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Fourier Coefficients of a Class of Eta Quotients of Weight 12 with Level 12

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Abstract

Recently, Williams[18] and then Yao, Xia and Jin[15] discovered explicit formulas for the coefficients of the Fourier series expansions of a class of eta quotients. Williams expressed all coefficients of 126 eta quotients in terms of $\sigma(n), \sigma(\frac{n}{2}), \sigma(\frac{n}{3})$ and $\sigma(\frac{n}{6})$ and Yao, Xia and Jin, following the method of proof of Williams, expressed only even coefficients of 104 eta quotients in terms of $\sigma_3(n), \sigma_3(\frac{n}{2}), \sigma_3(\frac{n}{3})$ and $\sigma_3(\frac{n}{6})$. Here, we will express the even Fourier coefficients of 2 eta quotients i.e., the Fourier coefficients of the sum, f(q)+f(-q), of 2 eta quotients in terms of $\sigma_5(n), \sigma_5\left(\frac{n}{2}\right), \sigma_5\left(\frac{n}{3}\right), \sigma_5\left(\frac{n}{4}\right), \sigma_5\left(\frac{n}{6}\right), \sigma_5\left(\frac{n}{12}\right), \sigma_{11}(n), \sigma_{11}\left(\frac{n}{2}\right),$ $\sigma_{11}\left(\frac{n}{3}\right), \sigma_{11}\left(\frac{n}{4}\right), \sigma_{11}\left(\frac{n}{6}\right), \sigma_{11}\left(\frac{n}{12}\right), \tau(n)$ (tau function), $\tau\left(\frac{n}{2}\right), \tau\left(\frac{n}{3}\right), \tau\left(\frac{n}{4}\right), \tau\left(\frac{n}{6}\right), \tau\left(\frac{n}{12}\right)$ and the odd Fourier coefficients of 393 eta quotients in terms of $\sigma_5(n), \sigma_5\left(\frac{n}{2}\right), \sigma_5\left(\frac{n}{3}\right), \sigma_5\left(\frac{n}{4}\right)$, $\sigma_5\left(\frac{n}{6}\right), \sigma_5\left(\frac{n}{12}\right), \sigma_{11}(n), \sigma_{11}\left(\frac{n}{2}\right), \sigma_{11}\left(\frac{n}{3}\right), \sigma_{11}\left(\frac{n}{4}\right), \sigma_{11}\left(\frac{n}{6}\right), \sigma_{11}\left(\frac{n}{12}\right), \tau(n), \tau\left(\frac{n}{2}\right), \tau\left(\frac{n}{3}\right), \tau\left(\frac{n}{4}\right), \tau\left(\frac{n}{6}\right), \tau\left(\frac{n}{12}\right)$ and $f_{13}, \cdots, f_{19}$.

Keywords: Dedekind eta function; eta quotients; Fourier series.

1 Introduction

The divisor function $\sigma_i(n)$ is defined for a positive integer i by

$$\sigma_i(n) := \sum_{d \text{ positive integer}, \ d|n} d^i, \quad \text{if } n \text{ is a positive integer, and} \tag{1}$$

$$\sigma_i(n) := 0 \quad \text{if n is not a positive integer.}$$

The Dedekind eta function is defined by

$$\eta(z) := q^{1/24} \prod_{n=1}^{\infty} (1 - q^n), \tag{2}$$

where

$$q := e^{2\pi iz}, z \in H = \{x + iy : y > 0\} \quad (3)$$

and an eta quotient of level n is defined by

$$f(z) := \prod_{m|n} \eta(mz)^{a_m}, n \in \mathbb{N}, a_m \in Z, a_n \neq 0. \quad (4)$$

It is interesting and important to determine explicit formulas of the Fourier coefficients of eta quotients since they are the building blocks of modular forms of level n and weight k. The book of Köhler [13] (Chapter 3, pg.39) describes such expansions by means of Hecke Theta series and develops algorithms for the determination of suitable eta quotients. One can find more information in [3], [6], [14], [16], [17]. I have determined the Fourier coefficients of the theta series associated to some quadratic forms, see [10], [11], [12], [7], [8] and [9].

Recently, Williams, see [18] discovered explicit formulas for the coefficients of Fourier series expansions of a class of 126 eta quotients in terms of $\sigma(n), \sigma(\frac{n}{2}), \sigma(\frac{n}{3})$ and $\sigma(\frac{n}{6})$. One example is as follows:

$$\frac{\eta^2(2z)\eta^4(4z)\eta^6(6z)}{\eta^2(z)\eta^2(3z)\eta^4(12z)}$$

gives the expansion found by Williams.

Then Yao, Xia and Jin [15] expressed the even Fourier coefficients of 104 eta quotients in terms of $\sigma_3(n), \sigma_3(\frac{n}{2}), \sigma_3(\frac{n}{3})$ and $\sigma_3(\frac{n}{6})$. One example is as follows:

$$\frac{\eta^{25}(2z)\eta^4(3z)}{\eta^{12}(z)\eta^5(4z)\eta^3(6z)\eta(12z)}$$

gives the even coefficients. Motivated by these two results, we can also express the even Fourier coefficients of 2 eta quotients in terms of $\sigma_5(n), \sigma_5\left(\frac{n}{2}\right), \sigma_5\left(\frac{n}{3}\right), \sigma_5\left(\frac{n}{4}\right), \sigma_5\left(\frac{n}{6}\right), \sigma_5\left(\frac{n}{12}\right), \sigma_{11}(n), \sigma_{11}\left(\frac{n}{2}\right),$

$\sigma_{11}\left(\frac{n}{3}\right), \sigma_{11}\left(\frac{n}{4}\right), \sigma_{11}\left(\frac{n}{6}\right), \sigma_{11}\left(\frac{n}{12}\right), \tau(n)$ (tau function), $\tau\left(\frac{n}{2}\right), \tau\left(\frac{n}{3}\right), \tau\left(\frac{n}{4}\right), \tau\left(\frac{n}{6}\right)$, and $\tau\left(\frac{n}{12}\right)$.

seeTable 2. One example is as follows:

$$\eta^{24}(2z)$$

gives result. We can also express the odd Fourier coefficients of 393 eta quotients in terms of $\sigma_5(n), \sigma_5\left(\frac{n}{2}\right), \sigma_5\left(\frac{n}{3}\right), \sigma_5\left(\frac{n}{4}\right), \sigma_5\left(\frac{n}{6}\right), \sigma_5\left(\frac{n}{12}\right), \sigma_{11}(n), \sigma_{11}\left(\frac{n}{2}\right),$

$\sigma_{11}\left(\frac{n}{3}\right), \sigma_{11}\left(\frac{n}{4}\right), \sigma_{11}\left(\frac{n}{6}\right), \sigma_{11}\left(\frac{n}{12}\right), \tau(n), \tau\left(\frac{n}{2}\right), \tau\left(\frac{n}{3}\right), \tau\left(\frac{n}{4}\right), \tau\left(\frac{n}{6}\right)$, and $\tau\left(\frac{n}{12}\right), f_{13}, \cdots, f_{19},$

see 40 of them in Table 3A and Table 3B. One example is as follows:

$$\frac{\eta^{20}(4z)\eta^4(6z)\eta^4(12z)}{\eta^4(2z)}$$

gives result. Now we can state our main Theorem:

Theorem 1 *Let* $b_1, b_2, \cdots, b_5$ *be non-negative integers satisfying*

$$b_1 + b_2 + \cdots + b_5 \le 24. \tag{5}$$

Define the integers $a_1, a_2, a_3, a_4, a_6, a_{12}$ by

$$a_1 := -b_1 + 2b_2 - 2b_3 - 4b_4 - b_5 + 24, \tag{6}$$

$$a_2 := 3b_1 + b_2 + 3b_3 + 10b_4 + b_5 - 60, \tag{7}$$

$$a_3 := 3b_1 + 2b_2 + 6b_3 + 4b_4 + 3b_5 - 72, \tag{8}$$

$$a_4 := -2b_1 - b_2 - b_3 - 4b_4 + 2b_5 + 24, \tag{9}$$

$$a_6 := -9b_1 - 7b_2 - 9b_3 - 10b_4 - 7b_5 + 180, \tag{10}$$

$$a_{12} := 6b_1 + 3b_2 + 3b_3 + 4b_4 + 2b_5 - 72. \tag{11}$$

Let

$$f_1 := \sum_{n=0}^{\infty} f_1(n) = \frac{\eta^{20}(4z)\eta^{10}(6z)\eta^4(12z)}{\eta^{10}(2z)},$$

$$f_2 := \sum_{n=0}^{\infty} f_2(n) = \frac{\eta^{15}(4z)\eta^{15}(6z)\eta^3(12z)}{\eta^9(2z)},$$

$$f_3 := \sum_{n=0}^{\infty} f_3(n) = \frac{\eta^{10}(4z)\eta^{20}(6z)\eta^2(12z)}{\eta^8(2z)},$$

$$f_4 := \sum_{n=0}^{\infty} f_4(n) = \eta^5(2z)\eta^5(4z)\eta^{13}(6z)\eta(12z),$$

$$f_5 := \sum_{n=0}^{\infty} f_5(n) = \frac{\eta^{16}(4z)\eta^2(6z)\eta^8(12z)}{\eta^2(2z)},$$

$$f_6 := \sum_{n=0}^{\infty} f_6(n) = \frac{\eta^9(2z)\eta^9(6z)\eta^9(12z)}{\eta^3(4z)},$$

$$f_7 := \sum_{n=0}^{\infty} f_6(n) = \frac{\eta^{11}(2z)\eta^{11}(4z)\eta^7(12z)}{\eta^5(6z)},$$

$$f_8 := \sum_{n=0}^{\infty} f_6(n) = \frac{\eta^{12}(6z)\eta^{18}(12z)}{\eta^6(4z)},$$

$$f_9 := \sum_{n=0}^{\infty} f_6(n) = \frac{\eta^{12}(6z)\eta^{18}(12z)}{\eta^6(4z)},$$

$$f_{10} := \sum_{n=0}^{\infty} f_6(n) = \frac{\eta^6(2z)\eta^{12}(4z)\eta^{18}(6z)}{\eta^{12}(12z)},$$

$$f_{11} := \sum_{n=0}^{\infty} f_6(n) = \frac{\eta^{12}(2z)\eta^{18}(4z)}{\eta^6(12z)},$$

$$f_{12} := \sum_{n=0}^{\infty} f_6(n) = \frac{\eta^{16}(4z)\eta^{14}(6z)\eta^8(12z)}{\eta^{14}(2z)},$$

$$f_{13} := \sum_{n=0}^{\infty} f_{19}(n) = \frac{\eta^{20}(4z)\eta^4(6z)\eta^4(12z)}{\eta^4(2z)},$$

$$f_{14} := \sum_{n=0}^{\infty} f_{20}(n) = \frac{\eta^{14}(6z)\eta^{14}(12z)}{\eta^2(2z)\eta^2(4z)},$$

$$f_{15} := \sum_{n=0}^{\infty} f_{21}(n) = \frac{\eta^{19}(4z)\eta^{17}(6z)}{\eta^{11}(2z)\eta(12z)},$$

$$f_{16} := \sum_{n=0}^{\infty} f_{22}(n) = \frac{\eta^{18}(4z)\eta^{18}(6z)}{\eta^6(2z)\eta^6(12z)},$$

$$f_{17} := \sum_{n=0}^{\infty} f_{23}(n) = \frac{\eta^{16}(4z)\eta^8(6z)\eta^8(12z)}{\eta^8(2z)},$$

$$f_{18} := \sum_{n=0}^{\infty} f_{24}(n) = \frac{\eta^{11}(4z)\eta^{13}(6z)\eta^7(12z)}{\eta^7(2z)},$$

$$f_{19} := \sum_{n=0}^{\infty} f_{25}(n) = \frac{\eta^{20}(4z)\eta^{16}(6z)\eta^4(12z)}{\eta^{16}(2z)}.$$

They are functions of q by (3). Now define rational numbers

$$k_0, k_1, k_2, k_3, k_4, k_5, k_6, k_7, k_8, k_9, k_{10,} k_{11}, k_{12},$$

$$k_{13}, k_{14}, k_{15}, k_{16}, k_{17}, k_{18}, k_{19}, k_{20}, k_{21}, k_{22}, k_{23,} k_{24}$$

by

$$\frac{1}{2^{b_1+b_5}} x^{b_1}(1-x)^{b_2}(1+x)^{b_3}(1+2x)^{b_4}(2+x)^{b_5} \tag{12}$$

$$= k_0 + k_1 x + k_2 x^2 + k_3 x + k_4 x^4 + k_5 x^5 + k_6 x^6 + k_7 x^7 + k_8 x^8 + k_9 x^9$$

$$+ k_{10} x^{10} + k_{11} x^{11} + k_{12} x^{12} + k_{13} x^{13} + k_{14} x^{14} + k_{15} x^{15} + k_{16} x^{16} + k_{17} \quad (13)$$

$$x^{17} + k_{18} x^{18} + k_{19} x^{19} + k_{20} x^{20} + k_{21} x^{21} + k_{22} x^{22} + k_{23} x^{23} + k_{24} x^{24}. \quad (14)$$

Define the rational numbers

$$c_1, c_2, c_3, c_4, c_6, c_{12}, r_1, r_2, r_3, r_4, r_5, r_6, r_7,$$

$$r_8, r_9, r_{10}, r_{11}, r_{12}, r_{13}, r_{14}, r_{15}, r_{16}, r_{17}, r_{18} \text{ and } r_{19}$$

as in Table 1.

