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Parameterized Soft Complex Fuzzy Sets

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Abstract

In this paper, we presents a new concept of parameterized soft complex fuzzy set *(pscf-set)* which is generalized from the innovative concept of a complex fuzzy set by adding phase term to the definition of fuzzy set. Various operators with examples of *pscf-set* provided in this paper. We then define a *pscf-set* aggregation operator

Keywords Fuzzy set; Fuzzy soft set;Fuzzy parameterized fuzzy soft set; Complex fuzzy set (CFS) ;Soft complex fuzzy set; Parameterized soft complex fuzzy set *(pscf-set).*

1. INTRODUCTION

Many fields deal daily with the uncertainity and periodicity data, that may not be successfully modeled by the classical mathematics. There are some mathematical tools for dealing with uncertainties and periodicities; three of them are fuzzy set theory, developed by Zadeh (1965), soft set theory, introduced by Molodtsov (1999), and complex fuzzy set, developed by Ramot et al(2002) that are related to this work.

Fuzzy sets [9] provide a robust mathematical framework for dealing with "real-world" imprecision and nonstatistical uncertainty. Qualitative "linguistic" variables allow one to represent a range of numerical values as a single descriptive term that is described by a fuzzy set. Given that the present day complex networks are dynamic, that there is great uncertainty associated with the input traffic and other environmental parameters, that they are subject to unexpected overloads, failures and perturbations, and that they defy accurate analytical modeling, fuzzy logic appears to be a promising approach to address key aspects of networks. The ability to model networks in the continuum mathematics of fuzzy sets rather than with traditional discrete values, coupled with extensive simulation, offers a reasonable compromise between rigorous analytical modeling and purely qualitative simulation.

Soft set theory is a generalization of fuzzy set theory, which was proposed by Molodtsov [6] in 1999 to deal with uncertainty in a non-parametric manner. One of the most important steps for the theory of Soft Sets was to define mappings on soft sets, this was achieved in 2009 by mathematician Athar Kharal, though the results were published in 2011. Soft sets have also been applied to the problem of medical diagnosis for use in medical expert systems. Fuzzy soft sets have also been introduced in [3].

Complex fuzzy set (CFS) [4]-[5] is a new development in the theory of fuzzy systems. The concept of CFS is an extension of fuzzy set, by which the membership for each element of a complex fuzzy set is extended to complex-valued state.

In this paper, we define parameterized soft complex fuzzy sets (*pscf*-sets) in which the approximate functions are defined from complex fuzzy parameters set to the complex fuzzy subsets of universal set. We also defined their operations and soft aggregation operator. We also present examples which show that the methods can be successfully applied to many problems that contain uncertainties and periodicities.

2. PRELIMINARIES

Definition: 2.1 Let U be a universe. A fuzzy set X over U is a set defined by a function μ_x representing a mapping

$$\mu_x : U \rightarrow [0, 1]$$

Here, μ_x called membership function of X, and the value μ_x (u) is called the grade of membership of $u \in U$. The value represents the degree of u belonging to the fuzzy set X. Thus, a fuzzy set X over U can be represented as follow,

$$X = \{(\mu_x(u)/u) : u \in U, \mu_x(u) \in [0,1]\}$$

Note that the set of all the fuzzy sets over U will be denoted by F(U). **[9]**

Definition: 2.2 Let U be an initial universe, P(U) be the power set of U, E be the set of all parameters and $A \subseteq E$. Then, a soft set **[6]**F_A over U is a set defined by a function f_A representing a mapping

$$f_A: E \rightarrow P(U) \text{ such that } f_A(x)=\phi \text{ if } x \notin A.$$

Here, f_A is called approximate function of the soft set F_A, and the value $f_A(x)$ is a set called x-element of the soft set for all $x \in E$. It is worth noting that the sets $f_A(x)$ may be arbitrary. Some of them may be empty, some may have nonempty intersection. Thus, a soft set F_A over U can be represented by the set of ordered pairs

$$F_A = \{(x, f_A(x)) : x \in E, f_A(x) \in P(U)\}$$

Note that the set of all soft sets over U will be denoted by S (U).

