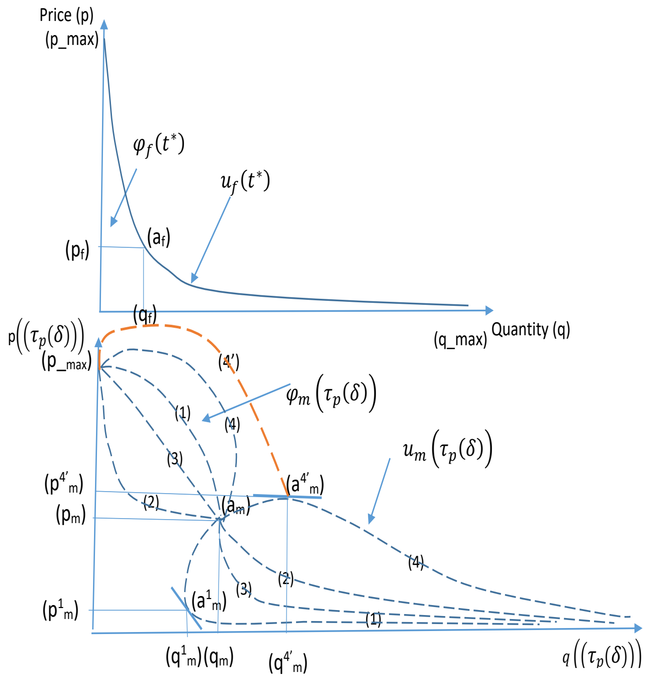

Figure 6c. Fixed and moving utility and consumer surplus when $(q_m(X) > q_f(X))$

Figure 6c demonstrates the case $(q_m(X) > q_f(X))$, when in the CTA based utility and consumer surplus, consumer buys more units of product(X), (q_m) at a higher price,(p_m). In curve (1), consumer derives (medium level) satisfaction in consuming product (X), region $(p_{max}a_mp_m)$. Therefore, consumer surplus is medium high, and this is confirmed by looking at the lower segment of curve (1), the diminishing returns region which shows a diminishing returns if consumer consumed less. In curve (2), the consumer surplus region shows that consumer appreciates product (X) much less than in curve (1), and is content with the purchase of (q_m^2) quantity of product (X), after which he perceives diminishing returns. In curve (3), consumer appreciates product (X) at quantity (q_m) and is not willing to purchase any more units of (X). In curve (4), consumer appreciates product (X), and would be willing to purchase more at a slightly higher price $(p_m^{4'})$ in order to perceive more satisfaction shown as region $(p_{max}a_m^{4'}p_m^{4'})$.

Another element affecting mental processes of a consumer is intention that leads to the act of purchasing. Intention is based on belief and desire, at the time when intention presents itself. (ι) represents the force of beliefs and desires. This can be called the representational state of mind at instant, $(t^* - \Delta t)$, where (t^*) is the instant of a purchase, and (Δt) is a time lapse. The representational state of mind is the origin of the utility, $(u(\tau_p(\delta), t^*))$, function based on the CTA. It is desire, and belief that provide a pre-utility function. Pre-utility, $(u(\iota, (t^* - \Delta t))\backslash u(\tau_p(\delta), t^*); \Delta t = 1, \cdots, T)$, is $(u(\iota, (t^* - \Delta t)) modulo (u(\tau_p(\delta), t^*)))$, and (T) is the limit of time lapse from the instant of purchase, is the utility that does not include the impact of action which, in this case is buying $(q_{(X)})$ quantity of product (X) which includes either spatial displacement efforts or online efforts (search, and purchase online). Figure 7, gives an outline of intention. Intention has two compartments, beliefs, and desires. These forces create events that lead to pre-action. Pre-action is the mental assessment of action through the lens of beliefs, and desires that give value to the pre-utility function. Pre-utility is the perceived utility before the actual actions required to make a purchase. Actions related to pre-utility are perceived actions of two categories. Actions are movements of human body that result in two categories. Category 1) is bodily movements that are macro movements that result in large spatial displacement that represent monetary costs. Monetary costs are transportation costs, and physical energy costs of displacement. Category 2) is bodily movements that are micro movements that represent non-monetary costs. An example of non-monetary costs is the time required to execute a search on internet. Non-monetary costs are costs that are not measured in terms of monetary units, but measured as the levels of emotional stress and modifications of the belief, desire, that are part of the choice, decision, and deliberation process.

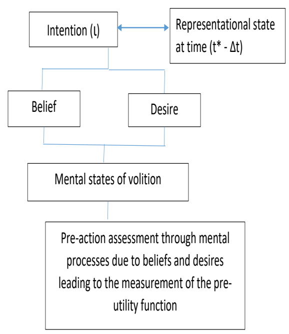

Figure 7. Intention

Let the set of beliefs be denoted as ($A_b = b_1, \cdots, b_F$) where (F) is the maximum number of beliefs and ($A_b \in \Re^n$). Let the set of desires be denoted as ($A_d = d_1, \cdots, d_E$) where (E) is the maximum number of desires, and ($A_d \in \Re^n$). Let the interaction of beliefs, and desires that shape the mental states of volition that leads to physical movements, be represented as ($(A_b \cup A_d) = A_{bd}$) where ($A_{bd} = (b \circ d)_1, \cdots, (b \circ d)_N$) where ($N = F \times E$). The set ($(A_b \cup A_d) = A_{bd}$) is used to constitute the pre-utility function. (A_{bd}) is a matrix of size ($F \times E$). The pre-utility function is denoted as a function of the beliefs, and the desires, formulated as $(u(\iota, (t^* - \Delta t)) \backslash u(\tau_p(\delta), t^*)) = u(\iota(A_{bd}), (t^* - \Delta t)) \backslash u(\tau_p(\delta), t^*))(A_{bd}))$. In the pre-utility, only beliefs, and desires are considered which translates into considering those benefits and advantages that own high weights. The disadvantages of a purchase are shadowed by advantages in the pre-utility function. The disadvantages are overshadowed by advantages in pre-utility functions. This implies that the maximum benefit is reached, once the purchase of ($q_{(X)}$) units of product (X) is done. At this point consumer surplus is minimum since consumer already receives maximum satisfaction from purchasing ($q_{(X)}$) units of product (X), at price ($p_{(X)}$). This constitutes triangles (3), and (4) in Figure 2. This effect is due to incomplete information. At this stage there is no knowledge of causal history and true causal chain. The pre-utility concept leads to actions such as displacement (taking a means of transport) involving sub-actions like (walking to car, or metro, driving to a mall or purchasing ticket to take metro), enter a shop and purchase an item, or open laptop computer, connect to the internet, and purchase the item. There are also micro-actions involved. These micro actions refer to body parts movements that are required at every stage when macro actions such as displacement and purchase are performed. Micro-actions are for example the pattern of arm and leg movements involved in driving a car, or purchasing a ticket. Both macro-actions, and micro-actions represent costs. Cost are both physical energy costs and monetary costs. At the pre-utility stage consumer is unaware of action costs or simply ignores them since desires and beliefs override action costs.