Here $f_1, f_2, \ldots, f_{19}$ are in $S_{12}(\Gamma_0(12))$, except f_9, f_{12}, f_{19} which are in $M_{12}(\Gamma_0(12))$ and

$$\eta^{a_1}(z)\eta^{a_2}(2z)\eta^{a_3}(3z)\eta^{a_4}(4z)\eta^{a_6}(6z)\eta^{a_{12}}(12z) = \delta(b_1) + \sum_{n=1}^{\infty} c(n) q^n,$$

where for $n \in \mathbb{N}$,

$$c(n) = -((c_1(\sigma_5(n) - 252W_1^5(n)) + c_2(\sigma_5\left(\frac{n}{2}\right) - 252W_1^5\left(\frac{n}{2}\right))$$

$$c_3(\sigma_5\left(\frac{n}{3}\right) - 252W_1^5\left(\frac{n}{3}\right)) + c_4(\sigma_5\left(\frac{n}{4}\right) - 252W_1^5\left(\frac{n}{4}\right))$$

$$+ c_6(\sigma_5\left(\frac{n}{6}\right) - 252W_1^5\left(\frac{n}{6}\right)) + c_{12}(\sigma_5\left(\frac{n}{12}\right) - 252W_1^5\left(\frac{n}{12}\right)))$$

$$+ r_1 f_1(n) + ... + r_{19} f_{19}(n).$$

$$c(2n) = -((c_1(\sigma_5(2n) - 252W_1^5(2n)) + c_2(\sigma_5(n) - 252W_1^5(n))$$

$$c_3(33\sigma_5\left(\frac{n}{3}\right) - 32\sigma_5\left(\frac{n}{6}\right) - 252W_1^5\left(\frac{2n}{3}\right)) + c_4(\sigma_5\left(\frac{n}{2}\right) - 252W_1^5\left(\frac{n}{2}\right))$$

$$+ c_6(\sigma_5\left(\frac{n}{3}\right) - 252W_1^5\left(\frac{n}{3}\right)) + c_{12}(\sigma_5\left(\frac{n}{6}\right) - 252W_1^5\left(\frac{n}{6}\right)))$$

$$+ r_{13} f_{13}(2n) + r_{14} f_{14}(2n) + .. + r_{19} f_{19}(2n),$$

$$c(2n-1) = -((c_1(\sigma_5(2n-1) - 252W_1^5(2n-1))$$

$$c_3(\sigma_5\left(\frac{2n-1}{3}\right) - 252W_1^5\left(\frac{2n-1}{3}\right))$$

$$+ r_1 f_1(2n-1) + r_2 f_2(2n-1) + ... + r_{12} f_{12}(2n-1),.$$

and, for $n = 1, 2, \ldots,$

$$f_{13}(2n-1) = f_{14}(2n-1) = ... f_{19}(2n-1) = 0,$$

$$f_1(2n) = f_2(2n) = ... = f_{12}(2n) = 0.$$

Proof. It follows from (6-11) that

$$a_1 + 2a_2 + 3a_3 + 4a_4 + 6a_6 + 12a_{12} = 24b_1 \tag{15}$$

$$a_1 + a_2 + a_3 + a_4 + a_6 + a_{12} = 24, \tag{16}$$

$$-\frac{a_1}{6} - \frac{a_2}{3} - \frac{a_3}{6} - 2\frac{a_4}{3} - \frac{a_6}{3} - 2\frac{a_{12}}{3} = -b_1 - b_5.$$

Now we will use p-k parametrization of Alaca, Alaca and Williams, see [1]:

$$p(q) := \frac{\varphi^2(q) - \varphi^2(q^3)}{2\varphi^2(q^3)}, k(q) := \frac{\varphi^3(q^3)}{\varphi(q)}, \tag{17}$$

where the theta function $\varphi(q)$ is defined by

$$\varphi(q) = \sum_{-\infty}^{\infty} q^{n^2}.$$

Setting x=p in (12), and multiplying both sides by k^{12}, we obtain

$$\frac{k^{12}}{2^{b_1+b_5}} p^{b_1}(1-p)^{b_2}(1+p)^{b_3}(1+2p)^{b_4}(2+p)^{b_5}$$
$$= (k_0 + k_1 p + k_2 p^2 + k_3 p^3 + k_4 p^4 + k_5 p^5 + k_6 p^6 + k_7 p^7$$
$$+ k_8 p^8 + k_9 p^9 + k_{10} p^{10} + k_{11} p^{11} + k_{12} p^{12} + k_{13} x^{13} + k_{14} x^{14} + k_{15} x^{15} + k_{16} x^{16}$$
$$+ k_{17} x^{17} + k_{18} x^{18} + k_{19} x^{19} + k_{20} x^{20} + k_{21} x^{21} + k_{22} x^{22} + k_{23} x^{23} + k_{24} x^{24}) k^{12}.$$

Alaca, Alaca and Williams [2] have established the following representations in terms of p and k:

$$\eta(q) = 2^{-1/6} p^{1/24} (1-p)^{1/2} (1+p)^{1/6} (1+2p)^{1/8} (2+p)^{1/8} k^{1/2}, \tag{18}$$

$$\eta(q^2) = 2^{-1/3} p^{1/12} (1-p)^{1/4} (1+p)^{1/12} (1+2p)^{1/4} (2+p)^{1/4} k^{1/2}, \tag{19}$$

$$\eta(q^3) = 2^{-1/6} p^{1/8} (1-p)^{1/6} (1+p)^{1/2} (1+2p)^{1/24} (2+p)^{1/24} k^{1/2}, \tag{20}$$

$$\eta(q^4) = 2^{-2/3} p^{1/6} (1-p)^{1/8} (1+p)^{1/24} (1+2p)^{1/8} (2+p)^{1/2} k^{1/2}, \tag{21}$$

$$\eta(q^6) = 2^{-1/3} p^{1/4} (1-p)^{1/12} (1+p)^{1/4} (1+2p)^{1/12} (2+p)^{1/12} k^{1/2}, \tag{22}$$

$$\eta(q^{12}) = 2^{-2/3} p^{1/2} (1-p)^{1/24} (1+p)^{1/8} (1+2p)^{1/24} (2+p)^{1/6} k^{1/2}, \tag{23}$$

$$E_6^2(q) \; : \; = \left(1 - 504 \sum_{n=1}^{\infty} \sigma_5(n) q^n \right)^2$$

$$= 1 - 1008\sum_{n=1}^{\infty}\sigma_5(n)q^n + 504^2\sum_{n=1}^{\infty}\left(\sum_{k=1}^{n-1}\sigma_5(k)\sigma_5(n-k)\right)q^n$$

$$= 1 - 1008\sum_{n=1}^{\infty}\left(\sigma_5(n) - 252\left(\sum_{k=1}^{n-1}\sigma_5(k)\sigma_5(n-k)\right)\right)q^n$$

$$= E_{12} - \frac{1309320}{691}\Delta$$

since

$$E_6^2(q) = E_{12} + c\Delta, c = \frac{24}{B_{12}} - 1008\sigma_5(1) = -\frac{2730}{691}24 - 1800 = -\frac{1309320}{691}$$

So

$$-1008\sigma_5(n) + 504^2 W_1^5(n) = -24(-\frac{2730}{691})\sigma_{11}(n) - \frac{1309320}{691}\tau(n)$$

$$= \frac{65520}{691}\sigma_{11}(n) - \frac{1309320}{691}\tau(n),$$

where

$$W_1^5(n) := \left(\sum_{k=1}^{n-1}\sigma_5(k)\sigma_5(n-k)\right).$$

So

$$W_1^5(n) = \frac{65520}{504^2 * 691}\sigma_{11}(n) - \frac{1309320}{504^2 * 691}\tau(n) + \frac{1}{252}\sigma_5(n)$$

$$= \frac{65}{174132}\sigma_{11}(n) - \frac{18185}{2437848}\tau(n) + \frac{1}{252}\sigma_5(n).$$

Now,

$$E_6^2(q) = p^{24} - 492p^{23} + 49452p^{22} + 2644516p^{21} + 49330374p^{20} + 493274676p^{19}$$
$$+ 3123900892p^{18} + 13679546148p^{17} + 43632281439p^{16} + 104744745160p^{15}$$
$$+ 193283821560p^{14} + 277798369704p^{13} + 313258396084p^{12} +$$
$$277798369704p^{11} + 193283821560p^{10} + 104744745160p^{9} + 43632281439p^{8} +$$
$$13679546148p^{7} + 3123900892p^{6} + 493274676p^{5} + 49330374p^{4} +$$
$$2644516p^{3} + 49452p^{2} - 492p + 1,$$

$$E_6^2(q^2) = p^{24} + 12p^{23} - 192p^{22} - 2618p^{21} + 1437p^{20} + 109542p^{19}$$
$$+ 790615p^{18} + 3442455p^{17} + 42829821/4\,p^{16} + 25362070p^{15}$$
$$+ 46891506p^{14} + 67943661p^{13} + 153857201/2\,p^{12} + 67943661p^{11}$$

$$+46891506p^{10}+25362070p^{9}+42829821/4p^{8}+3442455p^{7}$$

$$+790615p^{6}+109542p^{5}+1437p^{4}-2618p^{3}-192p^{2}+12p+1$$

$$E_6^2(q^3)=p^{24}+12p^{23}+60p^{22}+28p^{21}-1146p^{20}-5748p^{19}-9044p^{18}+$$

$$20988p^{17}+122463p^{16}+252664p^{15}+351768p^{14}+477528p^{13}$$

$$+566836p^{12}+477528p^{11}+351768p^{10}+252664p^{9}+122463p^{8}$$

$$+20988p^{7}-9044p^{6}-5748p^{5}-1146p^{4}+28p^{3}+60p^{2}+12p+1$$

$$E_6^2(q^4)=1/4096p^{24}+129/1024p^{23}+15261/1024p^{22}-178759/512p^{21}+$$

$$1525227/1024p^{20}+2317461/512p^{19}-447559/128p^{18}-2363787/256p^{17}$$

$$+14298273/1024p^{16}+3635987/128p^{15}-247071/128p^{14}-696723/32p^{13}$$

$$+488023/64p^{12}+78285/2p^{11}+140205/4p^{10}+25457/2p^{9}-40401/16p^{8}$$

$$-22455/4p^{7}-12299/4p^{6}-708p^{5}+114p^{4}+154p^{3}+60p^{2}+12p+1,$$

$$E_6^2(q^6)=p^{24}+12p^{23}+60p^{22}+154p^{21}+177p^{20}-78p^{19}-539p^{18}-747p^{17}-$$