Definition: 2.3 Let *U* be an initial universe, *E* be the set of all parameters, $A \subseteq E$ and $\gamma_A(x)$ be a fuzzy set over *U* for all $x \in E$. Then, an Fuzzy Soft (*fs*-set) Γ_A over *U* is a set defined by a function γ_A representing a mapping

$$\gamma_A : E \rightarrow F(U) \text{ such that } \gamma_A(x)=\Phi; \text{ if } x \notin A.$$

Here, γ_A is called fuzzy approximate function of the *fs*-set Γ_A, and the value $\gamma_A(x)$ is a fuzzy set called *x*-element of the *fs*-set for all $x \in E$. Thus, an *fs*-set Γ_A over *U* can be represented by the set of ordered pairs

$$\Gamma_A = \{(x, \gamma_A(x)) : x \in E, \gamma_A(x) \in F(U)\}$$

Note that the set of all the fuzzy sets over U will be denoted by F(U) and from now on the sets of all *fs*-sets over *U* will be denoted by *FS (U)*. **[2],[3]**

Definition: 2.4 Let U be an initial universe, E be the set of all parameters and X be a fuzzy set over E with membership function $\mu_x(x) : E \rightarrow [0,1]$ and $\gamma_x(x)$ be a fuzzy set over U for all $x \in E$. Then, an fuzzy parameterized fuzzy soft set (fpfs-set) Γ_x over U is a set defined by a function $\gamma_x(x)$ representing a mapping

$$\gamma_x : E \rightarrow F(U) \text{ such that } \gamma_x(x)=\Phi; \text{ if } \mu_x(x)=0 .$$

Here, γ_x is called fuzzy approximate function of the fpfs-set Γ_x, and the value $\gamma_x(x)$ is a fuzzy set called x-element of the fpfs-set**[1]** for all $x \in E$. Thus, an fpfs-set Γ_x over U can be represented by the set of ordered pairs

$$\Gamma_x = \{(\mu_x(x)/x, \gamma_x(x)) : x \in E, \gamma_x(x) \in F(U), \mu_x(x) \in [0,1]\}$$

It must be noted that the sets of all *fpfs*-sets over U will be denoted by *fpfs*(U).

Definition: 2.5 Ramot et al. [3] recently proposed an important extension of these ideas, the *Complex Fuzzy Sets***[4]**, S, is characterized by a membership function, $\mu_S(x)$ is a complex-valued function that assigns a grade of membership of the form $r_S(x).e^{j\varpi_S(x)}$; ($j=\sqrt{-1}$) to any element x in the universe of discourse. The Value of $\mu_S(x)$ is defined by the two variables, $r_S(x)$ and $\varpi_S(x)$, both real valued, with $r_S(x) \in [0,1]$.

3. Parameterized Soft Complex Fuzzy Sets

Definition: 3.1 [8] Let U be an initial universe, E be the set of all parameters, $A \subseteq E$ and $\psi_A(x)$ be a complex fuzzy set over U for all $x \in E$. Then, an Soft complex fuzzy set χ_A over U is a set defined by a function ψ_A representing a mapping

$$\chi_A : E \to C(U) \text{ such that } \psi_A(x) = \Phi; \text{if } x \in A.$$

Here, ψ_A is called complex fuzzy approximate function of the *soft complex fuzzy set* χ_A, and the value $\psi_A(x)$ is a complex fuzzy set called *x*-element of the *soft complex fuzzy set* for all $x \in E$. Thus, an *soft complex fuzzy set* χ_A over *U* can be represented by the set of ordered pairs

$$\chi_A = \{(x, \psi_A(x)) : x \in E, \psi_A(x) \in C(U)\}$$

Note that the set of all the complex fuzzy sets over U will be denoted by C(U) .

Example: 3.2 [7] Let U= {h_1 (India), h_2(Australia), h_3(UK), h_4(USA)} be an initial set,

consider E= {e_1(Inflation rate), e_2(population growth), e_3(Unemployment rate), e_4(share market index)} be an country's growth parameters set and $A \subseteq E$, A={e_1,e_3},

$$\psi_A(e_1) = \left\{ \frac{0.4e^{j0.5\pi}}{h_1}, \frac{0.8e^{j0.6\pi}}{h_2}, \frac{0.8e^{j0.8\pi}}{h_3}, \frac{1.0e^{j0.75\pi}}{h_4} \right\} \quad \text{and}$$

$$\psi_A(e_3) = \left\{ \frac{0.6e^{j0.7\pi}}{h_1}, \frac{0.9e^{j0.9\pi}}{h_2}, \frac{0.7e^{j0.95\pi}}{h_3}, \frac{0.75^{j0.95\pi}}{h_4} \right\}$$

then **soft complex fuzzy set** χ_A is written by

$$\chi_A = \left\{ \left(e_1, \frac{0.4e^{j0.5\pi}}{h_1}, \frac{0.8e^{j0.6\pi}}{h_2}, \frac{0.8e^{j0.8\pi}}{h_3}, \frac{1.0e^{j0.75\pi}}{h_4} \right), \left(e_3, \frac{0.6e^{j0.7\pi}}{h_1}, \frac{0.9e^{j0.9\pi}}{h_2}, \frac{0.7e^{j0.95\pi}}{h_3}, \frac{0.75^{j0.95\pi}}{h_4} \right) \right\}$$