But once at the point of purchase, the utility changes from pre-utility,$(u(\iota(A_{bd}), (t^* - \Delta t)) \backslash u(\tau_p(\delta), t^*))(A_{bd}))$ to the CTA utility, $(u(\tau_p(\delta), t^*))(A_{bd}))$. $(u(\tau_p(\delta), t^*))(A_{bd}))$ is the utility at the time of purchase, (t^*), when the forces causing micro-macro movements are ($\tau_p(\delta)$) brought on by the forces of intention ($\iota(A_{bd})$). At this point, consumer, has had feedback information, mainly the dis-utilities of purchasing ($q_{(X)}$) units of product (X), namely, action costs, and budgetary restrictions, and other monetary problems. Budgetary, and monetary costs are considered to be Parallax causality in a parallax gap. Parallax gap is the space of all parallax causal elements. The CTA utility is a function of micro-macro movements $(\tau_p(\delta))$ spurred on by $(\iota(A_{bd}))$, including dis-benefits, $(u(\tau_p(\delta), t^*))(A_{bd}) = u(\tau_p(\delta), t^*))(A_{bd}, (C_a, C_p)))$, where (C_a) is the matrix of action costs, of size $(m_c^1 \times m_c^2)$. (m_c^1) is the number of micro actions, and (m_c^2) is the number of macro actions. Matrix (C_a) represents interaction between micro and macro actions considered as non-monetary costs. (C_p) represents interactions among monetary costs, and is of size $(m_c^1 \times m_c^2)$. The CTA utility function at the time of the purchase, (t^*) is formulated as $(u(\tau_p(\delta), t^*))(A_{bd}, (C_a, C_p)) = e^{-u(\tau_p(\delta),t^*))(A_{bd},(C_a,C_p))} \div e^{-u(\iota(A_{bd}),(t^*-\Delta t))})$, where the function $(u(\tau_p(\delta), t^*))(A_{bd}(C_a, C_p)))$ is formulated as $(u(\tau_p(\delta), t^*))(A_{bd}, (C_a, C_p)) = \tau_p(\delta) \otimes (A_{bd} \ominus i \otimes (C_a \oplus C_p))$. The pre-utility function is formulated as $(u(t^*-\Delta t)\backslash u(t^*) = u(t^*-\Delta t)\backslash u(t^*)(A_{bd}) = e^{-u(\iota(A_{bd}),(t^*-\Delta t))})$. Formulation of utility as a complex function sheds some light on the behavior of the CTA utility. In the standard complex theory, complex functions obey very rigid laws. For example within the zone of a very small disk of radius $(r \geq 0)$ all complex functions of (z), $(f(z))$ are homeomorphic to (z), $(z \to f(z))$ where (z) is a complex number. This fixed property leads to a fixed range of utility functions and fixed range of consumer surpluses. Due to causality, the complex formulation allows for homeomorphism in a much wider range.

3 Utility and Consumer surplus from Tversky to the CTA

Consumer surplus based on Tversky is fundamentally different from the CTA consumer surplus. Consumer surplus is directly linked to utility. In Tversky utility formulation is based on choice behavior which is measured using psychological measurement constituting the normative theory of choice. Normative theory of choice states that utility functions are not transitive and thus the transitivity property is violated due to measurable differences in psychological choice behavior. Transitivity of utility function points to the concept that no two utility functions are the same. Each individual possesses a unique utility function, $(u(x))$, where (x) represents individual (x), that is different from another individual's utility function, $(u(x') \neq u(x))$, where $((x'))$ is an individual different from (x). In Tversky's view of utility based on choice, an individuals' decision in making a particular choice, is based on his view of two events. Event one is conjunctive events (ϵ_c). Conjunctive events are those events that occur frequently in time and have many features in common. Similarity it is essential in forming knowledge, perception, and judgment of (x). The probability of the occurrence of conjunctive events depends on the availability of these events. The outcome of conjunctive events forms a probability of success of these events for (x). Disjunctive events, (ϵ_d) are those event that have no features in common, and do not occur frequently. Disjunctive events, represent risks. Generally, these events reduce the probability of making a positive decision. Therefore, individuals are not one dimensional decision makers, but rather a complex system made of conjunctive, and disjunctive processors that are determinant in their decision making outcomes, $(u(x) = u(x(\epsilon_c, \epsilon_d)) =\mid n(\epsilon_c) - n(\epsilon_d) \mid)$, where $(n(\epsilon_c))$ is the number of conjunctive events, and $(n(\epsilon_d))$ is the number of disjunctive events within a length of time. Conjunctive, and disjunctive events provide a means of psychological measurements or norms that make up the normative theory of choice. Tversky's model is a linear psychological function of two variables, (ϵ_c), and (ϵ_d). Tversky's psychological function is an increasing function of (ϵ_c), and a decreasing function of (ϵ_d). Tversky's choice behavior is outlined in Figure 8. In Tversky's model of choice behavior, utility and therefore, consumer surplus depend on thought processes of (x). The quantity of a product consumed and price acceptance are directly linked to this thought process.

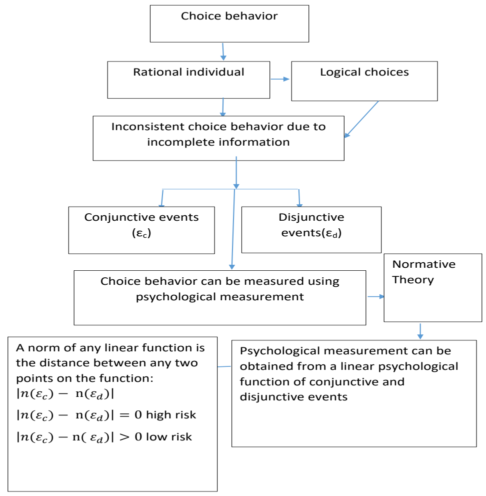

Figure 8. Tversky's Normative Theory

In the CTA model of utility and consumer surplus, thought process is shaped as a result of physical movement. There are two types of physical movements, micro movements, and macro movements. Micro movements, (ω_+), are those movements that are defined within a small distance. For example, any movements of fingers or arms are considered to be micro movements since they are defined within a small radius. Macro movements, (ω^+), are those bodily movements that can be defined within a large distance. An example of a macro movement is leaving a stationary place like (home), and walking to a train station and taking a train to a destination. In macro movements, space can expand and reduce. Micro and macro movements together constitute Neanderthalian, (Nea) work, (ω_+^+). Neanderthals made complex tools that involved intricate physical movements that reveals intelligence and successful adaption to immediate environment using strength and extreme muscularity due to harsh living conditions. Neanderthalian work, (ω_+^+) is defined as any micro and macro movements that create added values and either create or provide access to goods that build added values for an individual himself and for others. This is in contrast to the traditional definition of work which is a broad concept in the sense that added value is created for those who can afford to ask for it and not necessarily for the individual. Traditional work is defined as any activity that produces goods with added values for someone else. Traditional work is assumed to consume the totality of an individual's activities in such a way that almost no energy is left for the individual to engage in activities that would create added values for himself. The Neanderthalian work, (ω_+^+) is not restricted to production, but is now related to demand and therefore, utility and consumer surplus. In the CTA approach, conjunctive events are transformed into $(\epsilon_c \to \omega_+^+)$ micro-macro movements, and disjunctive events are transformed into the dialectic micro-macro movements,$((\omega_+^+)^-)$, $(\epsilon_d \to (\omega_+^+)^-).(\omega_+^+)^-)$ is the Neandertalian work that deviates from the main goal of the work. This concept is explored in more details later. This transformation changes (2) major findings of Tverskys' utility theory. In Tversky, utility is not transitive, and thus every individual has a unique utility function and consumer surplus. In the CTA utility and consumer surplus given that utility and consumer surplus are functions of a set of $((\omega_+)^i; i = 1, \cdots, N)$, where (i) is a finite number of micro movements and $((\omega^+)^j; i = 1, \cdots, M)$, where (j) is a finite number of macro movements and their dialectics $(((\omega_+)^-))^{i'}; i' = 1, \cdots, N')$, and $(((\omega^+)^-))^{j'}; j' = 1, \cdots, M')$. Therefore, there exists a set of unique utilities and consumer surpluses for each individual. This is due to the fact that each (ω_+^+), and $((\omega_+^+)^-)$ are tensors, and variations in (ω_+^+), and $((\omega_+^+)^-)$ create different forms of utilities and consumer surpluses. (ω_+^+) can be formulated as $(\omega_+^+ = \omega_+ \otimes \omega^+)$. (ω_+^+) constitutes the elements of matrix (C_p) in the previous section. The CTA model of utility and consumer surplus can be formulated as follows: utility is formulated as $(u(x) = u(x(\omega_+^+)) = e^{-(i \otimes \omega_+^+)} = e^{-(i \otimes C_p)})$, and consumer surplus is formulated as tangent bundles, $(\frac{\partial(u(x(\omega_+^+)))}{\partial(\omega_+^i)} \otimes \frac{\partial(u(x(\omega_+^+)))}{\partial(\omega_j^+)})$. The complex field formulation of consumer surplus and utility gives rise to more exotic consumer surplus forms as is shown in Figure 9.