$$2115/4p^{16}-218p^{15}+228p^{14}+1059p^{13}+3137/2p^{12}+1059p^{11}+$$

$$228p^{10}-218p^{9}-2115/4p^{8}-747p^{7}-539p^{6}-78p^{5}+177p^{4}+$$

$$154p^{3}+60p^{2}+12p+1,$$

$$E_6^2(q^{12})=1/4096p^{24}+3/1024p^{23}+15/1024p^{22}+35/512p^{21}+375/1024p^{20}$$

$$+699/512p^{19}+931/256p^{18}+2493/256p^{17}+24993/1024p^{16}$$

$$+3785/128p^{15}-5781/128p^{14}-7125/32p^{13}-18119/64p^{12}+267/4p^{11}$$

$$+606p^{10}+1265/2p^{9}-333/16p^{8}-2421/4p^{7}-2093/4p^{6}-78p^{5}$$

$$+177p^{4}+154p^{3}+60p^{2}+12p+1$$

It is easy to check the following expressions by (18-23)

$$f_1 \quad : \quad =\sum_{n=0}^{\infty} f_1(n)=\frac{\eta^{20}(4z)\eta^{10}(6z)\eta^{4}(12z)}{\eta^{10}(2z)}$$

$$=(-1/32768p^{21}-41/65536p^{20}-95/16384p^{19}-131/4096p^{18}-1901/16384p^{17}$$

$$-18839/65536p^{16}-15831/32768p^{15}-2079/4096p^{14}-423/2048p^{13}$$

$$+559/2048p^{12}+575/1024p^{11}+125/256p^{10}+31/128p^{9}+17/256p^{8}+1/128p^{7})k^{12},$$

$$f_2 \quad : \quad =\sum_{n=0}^{\infty} f_2(n)=\frac{\eta^{15}(4z)\eta^{15}(6z)\eta^{3}(12z)}{\eta^{9}(2z)}$$

$= (-1/8192 p^{20} - 35/16384 p^{19} - 273/16384 p^{18} - 311/4096 p^{17} - 227/1024$

$p^{16} - 6939/16384 p^{15} - 8217/16384 p^{14} - 2151/8192 p^{13} + 819/4096$

$p^{12} + 1057/2048 p^{11} + 485/1024 p^{10} + 123/512 p^{9} + 17/256 p^{8} + 1/128 p^{7})k^{12},$

$$f_3 \quad : \quad = \sum_{n=0}^{\infty} f_3(n) = \frac{\eta^{10}(4z)\eta^{20}(6z)\eta^{2}(12z)}{\eta^{8}(2z)}$$

$= (-1/2048 p^{19} - 29/4096 p^{18} - 23/512 p^{17} - 665/4096 p^{16} - 185/512 p^{15}$

$-1991/4096 p^{14} - 159/512 p^{13} + 533/4096 p^{12} + 965/2048 p^{11} + 235/512 p^{10}$

$+61/256 p^{9} + 17/256 p^{8} + 1/128 p^{7})k^{12},$

$$f_4 \quad : \quad = \sum_{n=0}^{\infty} f_4(n) = \eta^{5}(2z)\eta^{5}(4z)\eta^{13}(6z)\eta(12z)$$

$= (-1/128 p^{20} - 25/256 p^{19} - 257/512 p^{18} - 1327/1024 p^{17} - 1441/1024 p^{16}$

$+995/1024 p^{15} + 4807/1024 p^{14} + 4761/1024 p^{13} - 819/1024 p^{12}$

$-5515/1024 p^{11} - 4097/1024 p^{10} + 17/512 p^{9} + 221/128 p^{8} + 35/32 p^{7}$

$+19/64 p^{6} + 1/32 p^{5})k^{12},$

$$f_5 \quad : \quad = \sum_{n=0}^{\infty} f_5(n) = \frac{\eta^{16}(4z)\eta^{2}(6z)\eta^{8}(12z)}{\eta^{2}(2z)}$$

$= (1/16384 p^{22} + 19/16384 p^{21} + 641/65536 p^{20} + 1565/32768 p^{19}$

$+4801/32768 p^{18} + 2297/8192 p^{17} + 18505/65536 p^{16} - 1239/32768 p^{15}$

$-2211/4096 p^{14} - 1475/2048 p^{13} - 689/2048 p^{12} + 199/1024 p^{11} + 95/256 p^{10}$

$+29/128 p^{9} + 17/256 p^{8} + 1/128 p^{7})k^{12},$

$$f_6 \quad : \quad = \sum_{n=0}^{\infty} f_6(n) = \frac{\eta^{9}(2z)\eta^{9}(6z)\eta^{9}(12z)}{\eta^{3}(4z)}$$

$= (-1/128 p^{20} - 17/256 p^{19} - 105/512 p^{18} - 215/1024 p^{17} + 259/1024 p^{16}$

$+819/1024 p^{15} + 495/1024 p^{14} - 495/1024 p^{13} - 819/1024 p^{12}$

$-259/1024 p^{11} + 215/1024 p^{10} + 105/512 p^{9} + 17/256 p^{8} + 1/128 p^{7})k^{12},$

$$f_7 \quad : \quad = \sum_{n=0}^{\infty} f_6(n) = \frac{\eta^{11}(2z)\eta^{11}(4z)\eta^{7}(12z)}{\eta^{5}(6z)}$$

$= (1/1024 p^{23} + 17/1024 p^{22} + 247/2048 p^{21} + 485/1024 p^{20} + 16361/16384 p^{19}$

$+10703/16384 p^{18} - 8023/4096 p^{17} - 42317/8192 p^{16} - 56119/16384 p^{15}$

$+71643/16384p^{14}+74321/8192p^{13}+3679/1024p^{12}-595/128p^{11}$

$-2851/512p^{10}-317/256p^{9}+85/64p^{8}+67/64p^{7}+19/64p^{6}+1/32p^{5})k^{12},$

$$f_8 \quad : \quad =\sum_{n=0}^{\infty} f_6(n)=\frac{\eta^{12}(6z)\eta^{18}(12z)}{\eta^{6}(4z)}$$

$=(-1/2048p^{19}-13/4096p^{18}-1/128p^{17}-33/4096p^{16}+33/4096p^{14}$

$+1/128p^{13}+13/4096p^{12}+1/2048p^{11})k^{12},$

$$f_9 \quad : \quad =\sum_{n=0}^{\infty} f_6(n)=\frac{\eta^{12}(6z)\eta^{18}(12z)}{\eta^{6}(4z)}$$

$=(1/4096p^{24}+1/256p^{23}+215/8192p^{22}+755/8192p^{21}+10321/65536$

$p^{20}+191/32768p^{19}-16237/32768p^{18}-815/1024p^{17}-3959/65536p^{16}$

$+37801/32768p^{15}+4565/4096p^{14}-443/2048p^{13}-1937/2048$

$p^{12}-457/1024p^{11}+35/256p^{10}+25/128p^{9}+17/256p^{8}+1/128p^{7})k^{12},$

$$f_{10} \quad : \quad =\sum_{n=0}^{\infty} f_6(n)=\frac{\eta^{6}(2z)\eta^{12}(4z)\eta^{18}(6z)}{\eta^{12}(12z)}$$

$=(1/16p^{20}+p^{19}+219/32p^{18}+811/32p^{17}+12881/256p^{16}+3783/128p^{15}$

$-779/8p^{14}-3937/16p^{13}-21957/128p^{12}+11229/64p^{11}+26643/64p^{10}$

$+3603/16p^{9}-38879/256p^{8}-34777/128p^{7}-7467/64p^{6}+1091/32p^{5}$

$+931/16p^{4}+213/8p^{3}+23/4p^{2}+1/2p)k^{12},$

$$f_{11} \quad : \quad =\sum_{n=0}^{\infty} f_6(n)=\frac{\eta^{12}(2z)\eta^{18}(4z)}{\eta^{6}(12z)}$$

$=(-1/128p^{23}-41/256p^{22}-369/256p^{21}-3755/512p^{20}-45365/2048p^{19}$

$-142317/4096p^{18}+2541/1024p^{17}+523731/4096p^{16}+238005/1024p^{15}$

$+289525/4096p^{14}-358967/1024p^{13}-2160639/4096p^{12}-178493/2048p^{11}$

$+503675/1024p^{10}+246645/512p^{9}-51/128p^{8}-18129/64p^{7}-5643/32p^{6}$

$+25/16p^{5}+805/16p^{4}+207/8p^{3}+23/4p^{2}+1/2p)k^{12},$

$$f_{12} \quad : \quad =\sum_{n=0}^{\infty} f_6(n)=\frac{\eta^{16}(4z)\eta^{14}(6z)\eta^{8}(12z)}{\eta^{14}(2z)}$$

$=(1/65536p^{20}+9/32768p^{19}+73/32768p^{18}+11/1024p^{17}+2241/65536p^{16}$

$+2471/32768p^{15}+1925/16384p^{14}+1059/8192p^{13}+403/4096p^{12}$

$+101/2048p^{11}+15/1024p^{10}+1/512p^{9})k^{12},$

$$f_{13} \quad : \quad = \sum_{n=0}^{\infty} f_{19}(n) = \frac{\eta^{20}(4z)\eta^{4}(6z)\eta^{4}(12z)}{\eta^{4}(2z)}$$

$$= (1/16384\,p^{22} + 21/16384\,p^{21} + 793/65536\,p^{20} + 1103/16384\,p^{19} + 7931/32768\,p^{18}$$
$$+ 9395/16384\,p^{17} + 55257/65536\,p^{16} + 8633/16384\,p^{15} - 10083/16384\,p^{14}$$
$$- 1843/1024\,p^{13} - 3639/2048\,p^{12} - 245/512\,p^{11} + 389/512\,p^{10} + 31/32\,p^{9}$$
$$+ 133/256\,p^{8} + 9/64\,p^{7} + 1/64\,p^{6})k^{12}.$$

$$f_{14} \quad : \quad = \sum_{n=0}^{\infty} f_{20}(n) = \frac{\eta^{14}(6z)\eta^{14}(12z)}{\eta^{2}(2z)\eta^{2}(4z)}$$

$$= (-1/2048\,p^{19} - 17/4096\,p^{18} - 29/2048\,p^{17} - 97/4096\,p^{16} - 33/2048\,p^{15}$$
$$+ 33/4096\,p^{14} + 49/2048\,p^{13} + 77/4096\,p^{12} + 7/1024\,p^{11} + 1/1024\,p^{10})k^{12}.$$

$$f_{15} \quad : \quad = \sum_{n=0}^{\infty} f_{21}(n) = \frac{\eta^{19}(4z)\eta^{17}(6z)}{\eta^{11}(2z)\eta(12z)}$$

$$= (-1/8192\,p^{20} - 39/16384\,p^{19} - 343/16384\,p^{18} - 895/8192\,p^{17} - 765/2048\,p^{16}$$
$$- 14203/16384\,p^{15} - 22095/16384\,p^{14} - 81/64\,p^{13} - 333/1024\,p^{12}$$
$$+ 469/512\,p^{11} + 771/512\,p^{10} + 19/16\,p^{9} + 35/64\,p^{8} + 9/64\,p^{7} + 1/64\,p^{6})k^{12}.$$

$$f_{16} \quad : \quad = \sum_{n=0}^{\infty} f_{22}(n) = \frac{\eta^{18}(4z)\eta^{18}(6z)}{\eta^{6}(2z)\eta^{6}(12z)}$$

$$= (1/1024\,p^{20} + 19/1024\,p^{19} + 645/4096\,p^{18} + 1599/2048\,p^{17} + 10107/4096\,p^{16}$$
$$+ 1281/256\,p^{15} + 23867/4096\,p^{14} + 2587/2048\,p^{13} - 32175/4096\,p^{12}$$
$$- 3505/256\,p^{11} - 2479/256\,p^{10} + 3/16\,p^{9} + 819/128\,p^{8} + 93/16\,p^{7} + 21/8\,p^{6}$$
$$+ 5/8\,p^{5} + 1/16\,p^{4})k^{12}.$$