Definition: 3.3 Let U be an initial universe, E be the set of all parameters, and S be a fuzzy set over E with membership function $\mu_S(x) : E \to [0,1]$ and $\gamma_S(x)$ is a complex fuzzy set over U for all $x \in E$. Then, a *Parameterized Soft complex fuzzy set* Ω_S over *U* is a set defined by a function γ_S representing a mapping

$$\gamma_S : E \to C(U) \text{ such that } \gamma_S(x) = \Phi \text{ if } \mu_S(x) = 0$$

Here, γ_S is called complex fuzzy approximate function of the *Parameterized soft complex fuzzy set* Ω_S, and the value $\gamma_S(x)$ is a complex fuzzy set called *x*-element of the *Parameterized soft complex fuzzy set* for all $x \in E$. Thus, an *Parameterized soft complex fuzzy set* Ω_S over *U* can be represented by the set of ordered pairs

$$\Omega_S = \{(\mu_S(x), \gamma_S(x)) : x \in E, \gamma_S(x) \in C(U), \mu_S(x) \in [0,1]\}$$

Note that the set of all the complex fuzzy sets over U will be denoted by C(U), from now on the sets of all ***Parameterized soft complex fuzzy sets*** over *U* will be denoted by *pscf (U).*

Example: 3.4

Let U= {h_1,h_2,h_3,h_4} be an initial set, consider E= {e_1,e_2,e_3,e_4} is set of parameters

If $S = \left\{ \frac{0.6}{e_1}, \frac{0.4}{e_2} \right\}$ and $\gamma_S(e_1) = \left\{ \frac{0.2e^{0.5\pi}}{h_1}, \frac{0.8e^{j0.4\pi}}{h_4} \right\}$, $\gamma_S(e_2) = \left\{ \frac{0.3e^{0.4\pi}}{h_1}, \frac{0.7e^{j0.7\pi}}{h_2} \right\}$

Then **Parameterized soft complex fuzzy set** Ω_S is written by

$$\Omega_S=\left\{\left(\frac{0.6}{e_1},\left\{\frac{0.2e^{0.5\pi}}{h_1},\frac{0.8e^{j0.4\pi}}{h_4}\right\}\right),\left(\frac{0.4}{e_2},\left\{\frac{0.3e^{0.4\pi}}{h_1},\frac{0.7e^{j0.7\pi}}{h_2}\right\}\right)\right\}$$

Definition: 3.5 Let $\Omega_S \in pscf(U)$. If $\mu_S(x)=0$ *and* $\gamma_S(x)=\phi$ for all $x \in E$, then Ω_S is called an S-empty *pscf*-set, denoted by Ω_{ϕ_S}.

If S=ø, then the S-empty *pscf*-set is called empty *pscf*-set, denoted by Ω_ϕ

Definition: 3.6 Let $\Omega_S \in pscf(U)$. If $\mu_S(x)$=1and $\gamma_S(x)=U$ for all $x \in S$, then Ω_S is called an S-Universal *pscf*-set, denoted by $\Omega_{\tilde{S}}$. If S=E, then the S-universal *pscf*-set is called universal *pscf*-set, denoted by $\Omega_{\tilde{E}}$

Example: 3.7 Let U= $\{h_1,h_2,h_3,h_4\}$ be an initial set, consider E= $\{e_1,e_2,e_3,e_4\}$ is set of parameters,

If $S_1=\left\{\frac{0.6}{e_1},\frac{0.4}{e_2}\right\}$ and $\gamma_S(e_1)=\left\{\frac{0.2e^{0.5\pi}}{h_1},\frac{0.8e^{j0.4\pi}}{h_4}\right\}$, $\gamma_S(e_2)=\left\{\frac{0.3e^{0.4\pi}}{h_1},\frac{0.7e^{j0.7\pi}}{h_2}\right\}$