Figure 9, exhibits examples of the various forms of utility and consumer surplus functions. Figure 9(a), and 9(b) illustrate two possible CTA utility curves and the corresponding CTA consumer surpluses. In both graphs, 9(a), and 9(b), consumer surplus is the area under the blue and red curves above the inflection point point (a^*), and at price (p^*). Both these curves (blue, and red) are valid and can occur depending on the coalgebra of (ω_+^+) tensors, and their dialectics tensors ($(\omega_+^+)^-$). In Figure 9(c), CTA utility contains a (1) loop above the inflection point (a^*), consumer surplus is a one loop curve. In this case, the CTA consumer surplus calculation becomes more complicated. The area under the utility curve above the purchase point (a^*), and at price (p^*) contains a region and one loop. The loop is the manifestation of dialectic in the CTA consumer surplus. Dialectic in the CTA in general refers to the fact that some micro and/or macro movements are contradictory to the continuity of causality that leads to purchase of a good at price (p^*). An example of dialectic in the CTA consumer surplus, would be that some of the physical movements may be unnecessary in view of direct and continuous causality such as taking metro to go to a mall, but first walking to a coffee shop to look at the menu before taking the metro. This causes a fork and creates a loop in the demand curve. The number of loops depends on the number of dialectic in the CTA, so a (2) loop consumer surplus represents (2) dialectic micro and/or macro movements in the CTA utility function, Figure 9(d). Figure 9(e) is a triangular dialectic CTA which represents a (3) point dialectic CTA.

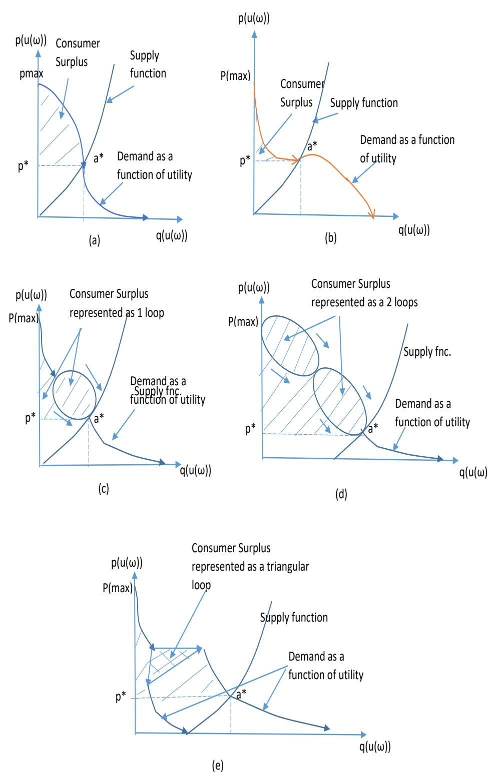

Figure 9. Various forms of utility functions and corresponding consumer surpluses as functions of the CTA

4 Pricing in the CTA model of utility and consumer surplus

In this section pricing based on the CTA approach are studied. Let's look at and analyze (3) examples of CTA equilibrium pricing, in order to explain the idea behind the CTA equilibrium pricing mechanism. The CTA equilibrium pricing and its' evolution based on Nea work, (ω_{+}^{+}), and its' dialectic $(\omega_{+}^{+})^{-})$ is shown in Figures 10a, 10b, and 10c. In these Figures, the inflection point which is the point of transition from the consumer surplus region to the consumer diminishing return region is also the intersection point of supply and demand curves. In Figure 10a, the equilibrium point represents the point where $(q(\omega_{+},(\omega_{+})^{-})$ is purchased at price $(p(\omega^{+},(\omega^{+})^{-})$. (ω_{+}) represents Nea work (micro movements), and $((\omega_{+})^{-})$ represents the dialectic of the micro movements. (ω_{+}) and its' dialectic, $((\omega_{+})^{-})$ are associated with the quantity purchased, (q) while macro movements, $((\omega^{+})$ and its' dialectic, $((\omega^{+})^{-})$ are associated with price. The reason for this assumption is that micro, and macro movements directly impact the consumer's desire and determination to consume as is demonstrated in the previous section. This dependency is the reason for a set of utilities and consumer surpluses. Each individual in the CTA environment possesses a set of possible utilities and consumer surpluses as is shown in the previous section. All groups of individual consumers sharing the same pattern and quantity of Nea work (ω_{+}^{+}) can be represented by a specific set of utilities and consumer surpluses as is shown in Figure 10a. The equilibrium point (e_1) is given for a fixed $(\omega_{+}^{+}) = (\omega_{+},(\omega_{+})^{-}),(\omega^{+},(\omega^{+})^{-}))$. The point (e_1), is unique since any other combination of micro-macro movements leads to a different equilibrium point, which also means a different price and quantity.

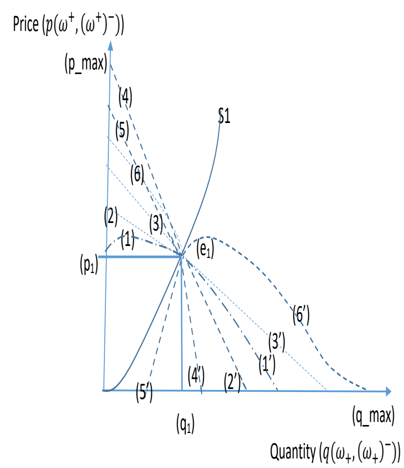

Figure 10a. The CTA equilibrium pricing as a function of Nea work,(ω_{+}^{+}), and its' dialectic $(\omega_{+}^{+})^{-})$