$$f_{17} \quad : \quad = \sum_{n=0}^{\infty} f_{23}(n) = \frac{\eta^{16}(4z)\eta^{8}(6z)\eta^{8}(12z)}{\eta^{8}(2z)}$$

$$= (-1/32768\,p^{21} - 37/65536\,p^{20} - 153/32768\,p^{19} - 371/16384\,p^{18} - 1159/16384\,p^{17}$$
$$- 9567/65536\,p^{16} - 783/4096\,p^{15} - 513/4096\,p^{14} + 45/1024\,p^{13} + 379/2048\,p^{12}$$
$$+ 49/256\,p^{11} + 27/256\,p^{10} + 1/32\,p^{9} + 1/256\,p^{8})k^{12}.$$

$$f_{18} \quad : \quad = \sum_{n=0}^{\infty} f_{24}(n) = \frac{\eta^{11}(4z)\eta^{13}(6z)\eta^{7}(12z)}{\eta^{7}(2z)}$$

$$= (-1/8192\,p^{20} - 31/16384\,p^{19} - 211/16384\,p^{18} - 411/8192\,p^{17} - 497/4096\,p^{16}$$
$$- 2963/16384\,p^{15} - 2291/16384\,p^{14} + 35/2048\,p^{13} + 679/4096\,p^{12}$$

$$+189/1024\,p^{11}+107/1024\,p^{10}+1/32\,p^{9}+1/256\,p^{8})k^{12}.$$

$$f_{19}\quad : \quad \sum_{n=0}^{\infty} f_{25}(n)=\frac{\eta^{20}(4z)\eta^{16}(6z)\eta^{4}(12z)}{\eta^{16}(2z)}$$

$$=(1/65536\,p^{20}+5/16384\,p^{19}+91/32768\,p^{18}+249/16384\,p^{17}+3649/65536\,p^{16}$$

$$+589/4096\,p^{15}+1099/4096\,p^{14}+373/1024\,p^{13}+731/2048\,p^{12}+63/256\,p^{11}$$

$$+29/256\,p^{10}+1/32\,p^{9}+1/256\,p^{8})k^{12}.$$

Obviously, $f_1,\dots f_{19}$ are functions of q , see (3),(17). Here $f_1, f_2,\dots, f_{19}$ are in $S_{12}(\Gamma_0(12))$, except f_9, f_{12}, f_{19} which are in $M_{12}(\Gamma_0(12))$ by [4]. Now

$$\eta^{a_1}(z)\eta^{a_2}(2z)\eta^{a_3}(3z)\eta^{a_4}(4z)\eta^{a_6}(6z)\eta^{a_{12}}(12z)$$

$$=q^{b_1}\prod_{n=1}^{\infty}\left(1-q^{n}\right)^{a_1}\left(1-q^{2n}\right)^{a_2}\left(1-q^{3n}\right)^{a_3}\left(1-q^{4n}\right)^{a_4}\left(1-q^{6n}\right)^{a_6}\left(1-q^{12n}\right)^{a_{12}}$$

$$=2^{-\frac{a_1}{6}-\frac{a_2}{3}-\frac{a_3}{6}-2\frac{a_4}{3}-\frac{a_6}{3}-2\frac{a_{12}}{3}}\,p^{\frac{a_1}{24}+\frac{a_2}{12}+\frac{a_3}{8}+\frac{a_4}{6}+\frac{a_6}{4}+\frac{a_{12}}{2}}(1-p)^{\frac{a_1}{2}+\frac{a_2}{4}+\frac{a_3}{6}+\frac{a_4}{8}+\frac{a_6}{12}+\frac{a_{12}}{24}}$$

$$(1+p)^{\frac{a_1}{6}+\frac{a_2}{12}+\frac{a_3}{2}+\frac{a_4}{24}+\frac{a_6}{4}+\frac{a_{12}}{8}}(1+2p)^{\frac{a_1}{8}+\frac{a_2}{4}+\frac{a_3}{24}+\frac{a_4}{8}+\frac{a_6}{12}+\frac{a_{12}}{24}}(2+p)^{\frac{a_1}{8}+\frac{a_2}{4}+\frac{a_3}{24}+\frac{a_4}{2}+\frac{a_6}{12}+\frac{a_{12}}{6}}$$

$$k^{\frac{a_1+a_2+a_3+a_4+a_6+a_{12}}{2}}=\frac{k^{12}}{2^{b_1+b_5}}\,p^{b_1}(1-p)^{b_2}(1+p)^{b_3}(1+2p)^{b_4}(2+p)^{b_5}$$

$$=k^{12}(k_0+k_1p+k_2p^2+k_3p^3+k_4p^4+k_5p^5+k_6p^6$$

$$+k_7p^7+k_8p^8+k_9p^9+k_{10}p^{10}+k_{11}p^{11}+k_{12}$$

$$p^{12}+k_{13}p^{13}+k_{14}p^{14}+k_{15}p^{15}+k_{16}p^{16}+k_{17}p^{17}+k_{18}$$

$$p^{18}+k_{19}p^{19}+k_{20}p^{20}+k_{21}p^{21}+k_{22}p^{22}+k_{23}p^{23}+k_{24}p^{24})$$

$$=\frac{c_1}{1008}(1-1008\sum_{n=1}^{\infty}(\sigma_5(n)-252W_1^5(n))q^{n})$$

$$+\frac{c_2}{1008}(1-1008\sum_{n=1}^{\infty}(\sigma_5(n)-252W_1^5(n))q^{2n})$$

$$+\frac{c_3}{1008}(1-1008\sum_{n=1}^{\infty}(\sigma_5(n)-252W_1^5(n))q^{3n})$$

$$+\frac{c_4}{1008}(1-1008\sum_{n=1}^{\infty}(\sigma_5(n)-252W_1^5(n))q^{4n}))$$

$$+\frac{c_6}{1008}(1-1008\sum_{n=1}^{\infty}(\sigma_5(n)-252W_1^5(n))q^{6n})$$

$$+\frac{c_{12}}{1008}(1-1008\sum_{n=1}^{\infty}(\sigma_5(n)-252W_1^5(n))q^{12n})$$

$$+r_1q^7\prod_{n=1}^{\infty}\frac{(1-q^{4n})^{20}(1-q^{6n})^{10}(1-q^{12n})^4}{(1-q^{2n})^{10}}$$

$$+r_2q^7\prod_{n=1}^{\infty}\frac{(1-q^{4n})^{15}(1-q^{6n})^{15}(1-q^{12n})^3}{(1-q^{2n})^9}$$

$$+r_3q^7\prod_{n=1}^{\infty}\frac{(1-q^{4n})^{10}(1-q^{6n})^{20}(1-q^{12n})^2}{(1-q^{2n})^8}$$

$$+r_4q^5\prod_{n=1}^{\infty}(1-q^{2n})^5(1-q^{4n})^5(1-q^{6n})^{13}(1-q^{12n})$$

$$+r_5.q^7\prod_{n=1}^{\infty}\frac{(1-q^{4n})^{16}(1-q^{6n})^2(1-q^{12n})^8}{(1-q^{2n})^2}$$

$$+r_6q^7\prod_{n=1}^{\infty}\frac{(1-q^{2n})^9(1-q^{6n})^9(1-q^{12n})^9}{(1-q^{4n})^3}$$

$$+r_7q^5\prod_{n=1}^{\infty}\frac{(1-q^{2n})^{11}(1-q^{4n})^{11}(1-q^{12n})^7}{(1-q^{6n})^5}$$

$$+r_8q^{11}\prod_{n=1}^{\infty}\frac{(1-q^{6n})^{12}(1-q^{12n})^{18}}{(1-q^{4n})^6}$$

$$+r_9q^7\prod_{n=1}^{\infty}\frac{(1-q^{6n})^{12}(1-q^{12n})^{18}}{(1-q^{4n})^6}$$

$$+r_{10}q\prod_{n=1}^{\infty}\frac{(1-q^{2n})^6(1-q^{4n})^{12}(1-q^{6n})^{18}}{(1-q^{12n})^{12}}$$

$$+r_{11}q\prod_{n=1}^{\infty}\frac{(1-q^{2n})^{12}(1-q^{4n})^{18}}{(1-q^{12n})^6}$$

$$+r_{12}q^9\prod_{n=1}^{\infty}\frac{(1-q^{4n})^{16}(1-q^{6n})^{14}(1-q^{12n})^8}{(1-q^{2n})^{14}}$$

$$+r_{13}q^6\prod_{n=1}^{\infty}\frac{(1-q^{4n})^{20}(1-q^{6n})^4(1-q^{12n})^4}{(1-q^{2n})^4}$$

$$+r_{14}q^{10}\prod_{n=1}^{\infty}\frac{(1-q^{6n})^{14}(1-q^{12n})^{14}}{(1-q^{2n})^2(1-q^{4n})^2}$$

$$+r_{15}q^{6}\prod_{n=1}^{\infty}\frac{(1-q^{4n})^{19}(1-q^{6n})^{17}}{(1-q^{2n})^{11}(1-q^{12n})}$$

$$+r_{16}q^{4}\prod_{n=1}^{\infty}\frac{(1-q^{4n})^{18}(1-q^{6n})^{18}}{(1-q^{2n})^{6}(1-q^{12n})^{6}}$$

$$+r_{17}q^{8}\prod_{n=1}^{\infty}\frac{(1-q^{6n})^{8}(1-q^{4n})^{16}(1-q^{12n})^{8}}{(1-q^{2n})^{8}}$$

$$+r_{18}q^{8}\prod_{n=1}^{\infty}\frac{(1-q^{4n})^{11}(1-q^{6n})^{13}(1-q^{12n})^{7}}{(1-q^{2n})^{7}}$$

$$+r_{19}q^{8}\prod_{n=1}^{\infty}\frac{(1-q^{6n})^{16}(1-q^{4n})^{20}(1-q^{12n})^{4}}{(1-q^{2n})^{16}}$$

$$=\delta(b_1)-\sum_{n=1}^{\infty}(c_1(\sigma_5(n)-252W_1^5(n))+c_2(\sigma_5\left(\frac{n}{2}\right)-252W_1^5\left(\frac{n}{2}\right))$$

$$+c_3(\sigma_5\left(\frac{n}{3}\right)-252W_1^5\left(\frac{n}{3}\right))+c_4(\sigma_5\left(\frac{n}{4}\right)-252W_1^5\left(\frac{n}{4}\right))$$

$$+c_6(\sigma_5\left(\frac{n}{6}\right)-252W_1^5\left(\frac{n}{6}\right))+c_{12}(\sigma_5\left(\frac{n}{12}\right)-252W_1^5\left(\frac{n}{12}\right))$$

$$+r_1f_1(n)+....+r_{19}f_{19}(n),$$

where

$$\delta(b_1)=\begin{cases}0 & \text{if } b_1\neq 0\\ 1 & \text{if } b_1=0\end{cases}.$$