Then **Parameterized soft complex fuzzy set** Ω_{S1} is written by

$$\Omega_{S1}=\left\{\left(\frac{0.6}{e_1},\left\{\frac{0.2e^{0.5\pi}}{h_1},\frac{0.8e^{j0.4\pi}}{h_4}\right\}\right),\left(\frac{0.4}{e_2},\left\{\frac{0.3e^{0.4\pi}}{h_1},\frac{0.7e^{j0.7\pi}}{h_2}\right\}\right)\right\}$$

If $S_2=\left\{\frac{0.6}{e_1},\frac{0.4}{e_2}\right\}$ and $\gamma_S(e_1)=ø, \gamma_S(e_2)=ø$ then the *pscf*-set Ω_{S2} is a S_2-empty *pscf*-set

If $S_3=\left\{\frac{1}{e_1},\frac{1}{e_2}\right\}$ and $\gamma_S(e_1)=U, \gamma_S(e_2)=U$, then the *pscf*-set Ω_{S3} is a S_3-Universal *pscf*-set

4. Operations on Parameterized Soft Complex Fuzzy Sets

Definition: 4.1 Let $\Omega_S, \Omega_T \in PSCF(U)$. Then, Ω_S and Ω_T the *pscf*-equal, written as $\Omega_S = \Omega_T$, if and only if $\mu_S(x)=\mu_T(x)$ and $\gamma_S(x)=\gamma_T(x)$ for all $x \in E$

Definition: 4.2 Let $\Omega_A, \Omega_B \in PSCF(U)$. Then, union of Ω_A and Ω_B, denoted by $\Omega_A \cup \Omega_B$,is defined by

$\mu_{A\cup B}(x)=\max(\mu_A(x),\mu_B(x))$ and $\gamma_{A\cup B}(x)=[r_A(x)\oplus r_B(x)]e^{j\omega_{AUB}(\varepsilon)}$ for all $x \in E$; $[r_A(x)\oplus r_B(x)]$ are defined in [3],

ω_{AUB} is defined as follows.

a) (Sum) $\omega_{AUB}=\omega_A+\omega_B$

b) (Max) $\omega_{AUB}=\max\ (\omega_A,\omega_B)$

c) (Min) $\omega_{AUB}=\min\ (\omega_A,\omega_B)$

d) (Winner takes all) $\omega_{AUB}=\begin{cases}\omega_A, if ...r_A > r_B\\ \omega_B, if ..r_B \geq r_A\end{cases}$

Definition: 4.3 Let $\Omega_A, \Omega_B \in PSCF(U)$. Then, Intersection of Ω_A and Ω_B, denoted by $\Omega_A \cap \Omega_B$,is defined by $\mu_{A\cap B}(x) = \min(\mu_A(x), \mu_B(x))$ and $\gamma_{A\cap B}(x) = [r_A(x) * r_B(x)]e^{j\omega_{A\cup B}(\varepsilon)}$ for all $x \in E$; $[r_A(x) * r_B(x)]$ are defined in [3], $\omega_{A\cap B}$ is defined as in Definition 4.2

Definition: 4.4 Let $\Omega_S \in PSCF(U)$. Then *pscf*-aggregation operator, denoted by $PSCF_{agg}$, is defined by $PSCF_{agg}$: $F(E)\ X\ PSCF(U) \to C(U)$

$PSCF_{agg}(X, \Omega_X) = \Omega_X^*$ where $\Omega_X^* = \{\mu_{\Omega_X^*}(u)/u : u \in U\}$ which is a complex fuzzy set over U. The value Ω_X^* is called aggregate complex fuzzy set of the Ω_X .Here, the membership degree $\mu_{\Omega_X^*}(u)$ of u is defined as follows $\mu_{\Omega_X^*}(u) = \frac{1}{|E|}\sum_{x\in E}\mu_X(x).\mu_{\gamma_X(x)}(u)$, where |E| is the cardinality of E.

CONCLUSION:

A soft set is a mapping from parameter to the crisp subset of universe. In soft complex fuzzy sets, the soft set theory is extended to a complex fuzzy set. This paper presented a new concept of parameterized soft complex fuzzy set, which is generalized from the innovative concept of a soft complex fuzzy set. To develop the theory, in this work, we define *pscf-set* with their operations. Finally we define *pscf-set* Aggregation operator for develop decision making problems.

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His ongoing research focusing on the subject of complex fuzzy sets, and he has published papers in International and National level conferences.