In the example of Figure 10b, it is shown that different combinations of micro-macro movements, provide different utilities and consumer surpluses around the equilibrium points, (e_2).Comparison between the two equilibrium points, (e_2), and (e_1) shows that smaller number of micro movements requires a higher number of macro movements, which translates into an smaller quantity consumed at a higher price. In other words, $((\omega_{+}^{+})_2 \neq (\omega_{+}^{+})_1)$, the Nea works are not similar both in pattern and in quantity in the (2) cases chosen as an example here. In particular, $((\omega_{+},(\omega_{+})^{-})_2 < (\omega_{+},(\omega_{+})^{-})_1)$, and $(((\omega^{+},(\omega^{+})^{-})_2 > ((\omega^{+},(\omega^{+})^{-})_1)$, micro-movements and their dialectics are smaller in the second case as compared to the first case. It is assumed that the dialectic micro movements are proportional to the micro movements themselves. This is a logical assumption, while the macro-movements and their dialectics are larger in the second case as compared to the first case. Consequently, consumer appreciation varies as consumer applies different combinations of micro-macro movements in each case. Consumer appreciation can be quantified by integrating the region under the upper segment of the utility curve, which is consumer surplus, (φ). The area is denoted as $(\bigcirc(\varphi))$. From the comparison of the examples in Figures 10a, and 10b, the following can be observed: $((\bigcirc(\varphi))_2^1 \approx (\bigcirc(\varphi))_1^1)$, the two regions of consumer surpluses for the utility curve (1) are approximately the same size, (equivalent), which indicates that given the differences in the Nea works of the two cases, consumer appreciation does not change significantly. $((\bigcirc(\varphi))_2^2 > (\bigcirc(\varphi))_1^2)$,the consumer region of the utility curve (2) in case (2), is greater than the consumer region of the utility curve (2) in case (1). For the utility curve (2), the conclusion to be derived is that consumer derives more satisfaction from the purchase, when the Nea work contains more macro movements. $((\bigcirc(\varphi))_2^3 \approx (\bigcirc(\varphi))_1^3)$, the consumer region of the utility curve (3) in case (2), is approximately the same size as the consumer region of the utility curve (3) in case (1). Again, consumer appreciation is unchanged irrespective of the differences between the two Nea works. The same conclusion can be derived from utility curves (4), and (5), where $((\bigcirc(\varphi))_2^4 \approx (\bigcirc(\varphi))_1^4)$, and $((\bigcirc(\varphi))_2^5 \approx (\bigcirc(\varphi))_1^5)$. The consumer region of the utility curve (6), of case (1) is much larger than the utility curve (6), of case (2), $((\bigcirc(\varphi))_2^6 \gg (\bigcirc(\varphi))_1^6)$. As for pricing policies, this allows for a wide range of pricing possibilities as a function of the Nea work.

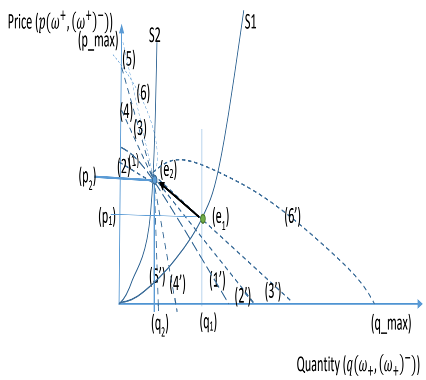

Figure 10b. The CTA equilibrium pricing evolution as a function of Nea work,(ω_+^+)
and its' dialectic $(\omega_+^+)^-)$ when it induces lower consumption

The example, in Figure 10c, shows an overall improvement in consumer appreciation, when micro movements are larger than macro movements. This translates into pricing strategy that is mainly based on micro movements.

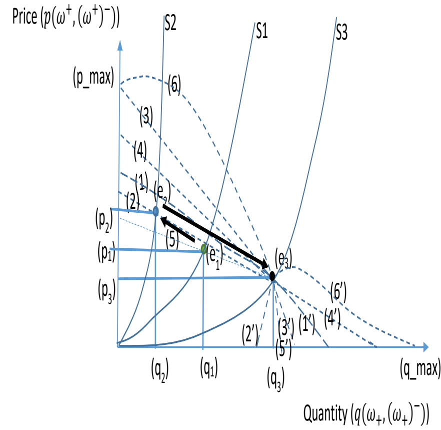

Figure 10c. The CTA equilibrium pricing evolution as a function of Nea work,(ω_+^+) and its' dialectic $(\omega_+^+)^-)$ when it induces higher consumption

5 Application of Hopf algebra in establishing the compatibility structure for (ω_+) and (ω^+) tensors

In order to generalize individual CTA utility function, which is a function of tensor fields of (ω_+) and (ω^+), must show that tensor fields for both (ω_+) and (ω^+) tensors, exist and own morphisms and have commutative and tensor coalgebra and comodules characteristics, and stay within the compatibility structure namely bialgebras and endomorphic characteristics by using Hopf algebra tools, [7], [8]. This will allow an individual CTA utility function to be generalized to represent an average CTA utility function, it is then that it can be used similar to a standard demand function that represents an average general function for consumer behavior. The application of the Hopf algebra to the CTA utility is discussed in the following paragraph.

Causal and dialectic causal micro movements are formulated as:

$$\omega_+(\alpha,\hat{\alpha}) = \begin{cases} \omega_+(\alpha) = x_+(\alpha) + i \times y_+(\alpha) \\ \hat{\omega}_+(\hat{\alpha}) = x_+(\alpha) - i \times \hat{y}_+(\hat{\alpha}) \end{cases} \tag{1}$$

where $(\omega_+(\alpha))$ represents the set of micro movements tensors as a function of a vector of causalities (α), where $(\alpha = (\alpha_1, \cdots, \alpha_{nn}))$, and (nn) is the number of elements in the causality vector. Vector $(x_+(\alpha))$ represents physical micro movements within a small closed space. Vector $(y_+(\alpha))$ represents physical micro movements that are complimentary to vector $(x_+(\alpha))$, and therefore, must have the same causality (α). $(\hat{\omega}_+(\hat{\alpha}))$ is the dialectic causal micro movements tensor, where $(\hat{\alpha} = (\hat{\alpha}_1, \cdots, \hat{\alpha}_{mm}))$, and (mm) is the number of elements in the dialectic causality vector. The causality for $(\hat{\omega}_+(\hat{\alpha}))$ tensor is a collection of causality factors that are opposite to the (α) causality, $(\hat{\alpha} \not\subseteq \alpha)$, since in the dialectic case, vector $(\hat{y}_+(\hat{\alpha}))$ is the vector of micro movements that render vector $(x_+(\alpha))$ less effective to achieve the final outcome of purchasing a product. The reason that the dialectic micro movement is included is that not all causal elements are positive impact causal elements, there exist negative causal elements, that are not perceived by an individual as negative, and only reveal their nature (positive, or negative) after the micro movements are performed. Therefore, the micro movements are a set of tensors that should be considered as a complete set since the set includes causal vectors, as well as dialectic causal vectors. It should be noted that the dialectic micro movements are not a conjugate of the $(\omega_+(\alpha))$ micro movements. Both the $(\omega_+(\alpha))$ and the dialectic $(\hat{\omega}_+(\hat{\alpha}))$, micro movements have their own corresponding conjugates not given here.

In a similar way causal macro movements can be represented as a set of complex tensors consisting of standard causal and dialectic causal macro movements given as the following set:

$$\omega^{+}(\bar{\alpha},\bar{\bar{\alpha}}) = \begin{cases} \omega^{+}(\bar{\alpha}) = X^{+}(\bar{\alpha}) + i \times Y^{+}(\bar{\alpha}) \\ \hat{\omega^{+}}(\bar{\bar{\alpha}}) = X^{+}(\bar{\alpha}) - i \times \hat{Y}^{+}(\bar{\bar{\alpha}}) \end{cases} \quad (2)$$