So

$$c(n)=-((c_1(\sigma_5(n)-252W_1^5(n))+c_2(\sigma_5\left(\frac{n}{2}\right)-252W_1^5\left(\frac{n}{2}\right))$$

$$c_3(\sigma_5\left(\frac{n}{3}\right)-252W_1^5\left(\frac{n}{3}\right))+c_4(\sigma_5\left(\frac{n}{4}\right)-252W_1^5\left(\frac{n}{4}\right))$$

$$+c_6(\sigma_5\left(\frac{n}{6}\right)-252W_1^5\left(\frac{n}{6}\right))+c_{12}(\sigma_5\left(\frac{n}{12}\right)-252W_1^5\left(\frac{n}{12}\right)))$$

$$+r_1f_1(n)+....+r_{19}f_{19}(n).$$

Therefore, for n=1,2,... ,

$$c(2n)=-((c_1(\sigma_5(2n)-252W_1^5(2n))+c_2(\sigma_5(n)-252W_1^5(n))$$

$$c_3(\sigma_5\left(\frac{2n}{3}\right)-252W_1^5\left(\frac{2n}{3}\right))+c_4(\sigma_5\left(\frac{n}{2}\right)-252W_1^5\left(\frac{n}{2}\right))$$

$$+c_6(\sigma_5\left(\frac{n}{3}\right)-252W_1^5\left(\frac{n}{3}\right))+c_{12}(\sigma_5\left(\frac{n}{6}\right)-252W_1^5\left(\frac{n}{6}\right))$$

$$+r_1f_1(2n)+....+r_{19}f_{19}(2n).$$

$$=-((c_1(\sigma_5(2n)-252W_1^5(2n))+c_2(\sigma_5(n)-252W_1^5(n))$$

$$c_3(33\sigma_5\left(\frac{n}{3}\right)-32\sigma_5\left(\frac{n}{6}\right)-252W_1^5\left(\frac{2n}{3}\right))+c_4(\sigma_5\left(\frac{n}{2}\right)-252W_1^5\left(\frac{n}{2}\right))$$

$$+c_6(\sigma_5\left(\frac{n}{3}\right)-252W_1^5\left(\frac{n}{3}\right))+c_{12}(\sigma_5\left(\frac{n}{6}\right)-252W_1^5\left(\frac{n}{6}\right))$$

$$+r_{13}f_{13}(2n)+r_{14}f_{14}(2n)+..+r_{19}f_{19}(2n),.$$

$$c(2n-1)=-((c_1(\sigma_5(2n-1)-252W_1^5(2n-1))+c_2(\sigma_5\left(\frac{2n-1}{2}\right)-252W_1^5\left(\frac{2n-1}{2}\right))$$

$$c_3(\sigma_5\left(\frac{2n-1}{3}\right)-252W_1^5\left(\frac{2n-1}{3}\right))+c_4(\sigma_5\left(\frac{2n-1}{4}\right)-252W_1^5\left(\frac{2n-1}{4}\right))$$

$$+c_6(\sigma_5\left(\frac{2n-1}{6}\right)-252W_1^5\left(\frac{2n-1}{6}\right))+c_{12}(\sigma_5\left(\frac{2n-1}{12}\right)-252W_1^5\left(\frac{2n-1}{12}\right))$$

$$+r_1f_1(2n-1)+r_2f_2(2n-1)+...+r_{12}f_{12}(2n-1),.$$

$$c(2n-1)=-((c_1(\sigma_5(2n-1)-252W_1^5(2n-1))$$

$$c_3(\sigma_5\left(\frac{2n-1}{3}\right)-252W_1^5\left(\frac{2n-1}{3}\right))$$

$$+r_1f_1(2n-1)+r_2f_2(2n-1)+...+r_{12}f_{12}(2n-1),$$

since it is easy to see that

$$\sigma_5\left(\frac{2n}{3}\right)=33\sigma_5\left(\frac{n}{3}\right)-32\sigma_5\left(\frac{n}{6}\right),$$

and, for n=1,2,... ,

$$f_{13}(2n-1)=f_{14}(2n-1)=....f_{19}(2n-1)=0,$$

$$f_1(2n)=f_2(2n)=...=f_{12}(2n)=0.$$

These formulas are valid for 112116 nontrivial eta quotients. Among them, we have found 2 eta quotients, see Table2, such that

$$c(2n)=-((c_1(\sigma_5(2n)-252W_1^5(2n))+c_2(\sigma_5(n)-252W_1^5(n))$$

$$c_3(\sigma_5\left(\frac{2n}{3}\right)-252W_1^5\left(\frac{2n}{3}\right))+c_4(\sigma_5\left(\frac{n}{2}\right)-252W_1^5\left(\frac{n}{2}\right))$$

$$+c_6(\sigma_5\left(\frac{n}{3}\right)-252W_1^5\left(\frac{n}{3}\right))+c_{12}(\sigma_5\left(\frac{n}{6}\right)-252W_1^5\left(\frac{n}{6}\right)),$$

$$c(2n-1)=0,$$

and 393 eta quotients, see Table 3A and Table 3B, such that

$$c(2n)=-((c_1(\sigma_5(2n)-252W_1^5(2n))+c_2(\sigma_5(n)-252W_1^5(n))$$

$$c_3(\sigma_5\left(\frac{2n}{3}\right)-252W_1^5\left(\frac{2n}{3}\right))+c_4(\sigma_5\left(\frac{n}{2}\right)-252W_1^5\left(\frac{n}{2}\right))$$

$$+c_6(\sigma_5\left(\frac{n}{3}\right)-252W_1^5\left(\frac{n}{3}\right))+c_{12}(\sigma_5\left(\frac{n}{6}\right)-252W_1^5\left(\frac{n}{6}\right)))$$

$$+r_{13}f_{13}(2n)+\cdots+r_{19}f_{19}(2n),$$

$$c(2n-1)=0.$$

Remark 2 *If* f *is an eta quotient, then* $f(-q)$ *is also an eta quotient, and the coefficients of* $\frac{1}{2}(f(q)+f(-q))$ *are exactly the even coefficients of f. In particular, it means that we have obtained all coefficients of the the sum of 2 eta quotients.*

Remark 3 $S_{12}(\Gamma_0(12))$ *is 19 dimensional, see [5] (Chapter 3, pg.87 and Chapter 5, pg.197), and generated by*

$$\Delta,\Delta(2z),\Delta(3z),\Delta(4z),\Delta(6z),\Delta(12z),\Delta_{3,12},\Delta_{3,12}(2z),\Delta_{3,12}(4z),\Delta_{4,12},\Delta_{4,12}(3z),$$

$$\Delta_{6,12,1},\Delta_{6,12,1}(2z),\Delta_{6,12,2},\Delta_{6,12,2}(2z),\Delta_{6,12,3},\Delta_{6,12,3}(2z),\Delta_{12,12,1},\Delta_{12,12,2}$$

where Δ is the unique cuspidal form in $S_{12}(\Gamma_0(1))$, $\Delta_{3,12}$ is the unique newform in $S_6(\Gamma_0(3))$, $\Delta_{4,12}$ is the unique newform in $S_{12}(\Gamma_0(4))$, $\Delta_{6,12,1},\Delta_{6,12,2},\Delta_{6,12,3}$ are all newforms in $S_{12}(\Gamma_0(6))$ and $\Delta_{12,12,1},\Delta_{12,12,2}$ are all newforms in $S_{12}(\Gamma_0(12))$. By simple calculation, we see that

$$f_1= -\frac{79}{18800640}\Delta(z)-\frac{79}{783360}\Delta(2z)+\frac{7353}{696320}\Delta(3z)-\frac{79}{9180}\Delta(4z)+\frac{22059}{87040}\Delta(6z)$$

$$+\frac{7353}{340}\Delta(12z)+\frac{401}{27260928}\Delta_{3,12}(z)-\frac{5213}{4543488}\Delta_{3,12}(2z)+\frac{401}{13311}\Delta_{3,12}(4z)$$

$$-\frac{13}{1400832}\Delta_{4,12}(z)-\frac{2079}{155648}\Delta_{4,12}(3z)-\frac{13}{1105920}\Delta_{6,12,1}(z)+\frac{13}{34560}\Delta_{6,12,1}(2z)$$

$$-\frac{1}{165888}\Delta_{6,12,2}(z)-\frac{1}{5184}\Delta_{6,12,2}(2z)+\frac{35}{4810752}\Delta_{6,12,3}(z)$$

$$+\frac{35}{150336}\Delta_{6,12,3}(2z)+\frac{5}{221184}\Delta_{12,12,1}(z)-\frac{7}{525312}\Delta_{12,12,2}(z),$$

$$f_2 = -\frac{13}{3133440}\Delta(z)-\frac{13}{130560}\Delta(2z)+\frac{3333}{348160}\Delta(3z)-\frac{13}{1530}\Delta(4z)$$
$$+\frac{9999}{43520}\Delta(6z)+\frac{3333}{170}\Delta(12z)+\frac{173}{13630464}\Delta_{3,12}(z)$$
$$-\frac{2249}{2271744}\Delta_{3,12}(2z)+\frac{346}{13311}\Delta_{3,12}(4z)-\frac{1}{126976}\Delta_{4,12}(z)$$
$$-\frac{1503}{126976}\Delta_{4,12}(3z)-\frac{11}{1105920}\Delta_{6,12,1}(z)+\frac{11}{34560}\Delta_{6,12,1}(2z)$$
$$-\frac{11}{1990656}\Delta_{6,12,2}(z)-\frac{11}{62208}\Delta_{6,12,2}(2z)+\frac{25}{3608064}\Delta_{6,12,3}(z)$$
$$+\frac{25}{112752}\Delta_{6,12,3}(2z)+\frac{205}{10285056}\Delta_{12,12,1}(z)-\frac{1}{82944}\Delta_{12,12,2}(z),$$

$$f_3 = -\frac{7}{1762560}\Delta(z)-\frac{7}{73440}\Delta(2z)+\frac{559}{65280}\Delta(3z)-\frac{56}{6885}\Delta(4z)$$
$$+\frac{559}{2720}\Delta(6z)+\frac{4472}{255}\Delta(12z)+\frac{7}{638928}\Delta_{3,12}(z)-\frac{91}{106488}\Delta_{3,12}(2z)$$
$$+\frac{896}{39933}\Delta_{3,12}(4z)-\frac{1}{150784}\Delta_{4,12}(z)-\frac{1565}{150784}\Delta_{4,12}(3z)$$
$$-\frac{7}{829440}\Delta_{6,12,1}(z)+\frac{7}{25920}\Delta_{6,12,1}(2z)-\frac{23}{4478976}\Delta_{6,12,2}(z)$$
$$-\frac{23}{139968}\Delta_{6,12,2}(2z)+\frac{107}{16236288}\Delta_{6,12,3}(z)+\frac{107}{507384}\Delta_{6,12,3}(2z)$$
$$+\frac{17}{964224}\Delta_{12,12,1}(z)-\frac{13}{1181952}\Delta_{12,12,2}(z),$$

$$f_4 = \frac{1}{32640}\Delta(z)+\frac{1}{1360}\Delta(2z)-\frac{63}{10880}\Delta(3z)+\frac{16}{255}\Delta(4z)-\frac{189}{1360}\Delta(6z)$$
$$-\frac{1008}{85}\Delta(12z)-\frac{1}{212976}\Delta_{3,12}(z)+\frac{13}{35496}\Delta_{3,12}(2z)-\frac{128}{13311}\Delta_{3,12}(4z)$$
$$-\frac{3}{75392}\Delta_{4,12}(z)-\frac{1161}{75392}\Delta_{4,12}(3z)-\frac{1}{138240}\Delta_{6,12,1}(z)+\frac{1}{4320}\Delta_{6,12,1}(2z)$$
$$-\frac{1}{82944}\Delta_{6,12,2}(z)-\frac{1}{2592}\Delta_{6,12,2}(2z)-\frac{1}{150336}\Delta_{6,12,3}(z)$$
$$-\frac{1}{4698}\Delta_{6,12,3}(2z)+\frac{1}{107136}\Delta_{12,12,1}(z)+\frac{1}{32832}\Delta_{12,12,2}(z),$$

$$f_5 = -\frac{19}{3760128}\Delta(z)-\frac{19}{156672}\Delta(2z)+\frac{405}{139264}\Delta(3z)-\frac{19}{1836}\Delta(4z)$$
$$+\frac{1215}{17408}\Delta(6z)+\frac{405}{68}\Delta(12z)-\frac{47}{18173952}\Delta_{3,12}(z)$$
$$+\frac{611}{3028992}\Delta_{3,12}(2z)-\frac{47}{8874}\Delta_{3,12}(4z)+\frac{157}{43425792}\Delta_{4,12}(z)$$
$$-\frac{2673}{4825088}\Delta_{4,12}(3z)+\frac{1}{147456}\Delta_{6,12,1}(z)-\frac{1}{4608}\Delta_{6,12,1}(2z)$$
$$-\frac{1}{442368}\Delta_{6,12,2}(z)-\frac{1}{13824}\Delta_{6,12,2}(2z)+\frac{5}{1603584}\Delta_{6,12,3}(z)$$
$$+\frac{5}{50112}\Delta_{6,12,3}(2z)+-\frac{5}{2285568}\Delta_{12,12,1}(z)-\frac{1}{700416}\Delta_{12,12,2}(z),$$