where $(\omega^{+}(\bar{\alpha}))$ represents a tensor of causal macro movements with a set of macro causality vectors, $(\bar{\alpha})$, $(\bar{\alpha} = (\bar{\alpha}_1, \cdots, \bar{\alpha}_{NN}))$, and (NN) is the number of elements in the macro causality vector. Vector $(X^{+}(\bar{\alpha}))$ represents physical movements in an open space. Vector $(Y_j^{+}(\bar{\alpha})$ represents physical movements that are complimentary to vector $(X^{+}(\bar{\alpha}))$, and therefore, must have the same causality vector, $(\bar{\alpha})$. $(\hat{\omega^{+}}(\bar{\bar{\alpha}})$ is the dialectic causal macro movements tensor, $(\bar{\bar{\alpha}} = (\bar{\bar{\alpha}}_1, \cdots, \bar{\bar{\alpha}}_{MM})$, and (MM) is the number of elements in the dialectic macro causality vector. The causality vector in the dialectic macro movements is the opposite of the macro movement causality vector, since it is the collection of causality that diverts macro movements,from the goal. An example of macro movement,(X^{+}) would be walking to a train station, and an example of the imaginary part, (Y^{+}) is facing a blockage on the path that would require extra physical movements such as taking a detour to avoid the obstacle. The imaginary part enhances the micro movements in the sense that they positively impact the analytic utility function. The conjugate micro movements are mirror movements except that the conjugate imaginary part, $(\tilde{y}_{+})$ is not the same as the original imaginary part, (y_{+}). This is due to causality, since $(\tilde{y}_{+})$ is a function of a set of causal parameters $(\tilde{\alpha})$, $(\tilde{y}_{+} = \tilde{y}_{+}(\tilde{\alpha}))$ which requires that the original imaginary part becomes also a function of parameters $(\tilde{\alpha})$, $(y_{+}(\tilde{\alpha}))$. $(\tilde{y}_{+}(\tilde{\alpha}))$ does not enhance the micro movements and thus they negatively impact the analytic utility function, $(u(\omega))$. It is for this reason that conjugates of (ω_{+}) and (ω^{+}) are eliminated in favor of the dialectics of (ω_{+}) and (ω^{+}). Given that the conjugate of (ω_{+}), $(\overline{\omega}_{+})$ is a mirror image, and not the dialectic image, then it must be shown that due to causality the automorphism characteristic applies to the totality of the set, (original, and dialectic), and not each individual segment. The same way the macro movements (ω^{+}) contain both the movements and its' dialectic counterpart. This is due to causality. The automorphism character also applies to the set, (original, and dialectic) of macro movements.

In this section the aim is to show that micro and macro movements and their dialectics are a bialgebra, and therefore, the dialectic aspect is an integral part of the CTA utility and consumer surplus. The intrinsic dialectic nature of CTA consumer surplus gives a dynamic metabolism character to both utility and consumer surplus. This is a defining quality of the CTA utility and consumer surplus. Therefore, it is important to show the bialgebra of micro and macro movements. Hopf algebra is used for this purpose. In particular, the coalgebra structure, compatibility relationship, and endomorphism property are studied.

Let (W_{+}) be the space of all micro movements, and let (k_{+}) be a field of $(\omega_{+}(\alpha))$ tensors. Let a k-algebra be defined as a triple (W_{+}, M, u), where$(M : \omega_{+} \otimes \omega_{+} \rightarrow \omega_{+})$, and $(u : k_{+} \rightarrow W_{+})$. It is assumed that there exists a unitary ring morphism (u), such that $(u : k_{+} \rightarrow W_{+})$, and $(Im(u) \subseteq Z(W_{+}))$ where $(Z(W_{+})$ is a unity ring, where the imaginary part $(Im(u))$ goes to unity, and away from zero. Let $(\hat{W}_{+})$ be the space of all dialectic micro movements, and let $(\hat{k_{+}})$ be a field of $(\omega_{+}(\hat{\alpha}))$ dialectic micro tensors. Let a dialectic k-algebra be defined as a triple $(\hat{W}_{+}, \hat{M}, \hat{u})$, where $(\hat{M} : \hat{\omega_{+}} \otimes \hat{\omega_{+}} \rightarrow \hat{\omega_{+}})$, and $(\hat{u} : \hat{k_{+}} \rightarrow \hat{W}_{+})$. It is assumed that there exists a unitary ring morphism $(\hat{u})$, such that $(\hat{u} : \hat{k_{+}} \rightarrow \hat{W}_{+})$, and $(Im(\hat{u}) \subseteq Z(\hat{W}_{+}))$ where $(Z(\hat{W}_{+}))$ is a unity ring, where the imaginary part $(Im(\hat{u}))$ goes to zero, and away from unity. A bialgebra can be established between (W_{+}, M, u), and $(\hat{W}_{+}, \hat{M}, \hat{u})$, by finding a coalgebra $(W_{+} \otimes \hat{W}_{+}, \Delta, \epsilon)$ shown in Figure 12.

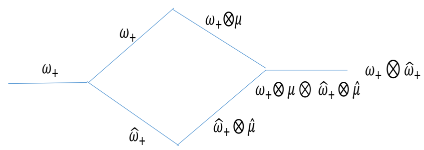

Figure 12. Micro and dialectic micro movements coalgebra

The same reasoning applies to macro and dialectic macro movements, Figure 13. Let (W^+) be the space of all macro movements, and let (k^+) be a field of $(\omega^+(\bar{\alpha}))$ tensors. Let a k-algebra for the macro and dialectic macro movements, be defined as a triple (W^+, M^+, u^+), where$(M^+ : \omega^+ \otimes \omega^+ \to \omega^+)$, and $(u^+ : k^+ \to W^+)$. It is assumed that there exists a unitary ring morphism (u^+), such that $(u^+ : k^+ \to W^+)$, and $(Im(u^+) \subseteq Z(W^+))$ where $(Z(W^+)$is an $(\alpha :\| \ Im(u^+) \ \| \leq \alpha)$ ring, where the imaginary part $(Im(u^+))$ goes to (α). Let $(\hat{W^+})$ be the space of all dialectic macro movements, and let $(\hat{k^+})$ be a field of $(\hat{\omega^+}(\bar{\bar{\alpha}}))$ dialectic macro tensors. Let a dialectic k-algebra be defined as a triple $(\hat{W^+}, \hat{M^+}, \hat{u^+})$, where $(\hat{M^+} : \hat{\omega^+} \otimes \hat{\omega^+} \to \hat{\omega^+})$, and $(\hat{u^+} : \hat{k^+} \to \hat{W^+})$. A bialgebra can be established between (W^+, M^+, u^+), $(\hat{W^+}, \hat{M^+}, \hat{u^+})$, $(W^+ \otimes \hat{W^+}, \Delta^+, \epsilon^+)$ shown in Figure 13.

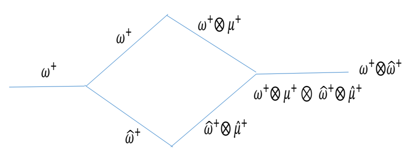

Figure 13. Macro and dialectic macro movements coalgebra

The rest of this section is dedicated to showing the existence of coalgebra between $(W_+ \otimes \hat{W}_+, \Delta, \epsilon)$, and $(W^+ \otimes \hat{W^+}, \Delta^+, \epsilon^+)$. It is shown that there exists a coalgebra structure between the two micro and macro spaces that includes their dialectic counterparts. The coalgebra structure is completed by showing compatibility relationship, and endomorphism property of the two spaces.

Theorem 1. *Let the two spaces $(W_+ \otimes \hat{W}_+, \Delta, \epsilon)$, and $(W^+ \otimes \hat{W^+}, \Delta^+, \epsilon^+)$ exist. For each $((\omega_+ \otimes \hat{\omega_+}) \in (W_+ \otimes \hat{W}_+))$, and $((\omega^+ \otimes \hat{\omega^+}) \in (W^+ \otimes \hat{W^+}))$, there exists a coalgebra structure. $(\omega_+ \otimes \hat{\omega_+})$ and $(\omega^+ \otimes \hat{\omega^+})$ are commutative, transitive and associative.*

Proof. Commutative property: Let $(F_1 = (\omega_+ \otimes \hat{\omega_+}))$ and $(F_2 = (\omega^+ \otimes \hat{\omega^+}))$ be two manifolds. Let there be a Riemann surface covering for the two manifolds (F_1) and (F_2). A Riemann covering for manifold (F_1) and (F_2) is a mapping from $(\Re^{N+M})$ to $(\Re^N)$. The imaginary space is a mapping from $(\mathbb{Z}^M)$ to a sphere in (M) dimension, (S^M) with radius (r=1). The Riemann surface covering possesses a set of curves (**c**) that connect a point on the covering to a corresponding point in the sphere.