$$f_6 = \frac{1}{32640}\Delta(z)+\frac{1}{1360}\Delta(2z)-\frac{63}{10880}\Delta(3z)+\frac{16}{255}\Delta(4z)-\frac{189}{1360}\Delta(6z)$$
$$-\frac{1008}{85}\Delta(12z)-\frac{1}{212976}\Delta_{3,12}(z)+\frac{13}{35496}\Delta_{3,12}(2z)-\frac{128}{13311}\Delta_{3,12}(4z)$$
$$+\frac{1}{75392}\Delta_{4,12}(z)+\frac{387}{75392}\Delta_{4,12}(3z)-\frac{1}{138240}\Delta_{6,12,1}(z)+\frac{1}{4320}\Delta_{6,12,1}(2z)$$
$$-\frac{1}{82944}\Delta_{6,12,2}(z)-\frac{1}{2592}\Delta_{6,12,2}(2z)-\frac{1}{150336}\Delta_{6,12,3}(z)-\frac{1}{4698}\Delta_{6,12,3}(2z)$$
$$-\frac{1}{321408}\Delta_{12,12,1}(z)-\frac{1}{98496}\Delta_{12,12,2}(z),$$

$$f_7 = \frac{211}{626688}\Delta(z)+\frac{211}{26112}\Delta(2z)-\frac{2187}{69632}\Delta(3z)+\frac{211}{306}\Delta(4z)$$
$$-\frac{6561}{8704}\Delta(6z)-\frac{2187}{34}\Delta(12z)+\frac{213}{504832}\Delta_{3,12}(z)$$
$$-\frac{8307}{252416}\Delta_{3,12}(2z)+\frac{426}{493}\Delta_{3,12}(4z)-\frac{3517}{7237632}\Delta_{4,12}(z)$$
$$-\frac{531441}{2412544}\Delta_{4,12}(3z)+\frac{3}{8192}\Delta_{6,12,1}(z)-\frac{3}{256}\Delta_{6,12,1}(2z)$$
$$+\frac{7}{73728}\Delta_{6,12,2}(z)+\frac{7}{2304}\Delta_{6,12,2}(2z)+\frac{11}{133632}\Delta_{6,12,3}(z)$$
$$+\frac{11}{4176}\Delta_{6,12,3}(2z)+-\frac{71}{126976}\Delta_{12,12,1}(z)-\frac{5}{19456}\Delta_{12,12,2}(z),$$

$$f_8 = \frac{1}{352512}\Delta(z)+\frac{1}{14688}\Delta(2z)-\frac{259}{39168}\Delta(3z)+\frac{8}{1377}\Delta(4z)$$
$$-\frac{259}{1632}\Delta(6z)-\frac{2072}{153}\Delta(12z)-\frac{5}{638928}\Delta_{3,12}(z)$$
$$+\frac{65}{106488}\Delta_{3,12}(2z)-\frac{640}{39933}\Delta_{3,12}(4z)-\frac{37}{12213504}\Delta_{4,12}(z)$$
$$-\frac{17731}{4071168}\Delta_{4,12}(3z)+\frac{1}{165888}\Delta_{6,12,1}(z)-\frac{1}{5184}\Delta_{6,12,1}(2z)$$
$$+\frac{7}{1492992}\Delta_{6,12,2}(z)+\frac{7}{46656}\Delta_{6,12,2}(2z)-\frac{31}{5412096}\Delta_{6,12,3}(z)$$
$$-\frac{31}{169128}\Delta_{6,12,3}(2z)+\frac{7}{964224}\Delta_{12,12,1}(z)-\frac{5}{1181952}\Delta_{12,12,2}(z),$$

$$f_9 = \frac{1543673}{43304140080}\Delta(z)+\frac{1543673}{180433920}\Delta(2z)-\frac{38051613}{481157120}\Delta(3z)$$
$$+\frac{1543673}{2114460}\Delta(4z)-\frac{1141548339}{60144640}\Delta(6z)-\frac{38051613}{234940}\Delta(12z)$$
$$+\frac{46641}{147410944}\Delta_{3,12}(z)-\frac{1818999}{73705472}\Delta_{3,12}(2z)+\frac{46641}{71978}\Delta_{3,12}(4z)$$
$$-\frac{63891}{14475264}\Delta_{4,12}(z)-\frac{767637}{4825088}\Delta_{4,12}(3z)+\frac{141}{573440}\Delta_{6,12,1}(z)$$
$$-\frac{141}{17920}\Delta_{6,12,1}(2z)+\frac{331}{1916928}\Delta_{6,12,2}(z)+\frac{331}{59904}\Delta_{6,12,2}(2z)$$
$$+\frac{563}{2672640}\Delta_{6,12,3}(z)+\frac{563}{83520}\Delta_{6,12,3}(2z)+-\frac{137}{253952}\Delta_{12,12,1}(z)$$
$$-\frac{25}{77824}\Delta_{12,12,2}(z)+\frac{1}{4457361592320}E_{12}(z)-\frac{683}{1485787197440}E_{12}(2z)$$
$$-\frac{177147}{1485787197440}E_{12}(3z)+\frac{1}{2176446090}E_{12}(4z)$$
$$+\frac{362974203}{1485787197440}E_{12}(6z)-\frac{177147}{725482030}E_{12}(12z),$$

$$f_{10} = \frac{2201}{12240}\Delta(z)+\frac{2201}{510}\Delta(2z)-\frac{50301}{1360}\Delta(3z)+\frac{281728}{765}\Delta(4z)$$
$$-\frac{150903}{170}\Delta(6z)-\frac{6438528}{85}\Delta(12z)+\frac{155}{986}\Delta_{3,12}(z)$$
$$-\frac{6045}{493}\Delta_{3,12}(2z)+\frac{158720}{493}\Delta_{3,12}(4z)+\frac{6475}{56544}\Delta_{4,12}(z)$$

$$+\frac{8007003}{18848}\Delta_{4,12}(3z)+\frac{79}{640}\Delta_{6,12,1}(z)-\frac{79}{20}\Delta_{6,12,1}(2z)$$

$$+\frac{109}{1152}\Delta_{6,12,2}(z)+\frac{109}{36}\Delta_{6,12,2}(2z)+\frac{233}{2088}\Delta_{6,12,3}(z)$$

$$+\frac{932}{261}\Delta_{6,12,3}(2z)+\frac{413}{2976}\Delta_{12,12,1}(z)+\frac{73}{912}\Delta_{12,12,2}(z),$$

$$f_{11}= -\frac{26}{765}\Delta(2z)-\frac{896}{765}\Delta(4z)-\frac{729}{85}\Delta(6z)+\frac{279936}{85}\Delta(12z)$$

$$+\frac{113}{2958}\Delta_{3,12}(2z)-\frac{1024}{1479}\Delta_{3,12}(4z)-\frac{1}{60}\Delta_{6,12,1}(2z)$$

$$+\frac{1}{36}\Delta_{6,12,2}(2z)-\frac{4}{261}\Delta_{6,12,3}(2z),$$

$$f_{12}= -\frac{1027}{12240}\Delta(2z)-\frac{112}{765}\Delta(4z)+\frac{2187}{1360}\Delta(6z)+\frac{34992}{85}\Delta(12z)$$

$$+\frac{39}{1972}\Delta_{3,12}(2z)-\frac{768}{493}\Delta_{3,12}(4z)+\frac{3}{160}\Delta_{6,12,1}(2z)$$

$$+\frac{7}{288}\Delta_{6,12,2}(2z)+\frac{11}{522}\Delta_{6,12,3}(2z),$$

$$f_{13}= -\frac{7}{24480}\Delta(2z)-\frac{26}{765}\Delta(4z)+\frac{2187}{2720}\Delta(6z)-\frac{729}{85}\Delta(12z)$$

$$-\frac{1}{5916}\Delta_{3,12}(2z)+\frac{113}{2958}\Delta_{3,12}(4z)+\frac{1}{3840}\Delta_{6,12,1}(2z)$$

$$+\frac{1}{2304}\Delta_{6,12,2}(2z)-\frac{1}{4176}\Delta_{6,12,3}(2z),$$

$$f_{14}= \frac{1}{36}\Delta(6z)+\frac{16}{9}\Delta(12z)-\frac{1}{17496}\Delta_{6,12,2}(2z)+\frac{1}{17496}\Delta_{6,12,3}(2z),$$

$$f_{15}= -\frac{7}{24480}\Delta(2z)-\frac{13}{170}\Delta(4z)+\frac{2187}{2720}\Delta(6z)+\frac{29997}{170}\Delta(12z)$$

$$-\frac{5}{1836}\Delta_{3,12}(2z)+\frac{98}{459}\Delta_{3,12}(4z)+\frac{89}{34560}\Delta_{6,12,1}(2z)$$

$$-\frac{13}{6912}\Delta_{6,12,2}(2z)+\frac{1}{432}\Delta_{6,12,3}(2z),$$

$$f_{16} = -\frac{7}{4080}\Delta(2z)+\frac{36}{85}\Delta(4z)+\frac{6561}{1360}\Delta(6z)-\frac{6804}{85}\Delta(12z)+\frac{11}{5916}\Delta_{3,12}(2z)$$
$$+\frac{176}{1479}\Delta_{3,12}(4z)+\frac{1}{240}\Delta_{6,12,1}(2z)-\frac{1}{232}\Delta_{6,12,3}(2z),$$

$$f_{17} = -\frac{1}{153}\Delta(4z)+\frac{378}{17}\Delta(12z)-\frac{1}{3132}\Delta_{3,12}(2z)+\frac{308}{13311}\Delta_{3,12}(4z)$$
$$+\frac{1}{3456}\Delta_{6,12,1}(2z)-\frac{1}{3456}\Delta_{6,12,2}(2z)+\frac{1}{3132}\Delta_{6,12,3}(2z),$$

$$f_{18} = -\frac{717}{110560}\Delta(2z)-\frac{189277}{58735}\Delta(4z)+\frac{216513}{110560}\Delta(6z)+\frac{37712628}{58735}\Delta(12z)$$
$$+\frac{297}{8468}\Delta_{3,12}(2z)-\frac{194697}{71978}\Delta_{3,12}(4z)+\frac{297}{8960}\Delta_{6,12,1}(2z)$$
$$-\frac{95}{3328}\Delta_{6,12,2}(2z)-\frac{77}{2320}\Delta_{6,12,3}(2z)+-\frac{1}{2176446090}E_{12}(2z)$$
$$+\frac{1}{2176446090}E_{12}(4z)+\frac{177147}{725482030}E_{12}(6z)-\frac{177147}{725482030}E_{12}(12z),$$

$$f_{19} = \frac{31}{1585845}\Delta(2z)+\frac{13067}{3171690}\Delta(4z)-\frac{3277}{58735}\Delta(6z)-\frac{1142579}{117470}\Delta(12z)$$
$$+\frac{49}{323901}\Delta_{3,12}(2z)-\frac{7909}{647802}\Delta_{3,12}(4z)-\frac{1}{6720}\Delta_{6,12,1}(2z)$$
$$+\frac{61}{606528}\Delta_{6,12,2}(2z)-\frac{13}{105705}\Delta_{6,12,3}(2z)+\frac{3}{725482030}E_{12}(2z)$$
$$-\frac{3}{725482030}E_{12}(4z)-\frac{3}{725482030}E_{12}(6z)+\frac{3}{725482030}E_{12}(12z).$$