Let the macro causality vector be represented as $(a^+ = a_l^+; l = 1, \cdots, m)$,and the dialectic macro causality vector be represented as $(\overset{+}{a} = \overset{+}{a_k}; k = 1, \cdots, m\prime)$. Let $(M = (m + m\prime))$, and $((a_l^+, \overset{+}{a_k}) = (a_l^+ \otimes \overset{+}{a_k}))$. let the micro causality vector be represented as $(a_+ = a_{+i}; i = 1, \cdots, n)$, and the dialectic micro causality vector be represented as $(\hat{a}_+ = \hat{a}_{+j}; j = 1, \cdots, n')$. Let $(N = (n + n'))$, and $((a_{+i}, \hat{a}_{+j}) = (a_{+i} \otimes \hat{a}_{+j}))$. Let matrix (E_{IJ}) be a matrix of size $(N \times M)$ representing the coverings of the two manifolds (F_1) and (F_2). Matrix (E_{IJ}) where $(I = 1, \cdots, N)$, and $(J = 1, \cdots, M)$ consists of four blocks $(E_I, E_{II}, E_{III}, E_{IV})$ each a covering based on the combination of the causal elements. For the case (i=j), and (k=l), the diagonal entries of the matrix (E_{IJ}) are entries relating to the union of the following sets: $((a_{+i}, \hat{a}_{+j}) \bigcap (a_l^+, \hat{a}_k^+); i = j, l = k)$, $(E_{dd} = E_{dd}((a_{+i}, \hat{a}_{+j}) \bigcap (a_l^+, \hat{a}_k^+)))$, where ($E_{dd}$) are the coverings of the the two manifolds (F_1), and (F_2) that corresponds to the set $((a_{+i}, \hat{a}_{+j}) \bigcap (a_l^+, \hat{a}_k^+); i = j, l = k)$. The first block matrix is the covering of the two manifolds relating to set $((a_+^{i-}, \hat{a}_+^j) \bigcap (a_{l-}^+, \hat{a}_k^+); i < j, l < k)$, thus $(E_I = E_I(a_+^{i-}, \hat{a}_+^j) \bigcap (a_{l-}^+, \hat{a}_k^+))$ for $(i < j, l < k)$. The second block of matrix (E_{IJ}) is the covering of the two manifolds relating to set $((a_+^{i-}, \hat{a}_+^j) \bigcap (a_{l+}^+, \hat{a}_k^+); i < j, l > k)$, thus $(E_{II} = E_{II}(a_+^{i-}, \hat{a}_+^j) \bigcap (a_{l+}^+, \hat{a}_k^+))$ for $(i < j, l > k)$. The third block of matrix (E_{IJ}) is the covering of the two manifolds relating to set $((a_+^{i+}, \hat{a}_+^j) \bigcap (a_{l-}^+, \hat{a}_k^+); i > j, l < k)$, thus $(E_{III} = E_{III}(a_+^{i+}, \hat{a}_+^j) \bigcap (a_{l-}^+, \hat{a}_k^+))$ for $(i > j, l < k)$. The fourth block of matrix (E_{IJ}) is the covering of the two manifolds relating to set $((a_+^{i+}, \hat{a}_+^j) \bigcap (a_{l+}^+, \hat{a}_k^+); i > j, l > k)$, thus $(E_{IV} = E_{IV}(a_{+i}^{i+}, \hat{a}_+^j) \bigcap (a_{l+}^+, \hat{a}_k^+))$ for $(i > j, l > k)$.

Each block of Riemann surface coverings $(E_I, E_{II}, E_{III}, E_{IV})$ of the two manifolds, (F_1) , (F_2) is modified by a curve (φ) that connects the real domain, to the complex domain, picked as a sphere of radius (r) as is shown as a 3D example in Figure 14. The number of connections or curves (φ) is equal to the number of (x^*, y^*) points. Let $(x^* = ((x_+^i, \hat{x}_+^i), (X_l^+, \hat{X}_k^+)) \to f((x_+^i, \hat{x}_+^i), (X_l^+, \hat{X}_k^+)))$ be a mapping from $(((x_+^i, \hat{x}_+^i), (X_l^+, \hat{X}_k^+)))$ to manifold $(f((x_+^i, \hat{x}_+^i), (X_l^+, \hat{X}_k^+)))$. Manifold (f) is a Riemann surface covering in ($\Re^N$). In a similar manner, let $(y^* = ((y_+^i, \hat{y}_+^i), (Y_l^+, \hat{Y}_k^+)) \to g((y_+^i, \hat{y}_+^i), (Y_l^+, \hat{Y}_k^+)))$ be a mapping from $((y_+^i, \hat{y}_+^i), (Y_l^+, \hat{Y}_k^+)))$ to a sphere $(g((y_+^i, \hat{y}_+^i), (Y_l^+, \hat{Y}_k^+)))$ in (M) dimension with radius (r=1), (S^M), $(g \in S^M)$. The curves connecting points (x^*) to (y^*) are represented as $(\varphi = \varphi(x^*, y^*))$. Therefore, each of the four blocks $(E_I, E_{II}, E_{III}, E_{IV})$ are formulated as $(E_I = x^* \otimes \varphi(x^*, y^*) \otimes y^*; \forall i < j, l < k)$, $(E_{II} = x^* \otimes \varphi(x^*, y^*) \otimes y^*; \forall i < j, l > k)$; $(E_{III} = x^* \otimes \varphi(x^*, y^*) \otimes y^*; \forall i > j, l < k)$; and $(E_{IV} = x^* \otimes \varphi(x^*, y^*) \otimes y^*; \forall i > j, l > k)$. To show the commutative property, let $(\Delta^I = E_I \oplus E_{II} \oplus E_{III} \oplus E_{IV})$, and let $(\Delta^{II} = (E_I \oplus E_{III}) \oplus (E_{II} \oplus E_{IV}))$, and let $(\Delta^{III} = (E_I \oplus E_{IV}) \oplus (E_{II} \oplus E_{III}))$, then $(\Delta^I \oplus \Delta^{II} \oplus \Delta^{III} = \Delta^{III} \oplus \Delta^I \oplus \Delta^{II})$. The associative property holds since $(\Delta^I \oplus \Delta^{II} \oplus \Delta^{III} = (\Delta^I \oplus \Delta^{II)} \oplus \Delta^{III})$.