TABLE1

$$c_1: \quad =-\frac{360988664}{16763355}k_0+\frac{4324842112}{217923615}k_1-\frac{562186922}{31131945}k_2+\frac{3542123518}{217923615}k_3$$
$$-\frac{3148554236}{217923615}k_4+\frac{2754984952}{217923615}k_5-\frac{2361415664}{217923615}k_6+\frac{1967846368}{217923615}k_7$$
$$-\frac{1574277056}{217923615}k_8+\frac{1180707712}{217923615}k_9-\frac{787138304}{217923615}k_{10}+\frac{393568768}{217923615}k_{11}$$
$$+\frac{1024}{217923615}k_{12}-\frac{393571328}{217923615}k_{13}+\frac{787142656}{217923615}k_{14}-\frac{1180716032}{217923615}k_{15}$$

$$+\frac{224899072}{31131945}k_{16}-\frac{1967879168}{217923615}k_{17}+\frac{2361481216}{217923615}k_{18}-\frac{2755116032}{217923615}k_{19}$$
$$+\frac{449830912}{31131945}k_{20}-\frac{506092544}{31131945}k_{21}+\frac{3936741376}{217923615}k_{22}-\frac{4724928512}{217923615}k_{23}$$
$$+\frac{9449857024}{217923615}k_{24},$$

$$c_2: \quad =-\frac{384870704}{76545}k_0+\frac{4370616022}{995085}k_1-\frac{562755542}{142155}k_2+\frac{3542123518}{995085}k_3$$
$$-\frac{3148554236}{995085}k_4+\frac{2754984952}{995085}k_5-\frac{2361415664}{995085}k_6+\frac{1967846368}{995085}k_7$$
$$-\frac{1574277056}{995085}k_8+\frac{1180707712}{995085}k_9-\frac{787138304}{995085}k_{10}+\frac{393568768}{995085}k_{11}$$
$$+\frac{1024}{995085}k_{12}-\frac{393571328}{995085}k_{13}+\frac{787142656}{995085}k_{14}-\frac{1180716032}{995085}k_{15}$$
$$+\frac{224899072}{142155}k_{16}-\frac{1967879168}{995085}k_{17}+\frac{2361481216}{995085}k_{18}-\frac{2755116032}{995085}k_{19}$$
$$+\frac{449830912}{142155}k_{20}-\frac{506092544}{142155}k_{21}+\frac{3936741376}{995085}k_{22}-\frac{4724928512}{995085}k_{23}$$
$$+\frac{9449857024}{995085}k_{24},$$

$$c_3: \quad =\frac{701893863 76}{155659725}k_0-\frac{16696101794}{40356225}k_1+\frac{410183903128}{1089618075}k_2$$
$$-\frac{369525945848}{1089618075}k_3+\frac{12177495704}{40356225}k_4-\frac{287938911068}{1089618075}k_5$$
$$+\frac{246897697048}{1089618075}k_6-\frac{1756980304}{9312975}k_7+\frac{163795356448}{1089618075}k_8$$
$$-\frac{121366395008}{1089618075}k_9+\frac{962640128}{13452075}k_{10}-\frac{33198637568}{1089618075}k_{11}$$
$$-\frac{1930843136}{155659725}k_{12}+\frac{776353792}{13452075}k_{13}-\frac{115793211392}{1089618075}k_{14}$$
$$+\frac{173298476032}{1089618075}k_{15}-\frac{26290597888}{121068675}k_{16}+\frac{307085396992}{1089618075}k_{17}$$
$$-\frac{55158388736}{155659725}k_{18}+\frac{1353300992}{3104325}k_{19}-\frac{574954668032}{1089618075}k_{20}$$

$$+\frac{98326537216}{155659725}k_{21}-\frac{30680999936}{40356225}k_{22}+\frac{1108589473792}{1089618075}k_{23}$$
$$-\frac{2217178947584}{1089618075}k_{24},$$

$$c_4: \quad =\frac{84647641088}{16763355}k_0-\frac{961489752064}{217923615}k_1+\frac{123805650944}{31131945}k_2$$
$$-\frac{779267178496}{217923615}k_3+\frac{692681940992}{217923615}k_4-\frac{606096707584}{217923615}k_5$$
$$+\frac{519511482368}{217923615}k_6-\frac{432926273536}{217923615}k_7+\frac{346341097472}{217923615}k_8$$
$$-\frac{259755986944}{217923615}k_9+\frac{173171007488}{217923615}k_{10}-\frac{86586290176}{217923615}k_{11}$$
$$+\frac{2097152}{217923615}k_{12}+\frac{86581047296}{217923615}k_{13}-\frac{173162094592}{217923615}k_{14}$$
$$+\frac{259738947584}{217923615}k_{15}-\frac{49472487424}{31131945}k_{16}+\frac{432859099136}{217923615}k_{17}$$
$$-\frac{519377231872}{217923615}k_{18}+\frac{605828255744}{217923615}k_{19}-\frac{98877865984}{31131945}k_{20}$$
$$+\frac{111170490368}{31131945}k_{21}-\frac{863704932352}{217923615}k_{22}+\frac{1034727931904}{217923615}k_{23}$$
$$-\frac{2069455863808}{217923615}k_{24},$$

$$c_6: \quad =-\frac{453628774143368}{51886575}k_0+\frac{107905593332732}{13452075}k_1-\frac{2650955865173324}{363206025}k_2$$
$$+\frac{2388138935772 64}{363206025}k_3-\frac{291468535976}{498225}k_4+\frac{1860653326020 04}{363206025}k_5$$
$$-\frac{159521875334984}{363206025}k_6+\frac{14753388140336}{40356225}k_7-\frac{105738360643424}{363206025}k_8$$
$$+\frac{78247072100224}{363206025}k_9-\frac{1855468758272}{13452075}k_{10}+\frac{21003878977024}{363206025}k_{11}$$
$$+\frac{103455579136}{3991275}k_{12}-\frac{1542430435328}{13452075}k_{13}+\frac{76294464679936}{363206025}k_{14}$$
$$-\frac{114082853067776}{363206025}k_{15}+\frac{17315619590144}{40356225}k_{16}-\frac{202483843429376}{363206025}k_{17}$$

$$+\frac{36424136869888}{51886575}k_{18}-\frac{3879008254976}{4484025}k_{19}+\frac{380974612873216}{363206025}k_{20}$$
$$-\frac{65270152322048}{51886575}k_{21}+\frac{20410760665088}{13452075}k_{22}-\frac{739489481363456}{363206025}k_{23}$$
$$+\frac{1478978962726912}{363206025}k_{24},$$

$$c_{12}:\quad =\frac{1361753348154368}{155659725}k_0-\frac{32354981896192}{40356225}k_1+\frac{794876575637504}{1089618075}k_2$$
$$-\frac{55082473443328}{83816775}k_3+\frac{23596773916672}{40356225}k_4-\frac{557908058804224}{1089618075}k_5$$
$$+\frac{478318728126464}{1089618075}k_6-\frac{44237323636736}{121068675}k_7+\frac{317051285848064}{1089618075}k_8$$
$$-\frac{234619848454144}{1089618075}k_9+\frac{1854506082304}{13452075}k_{10}-\frac{62978432487424}{1089618075}k_{11}$$
$$-\frac{4032838402048}{155659725}k_{12}+\frac{1541654368256}{13452075}k_{13}-\frac{228767647277056}{1089618075}k_{14}$$
$$+\frac{26313488740352}{83816775}k_{15}-\frac{51920588816384}{121068675}k_{16}+\frac{607144816480256}{1089618075}k_{17}$$
$$-\frac{109217358389248}{155659725}k_{18}+\frac{34893536432128}{40356225}k_{19}-\frac{1142351856664576}{1089618075}k_{20}$$
$$+\frac{195712979775488}{155659725}k_{21}-\frac{61202041397248}{40356225}k_{22}+\frac{2217383636320256}{1089618075}k_{23}$$
$$-\frac{4434767272640512}{1089618075}k_{24}$$

$$r_1:\quad =\frac{3003665227147516928}{1089618075}k_0-\frac{8254495271993852416}{3268854225}k_1+\frac{2503312582032718592}{1089618075}k_2$$
$$-\frac{83540275425188096}{40356225}k_3+\frac{6022117930030140416}{3268854225}k_4-\frac{1758210397223566592}{1089618075}k_5$$
$$+\frac{1507543698600796672}{1089618075}k_6-\frac{3763639968684860416}{3268854225}k_7+\frac{332669769731999744}{363206025}k_8$$
$$-\frac{8090035939459072}{11973825}k_9+\frac{1399973958868729856}{3268854225}k_{10}-\frac{186067151938281472}{1089618075}k_{11}$$
$$-\frac{36668294008979456}{363206025}k_{12}+\frac{1281709997065969664}{3268854225}k_{13}-\frac{772658319924543488}{1089618075}k_{14}$$
$$+\frac{1154656505019670528}{1089618075}k_{15}-\frac{4748606804540637184}{3268854225}k_{16}+\frac{689311643626479616}{363206025}k_{17}$$

$$-\frac{26209580558682849 28}{1089618075}k_{18}+\frac{97572941435894743 04}{3268854225}k_{19}-\frac{39716269955114106 88}{1089618075}k_{20}$$

$$+\frac{175716240714383 36}{3991275}k_{21}-\frac{17503217105311105024}{3268854225}k_{22}+\frac{7903951046871482368}{1089618075}k_{23}$$

$$-\frac{15767627299210919936}{1089618075}k_{24},$$

$$r_2:\quad =-\frac{229474709720062926016}{29419688025}k_0+\frac{2605263290736226144}{363206025}k_1-\frac{192050349205544015344}{29419688025}k_2$$

$$+\frac{172961716345500147824}{29419688025}k_3-\frac{5697617224585991872}{1089618075}k_4+\frac{19238894167926204272}{4202812575}k_5$$

$$-\frac{115452394083658527904}{29419688025}k_6+\frac{10682841315271062976}{3268854225}k_7-\frac{76706758491166497664}{29419688025}k_8$$

$$+\frac{57071206808948378624}{29419688025}k_9-\frac{105834765967806464}{83816775}k_{10}+\frac{16812844676910953984}{29419688025}k_{11}$$

$$+\frac{40991634697548431 36}{29419688025}k_{12}-\frac{1365057726953815 04}{155659725}k_{13}+\frac{48550345075087010816}{29419688025}k_{14}$$

$$-\frac{72663851397662972416}{29419688025}k_{15}+\frac{10944348539824605184}{3268854225}k_{16}-\frac{126452572522674443776}{29419688025}k_{17}$$

$$+\frac{156937930634831184896}{29419688025}k_{18}-\frac{7049870466264667648}{1089618075}k_{19}+\frac{227024565477717426176}{29419688025}k_{20}$$

$$-\frac{38237481012114293248}{4202812575}k_{21}+\frac{1000922376493475 84}{9312975}k_{22}-\frac{413001898707227524096}{29419688025}k_{23}$$

$$+\frac{824184358117245550592}{29419688025}k_{24},$$

$$r_3:\quad =\frac{138089266216997768}{14926275}k_0-\frac{4715103489802456}{552825}k_1+\frac{115871746885892222}{14926275}k_2$$

$$-\frac{104312298821721502}{14926275}k_3+\frac{3434566695563296}{552825}k_4-\frac{811488358775349 82}{14926275}k_5$$

$$+\frac{69558114946966652}{14926275}k_6-\frac{6439799189132648}{1658475}k_7+\frac{46344371624009552}{14926275}k_8$$

$$-\frac{4958546872853056}{2132325}k_9+\frac{284505803673472}{184275}k_{10}-\frac{11336680202674432}{14926275}k_{11}$$

$$-\frac{332645452456 96}{1148175}k_{12}+\frac{50554326401536}{61425}k_{13}-\frac{24247718948574208}{14926275}k_{14}$$

$$+\frac{363544246221639 68}{14926275}k_{15}-\frac{5404741198272512}{1658475}k_{16}+\frac{61154284065886208}{14926275}k_{17}$$

$$-\frac{73933025066702848}{14926275}k_{18}+\frac{3222981849800704}{552825}k_{19}-\frac{100454037793466368}{14926275}k_{20}$$

$$+\frac{16329644915216384}{2132325}k_{21}-\frac{4777557822859264}{552825}k_{22}+\frac{158315420309470208}{14926275}k_{23}$$

$$-\frac{316290948815343616}{14926275}k_{24},$$

$$r_4:\quad =\frac{1144148105604759 44}{1089618075}k_0-\frac{243517496688016 66}{251450325}k_1+\frac{960474628686020 36}{1089618075}k_2$$