Transitive property: Let the Radius of the sphere (r) be parameterized by introducing (τ), $(r = r(\tau))$ such that $(r(\tau))$ converges around $(\tau = 0)$, and $(\mid \tau \mid < \rho)$, for some $(\rho > 0)$ is a positive constant. $(r(\tau))$, (1) converges and has a modulus $(< p)$, for some integer (p). Given the parametrization of the radius of the sphere in ($\mathbb{Z}^M$), then both $(g(y^*(\tau)))$, and $(\varphi(x^*(\tau), y^*(\tau)))$ can be parameterized; (2) there exists different values of $(r(\tau))$ for different values of (τ), and for any two distinct values of (τ), (τ_1), and (τ_2), where $(\tau_1 \neq \tau_2)$,$(r(\tau))$ has different values. Thus in the neighborhood of $(\mid \tau \mid < \rho)$, $(g(y^*(\tau_1)) \neq g(y^*(\tau_2)))$, and $(\varphi(x^*(\tau_1), y^*(\tau_1)) \neq \varphi(x^*(\tau_2), y^*(\tau_2))$. The two conditions of transitivity are satisfied;, namely, 1) the inequality condition: for any distinct values of (τ),(τ_1),(τ_2), and (τ_3), such that $(\tau_1 < \tau_2 < \tau_2)$, then $(r(\tau_1) < r(\tau_2) < r(\tau_2))$, thus $(g(y^*(\tau_1)) < g(y^*(\tau_2)))$, $(g(y^*(\tau_2)) < g(y^*(\tau_3)))$, then $(g(y^*(\tau_1)) < g(y^*(\tau_3)))$. The connections do respect the transition condition since the size of the spheres affect the distances between any two points (x^*, y^*), then $(\varphi(x^*(\tau_1), y^*(\tau_1)) < \varphi(x^*(\tau_2), y^*(\tau_2))$, $(\varphi(x^*(\tau_2), y^*(\tau_2)) < \varphi(x^*(\tau_3), y^*(\tau_3))$, and thus $(\varphi(x^*(\tau_1), y^*(\tau_1)) < \varphi(x^*(\tau_3), y^*(\tau_3))$, and thus given that the covering (f) is fixed, then it can be concluded that $(E_*(\tau_1) < E_*(\tau_2))$, $(E_*(\tau_2) < E_*(\tau_3))$, and $(E_*(\tau_1) < E_*(\tau_3))$, for $(* = I, II, III, IV)$. 2) The equivalence condition: The pair $(g(y^*(\tau)))$, and $(\varphi(x^*(\tau), y^*(\tau)))$ are equivalent to the original pair $(g(y^*)$, and $(\varphi(x^*, y^*))$, and this is true for all values of $(r(\tau))$ as long as $(r(\tau))$ converges in the vicinity of $(\tau = 0)$, and $(\mid \tau \mid < \rho)$. For any fixed values of (τ) the equivalence relation is symmetric. Therefore, for any fixed value of (τ), the covering $(E_*(\tau))$is equivalent to itself, and if two pairs of coverings are equivalent to a third, then they are equivalent to each other.

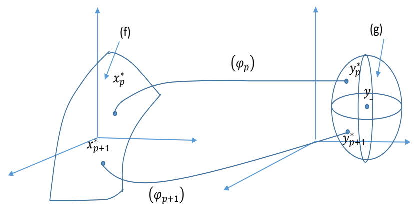

Figure 14. Demonstration of (c) curves

□

Theorem 2. *Let the two spaces $(W_+ \otimes \hat{W}_+, \Delta, \epsilon)$, and $(W^+ \otimes \hat{W}^+, \Delta^+, \epsilon^+)$ exist, and $((\omega_+ \otimes \hat{\omega_+}) \in (W_+ \otimes \hat{W}_+))$, and $((\omega^+ \otimes \hat{\omega^+}) \in (W^+ \otimes \hat{W}^+))$. Then $(\omega_+ \otimes \hat{\omega_+})$ and $(\omega^+ \otimes \hat{\omega^+})$ are compatible.*

Proof. Let the sub-spaces $((\omega_+ \otimes \hat{\omega_+}) = R_1)$ and $((\omega^+ \otimes \hat{\omega^+}) = R_2)$ be denoted by (R_1), and (R_2). The multiplicative subspace $(R_1 \otimes R_2)$ represents the Riemann coverings $(E_*; * = I, II, III, IV)$, then from Theorem 1, there exists sub-spaces $(\acute{R}_1)$, and $(\acute{R}_2)$ such that there exists a multiplicative subspace $(\acute{R}_1 \otimes \acute{R}_2)$ representing Riemann coverings $(\acute{E}_*; * = I, II, III, IV)$. Therefore, there exists a multiplicative subspace of the original multiplicative space.

□

Theorem 3. *Given multiplicative sub-spaces $(R_1 \otimes R_2)$, and $(\acute{R}_1 \otimes \acute{R}_2)$, then there exists a morphism of $(\acute{R}_1 \otimes \acute{R}_2)$ onto itself, and onto $(R_1 \otimes R_2)$.*

Proof. From Theorem 2 the two multiplicative sub-spaces $(R_1 \otimes R_2)$ and $(\acute{R}_1 \otimes \acute{R}_2)$ are compatible. Then, there exists a transition function $(\iota \sim \otimes 1)$, where (ι) is equivalent to identity such that $((\acute{R}_1 \otimes \acute{R}_2) \underset{\longrightarrow}{\iota} (\acute{R}_1 \otimes \acute{R}_2))$. Conversely, it is possible to have a morphism onto the original multiplicative space $(R_1 \otimes R_2)$ given a transitional function (ι_{-1}), $((\acute{R}_1 \otimes \acute{R}_2) \underset{\longrightarrow}{\iota_{-1}} (R_1 \otimes R_2))$as is shown in Figure 15.

□

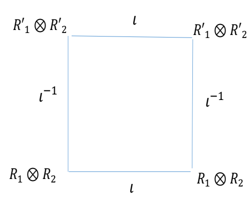

Figure 15. Endomorphism of multiplicative sub-spaces

6 Price evolution in the CTA model as compared to the Tversky model

In this section price evolution is analyzed with respective to the CTA price evolution scheme, and the Tversky price evolution scheme. Let's describe briefly the Tversky pricing scheme. The Tversky price evolution is demonstrated in Figure 16. In Figure 16, an example of pricing evolution based on Tversky's value function is given. Three hypothetical value functions, (vf_1, vf_2, vf_3) are depicted as well as (3) hypothetical indifference curves, (Ic_1, Ic_2, Ic_3) and three hypothetical supply curves, (s_1, s_2, s_3). The x-axis represents quantity consumed as well as gain, while the y-axis represents price and value. The indifference curves in this example, are shown as hyperbolas in order to show the dis-utility of price which is equivalent to the marginal diminishing returns. For each quantity and price (q,p), there exists a mirror image, (q^-, p^-) that measures the loss consumer is willing to accept when he makes the choice of (q,p). An example of how Figure 16, can be analyzed is as follows: Let's look at (s_1). (s_1) intersects (Ic_3), and (vf_3), at the unique point (q_{a1}, p_{a1}), and at a loss (q_{a1}^-, p_{a1}^-). The price evolution if the starting point is (a_1), is towards (b_1), and (b_2). Points (b_1), and (b_2) are the intersection points of (s_2) with (vf_2), and (Ic_1), and (Ic_3). Points (b_1), and (b_2) constitute a unique monotone affine function pointing to an evolution strictly from (b_1), to (b_2). The next evolution at this point is towards points (c_1), and (c_2), again with a unique monotone affine function. This example stops at points (d_1), and (d_2). In all cases presented in Figure 16, the price evolution is deterministic. Given that Figure 16, is a generic representation of the price evolution based on Tversky model, a general conclusion can be made that the Tversky price evolution is deterministic.

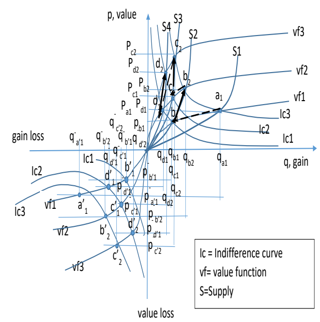

Figure 16. Tversky price evolution scheme

In contrast to the Tversky model, there is no element of risk associated with consumption in the CTA model. The quantity consumed and the corresponding price are functions of micor-macro movements, or Nea work. In contrast to the deterministic paths of the Tversky model, the CTA model produces dynamic paths related to the pattern and the combination of micro-macro movements, as is shown in Figure 17a. Figure 17a, depicts the three hypothetical equilibrium points, (e_1, e_2, e_3). These equilibrium points show a movement from point (e_1), to point (e_2), and point (e_3). This movement could be rotational depending on the homeomorphic characteristics of the bialgebra, $(\omega_+^+ = (\omega_+ \otimes \hat{\omega_+}) \otimes (\omega^+ \otimes \hat{\omega^+}) = (R_1 \otimes R_2))$ and in some cases isomorphic nature of the sub-groups of the Nea work, (ω_+^+), and the direction of rotation depends on the starting point. More equilibrium points can be found depending on the micro-macro movement patterns and combinations as is shown in Figure 17b. Eventually, it is possible to expand the features of the micro-macro movements so that in addition to pattern and combination, for example, one could include the time that it takes to execute micro-macro movements, $(t(\omega_+^+))$. An example of such expansion is shown in Figure 17c. Due to the rotational capability of the CTA model of price evolution, adding one more feature would produce a sphere shape price evolution. In Figre 17c, point (e_m) represents any price point on the surface of the unit sphere on a path beginning from a point $(e_{n'}; n' \neq n)$, or $(e_n; n \neq n')$, where point (e_n) represents any other price point inside of the sphere. The sphere is due to the limits of the Nea work, (ω_+^+) since the consumption $(q(\omega_+^+))$ is limited due to the law of the diminishing returns, and the price $(p(\omega_+^+))$, is also limited by the law of the diminishing returns. Eventually, $(t(\omega_+^+))$ the time of performing Nea work is limited by physical capacity (in this case time required to perform a task is defined as the standard capacity required to perform the task). In order to unify the limits on the major axis of example, in Figure 17c, the sphere is a unit sphere.

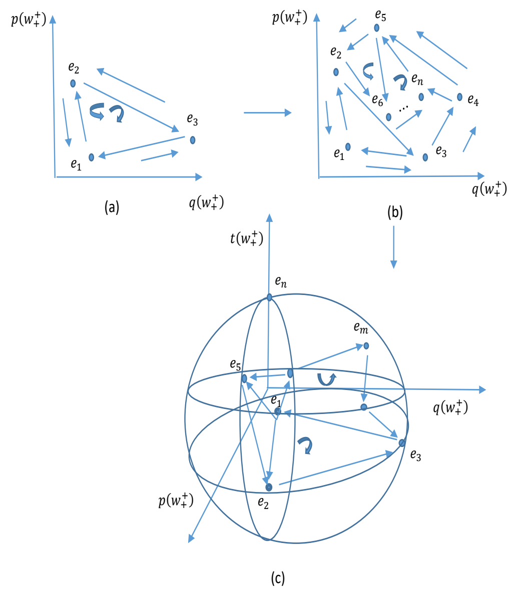

Figure 17. CTA price evolution scheme

Paths in the CTA model can be found on the Riemann coverings $(E_*; * = I, II, III)$, (IV) and are the result of rotational movements. This rotational movement is due to variations in the pattern and the combination of micro and macro movements that would cause change in price with respect to an initial price. A path is formed that starts from a price as a function of what can be considered to be a base group of micro-macro movements, and ends at a price that is the result of a sub-group of the base micro-macro movements group. The initial point can be considered as a fixed point. All representations of the rotation patterns of the CTA model price evolution are either homeomorphic, or isomorphic. This implies that it is possible to find a group of paths that are obtained by a group of original rotations. The original paths are irreducible and invariant paths. It is possible that paths are unique, and any transformation in this case is isomorphic. One advantage of finding group rotational paths based on a fixed point, and original rotation is to be able to predict future price movements. Representations of the CTA rotational paths, is different from the standard representations of rotations introduced by (Gelfend, Minlos, and Shapiro), [9]. The standard representation uses two elements: 1) a transition function $(g(\varphi_1, \theta, \varphi_2))$ and 2) Euler angles $(\varphi_1, \theta, \varphi_2)$ in three dimensional space. In the CTA version of representation of price paths, the angle of rotation, the same as the Euler angles are functions of causal elements, $(\alpha_+^+ = (\alpha_+, \hat{\alpha}_+) \cup (\bar{\alpha}^+, \bar{\bar{\alpha}}^+))$. The angles of rotation are determined by the most prominent causal element which is the element with the most impact on the angle, $(\varphi_1(\omega_+^+) \cong \varphi_1(\tilde{\alpha}_{1+}^+))$, where $(\tilde{\alpha}_{1+}^+)$ is the prominent causal element for angle (φ_1), and, $(\varphi_2(\omega_+^+) \cong \varphi_2(\tilde{\alpha}_{2+}^+))$, where $(\tilde{\alpha}_{2+}^+)$ is the prominent causal element for angle (φ_2),and $(\theta(\omega_+^+) \cong \theta(\tilde{\alpha}_{3+}^+));)$ where $(\tilde{\alpha}_{3+}^+)$ is the prominent causal element for angle (θ). The magnitudes of the rotational angles are proportional to the magnitude of the prominent causal elements. The transitions function $(g(\varphi_1, \theta, \varphi_2))$ is affected by the rotational angles. Thus the transitional function is a function of the patterns and the combination of the causal elements. Transition occurs by rotating the axis, using the transition function, $(q'(\omega_+^+) = g(\varphi_1, \theta, \varphi_2) \otimes q(\omega_+^+))$, $(p'(\omega_+^+) = g(\varphi_1, \theta, \varphi_2) \otimes p(\omega_+^+))$, and $(t'(\omega_+^+) = g(\varphi_1, \theta, \varphi_2) \otimes t(\omega_+^+))$ are the examples of rotation in the three dimensional space. The limits in representations, are important since they put restrictions on price forecasting.

7 Conclusion

In this paper the concept of the CTA based utility and consumer surplus is introduced. The CTA based utility and consumer surplus is different from the classical utility and consumer surplus. The difference is that in the CTA case, price is not the cause that defines utility and thus consumer surplus, it is rather the micro and macro movements required to accomplish the task of purchasing a good that ultimately define the shape of the consumer surplus. It is assumed that the real causality for the shape of the consumer surplus is the micro and macro movements required from an individual. Price is a shadow causality. The concept of dialectic in causality is introduced. Dialectic causality implies a duality within causality that needs to be addressed since it is embedded in the causality. The coalgebra structure of each set of micro and macro movements and the two sets together are analyzed for properties such as endomorphism and commutative properties. The duality property of price acceptability and evolution in the CTA utility and the CTA consumer surplus is compared to standard utility and consumer surplus defined by Tversky.

References

[1] Marshall, A. Principals of Economics, Pulgrave Classics publishers, Eight edition, England 1920.

[2] Child, W. Causality, Interpretation and the Mind, Clarendon Press, Oxford 1994.

[3] Davidson, D. Causal relations, Journal of Philosophy, vol.(64), 1967.

[4] Mellor, D. H. The Facts of Causation, Routledge Publishers, England, digital printing 2006.

[5] Tversky, A. The Essential Tversky, The MIT Press, Cambridge Massachusetts, USA. 2018.

[6] Hicks, J.R. Value and Capital: An inquiry into some fundamental principals of economic theory, Oxford university press, Second edition, England 1946.

[7] Underwood, R.G. An Introduction to Hopf Algebras, Springer New York, USA. 2011.

[8] Dascalescu, S., Nastasescu, C., Rainau, S. An Introduction to Hopf Algebras, Marcel Dekker, Inc., USA. 2001.

[9] Gelfand, I.M., Minlos, R.A., Shapiro, Z.Ya. Representations of the Rotation and Lorentz Groups and Their Applications, Dover Publications, Inc., New York, USA. 2018.