$$-\frac{1067417077839436}{13452075}k_3+\frac{230574176263147748}{3268854225}k_4-\frac{67252837695387446}{1089618075}k_5$$

$$+\frac{57645774123579676}{1089618075}k_6-\frac{144108786582005368}{3268854225}k_7+\frac{12807694276434032}{363206025}k_8$$

$$-\frac{4114918020319168}{155659725}k_9+\frac{57533230164875648}{3268854225}k_{10}-\frac{9539542991193856}{1089618075}k_{11}$$

$$-\frac{38271273978368}{363206025}k_{12}+\frac{2259527666200064}{251450325}k_{13}-\frac{19497262246882304}{1089618075}k_{14}$$

$$+\frac{29241566788946944}{1089618075}k_{15}-\frac{117103098372708352}{3268854225}k_{16}+\frac{16295642739524608}{363206025}k_{17}$$

$$-\frac{58811106522701824}{1089618075}k_{18}+\frac{206454920698452992}{3268854225}k_{19}-\frac{78919040036030464}{1089618075}k_{20}$$

$$+\frac{4244491776502784}{51886575}k_{21}-\frac{298736168535150592}{3268854225}k_{22}+\frac{120466398153124864}{1089618075}k_{23}$$

$$-\frac{240917175541526528}{1089618075}k_{24},$$

$$r_5:\quad =\frac{301461147086411264}{1089618075}k_0-\frac{828930437193319168}{3268854225}k_1+\frac{251404663921160576}{1089618075}k_2$$

$$-\frac{25168701364982144}{121068675}k_3+\frac{604740221949475328}{3268854225}k_4-\frac{176551761214246016}{1089618075}k_5$$

$$+\frac{151386451866332416}{1089618075}k_6-\frac{378043600395831808}{3268854225}k_7+\frac{33444435318244352}{363206025}k_8$$

$$-\frac{10599220767612928}{155659725}k_9+\frac{10936046779559936}{251450325}k_{10}-\frac{19632393196711936}{1089618075}k_{11}$$

$$-\frac{3152205141659648}{363206025}k_{12}+\frac{1210954813499678722}{3268854225}k_{13}-\frac{736973524742512 64}{1089618075}k_{14}$$

$$+\frac{1101741724438896 64}{1089618075}k_{15}-\frac{4518731662338334 72}{3268854225}k_{16}+\frac{6532102560699596 8}{363206025}k_{17}$$

$$-\frac{2471453438555299 84}{1089618075}k_{18}+\frac{9152213343804907 52}{3268854225}k_{19}-\frac{3705459301056471 04}{1089618075}k_{20}$$

$$+\frac{2119794765397606 4}{51886575}k_{21}-\frac{1241602283900477 44}{251450325}k_{22}+\frac{7235192353680547 84}{1089618075}k_{23}$$

$$-\frac{1444708745254535168}{1089618075}k_{24},$$

$$r_6:\quad =-\frac{205157345857041 92}{29419688025}k_0+\frac{77671005986386}{121068675}k_1-\frac{171738261524559 98}{29419688025}k_2$$

$$+\frac{154630411690748 38}{29419688025}k_3-\frac{509253927288644}{1089618075}k_4+\frac{132246929912008}{323293275}k_5$$

$$-\frac{103156170153699 68}{29419688025}k_6+\frac{954596664012512}{3268854225}k_7-\frac{685885929131052 8}{29419688025}k_8$$

$$+\frac{511389301798796 8}{29419688025}k_9-\frac{124092162060544}{1089618075}k_{10}+\frac{156032441267660 8}{29419688025}k_{11}$$

$$+\frac{267843277468672}{29419688025}k_{12}-\frac{11368965680128}{155659725}k_{13}+\frac{410096708001587 2}{29419688025}k_{14}$$

$$-\frac{614725144881459 2}{29419688025}k_{15}+\frac{923809375981568}{3268854225}k_{16}-\frac{817867147326464}{2263052925}k_{17}$$

$$+\frac{131338659859640 32}{29419688025}k_{18}-\frac{587109047346176}{1089618075}k_{19}+\frac{188169133029867 52}{29419688025}k_{20}$$

$$-\frac{315246433376153 6}{4202812575}k_{21}+\frac{317983602952192}{363206025}k_{22}-\frac{324396423769579 52}{29419688025}k_{23}$$

$$+\frac{614187297790935 04}{29419688025}k_{24},$$

$$r_7:\quad =-\frac{441191907265913 6}{1089618075}k_0+\frac{121566599764889 92}{3268854225}k_1-\frac{368761296119230 4}{1089618075}k_2$$

$$+\frac{158178812696624}{51886575}k_3-\frac{886541087732499 2}{3268854225}k_4+\frac{258750940862350 4}{1089618075}k_5$$

$$-\frac{221831960175606 4}{1089618075}k_6+\frac{554015512886099 2}{3268854225}k_7-\frac{6055361948288}{4484025}k_8$$

$$+\frac{109071523384422 4}{1089618075}k_9-\frac{300839662647296}{466979175}k_{10}+\frac{43099648625152}{155659725}k_{11}$$

$$+\frac{38233977935872}{363206025}k_{12}-\frac{1659564170029568}{3268854225}k_{13}+\frac{1021130097837056}{1089618075}k_{14}$$
$$-\frac{1527323039566336}{1089618075}k_{15}+\frac{6245630579396608}{3268854225}k_{16}-\frac{128378793606656}{51886575}k_{17}$$
$$+\frac{3381208528731136}{1089618075}k_{18}-\frac{12445953461954048}{3268854225}k_{19}+\frac{5008016514451456}{1089618075}k_{20}$$
$$-\frac{94901151683072}{17295525}k_{21}+\frac{3073847125648384}{466979175}k_{22}-\frac{1365466999794688}{155659725}k_{23}$$
$$+\frac{2728702461771776}{155659725}k_{24},$$

$$r_8: \quad =-\frac{70204760176}{5765175}k_0+\frac{5567913748}{498225}k_1-\frac{410578497328}{40356225}k_2$$
$$+\frac{370279262048}{40356225}k_3-\frac{12230639704}{1494675}k_4+\frac{290665198268}{40356225}k_5$$
$$-\frac{252063293848}{40356225}k_6+\frac{1840375504}{344925}k_7-\frac{182161922848}{40356225}k_8$$
$$+\frac{155803707008}{40356225}k_9-\frac{5268771584}{1494675}k_{10}+\frac{152581319168}{40356225}k_{11}$$
$$-\frac{29554699264}{5765175}k_{12}+\frac{12636289024}{1494675}k_{13}-\frac{618869444608}{40356225}k_{14}$$
$$+\frac{1149094304768}{40356225}k_{15}-\frac{234922790912}{4484025}k_{16}+\frac{3807025476608}{40356225}k_{17}$$
$$-\frac{952378968064}{5765175}k_{18}+\frac{32135595008}{114975}k_{19}-\frac{18232409325568}{40356225}k_{20}$$
$$+\frac{3931822889984}{5765175}k_{21}-\frac{151384119296}{166075}k_{22}$$
$$+\frac{36506138513408}{40356225}k_{23}+\frac{2217178947584}{40356225}k_{24},$$

$$r_9: \quad =\frac{2216738265088}{620865}k_0-\frac{54899497497344}{16763355}k_1+\frac{5550067870976}{1862595}k_2$$
$$-\frac{14999343555328}{5587785}k_3+\frac{40036019090944}{16763355}k_4-\frac{11686219632128}{5587785}k_5$$
$$+\frac{1431298468864}{798255}k_6-\frac{25018784006144}{16763355}k_7+\frac{948758902784}{798255}k_8$$

$$-\frac{1638323808256}{1862595}k_9+\frac{9442552981504}{16763355}k_{10}-\frac{146785564672}{620865}k_{11}$$

$$-\frac{588122048512}{5587785}k_{12}+\frac{7831681816576}{16763355}k_{13}-\frac{4784030369792}{5587785}k_{14}$$

$$+\frac{7153626007552}{5587785}k_{15}-\frac{29313552275456}{16763355}k_{16}+\frac{12693939832832}{5587785}k_{17}$$

$$-\frac{5327211075584}{1862595}k_{18}+\frac{59072164059136}{16763355}k_{19}-\frac{7957031819264}{1862595}k_{20}$$

$$+\frac{4088934004736}{798255}k_{21}-\frac{103548352649216}{16763355}k_{22}+\frac{46303276582912}{5587785}k_{23}$$

$$-\frac{92583665598464}{5587785}k_{24},$$

$$r_{10}: \quad =\frac{63555628979032}{1089618075}k_0-\frac{176958637635074}{3268854225}k_1+\frac{53756225621008}{1089618075}k_2$$

$$-\frac{16132771258616}{363206025}k_3+\frac{129066206613844}{3268854225}k_4-\frac{37644351130138}{1089618075}k_5$$

$$+\frac{32266602044228}{1089618075}k_6-\frac{80666413653704}{3268854225}k_7+\frac{7170311150096}{363206025}k_8$$

$$-\frac{177285190208}{11973825}k_9+\frac{32264429090944}{3268854225}k_{10}-\frac{5376434893568}{1089618075}k_{11}$$

$$-\frac{83949056}{40356225}k_{12}+\frac{16144252496896}{3268854225}k_{13}-\frac{10761172314112}{1089618075}k_{14}$$

$$+\frac{16141721196032}{1089618075}k_{15}-\frac{64569883744256}{3268854225}k_{16}+\frac{2989574660608}{121068675}k_{17}$$

$$-\frac{32290698447872}{1089618075}k_{18}+\frac{113031954674176}{3268854225}k_{19}-\frac{43066673364992}{1089618075}k_{20}$$

$$+\frac{177509333504}{3991275}k_{21}-\frac{161584385432576}{3268854225}k_{22}+\frac{64664289339392}{1089618075}k_{23}$$

$$-\frac{129328578678784}{1089618075}k_{24},$$

$$r_{11}: \quad =-\frac{63605244075992}{1089618075}k_0+\frac{177030047975204}{3268854225}k_1-\frac{53775902163278}{1089618075}k_2$$

$$+\frac{3724309568726}{83816775}k_3-\frac{129113434927384}{3268854225}k_4+\frac{4184236228322}{121068675}k_5$$

$$-\frac{10759469707516}{363206025}k_6+\frac{80695931349224}{3268854225}k_7-\frac{21518804835568}{1089618075}k_8$$

$$+\frac{16138855847488}{1089618075}k_9-\frac{32276236165504}{3268854225}k_{10}+\frac{5378402737408}{1089618075}k_{11}$$

$$+\frac{2266629632}{1089618075}k_{12}-\frac{16150156066816}{3268854225}k_{13}+\frac{3588369342464}{363206025}k_{14}$$

$$-\frac{6572089856}{443475}k_{15}+\frac{64593498146816}{3268854225}k_{16}-\frac{26916011341312}{1089618075}k_{17}$$

$$+\frac{32302505853952}{1089618075}k_{18}-\frac{113073281414656}{3268854225}k_{19}+\frac{43082417446912}{1089618075}k_{20}$$

$$-\frac{6925394469376}{155659725}k_{21}+\frac{161643436553216}{3268854225}k_{22}-\frac{21562637993984}{363206025}k_{23}$$

$$+\frac{43125275987968}{363206025}k_{24},$$

$$r_{12}: \quad =\frac{15952905573588864384}{50290065}k_0-\frac{60515318268024432}{206955}k_1+\frac{13382358974269672 96}{50290065}k_2$$

$$-\frac{12046089231577706496}{50290065}k_3+\frac{396593278794542 08}{1862595}k_4-\frac{9369701061195791 36}{50290065}k_5$$

$$+\frac{1147322910008596 48}{7184295}k_6-\frac{7436398104473190 4}{5587785}k_7+\frac{7648894093887692 8}{7184295}k_8$$

$$-\frac{4015678736767324 16}{50290065}k_9+\frac{9915277442828288}{1862595}k_{10}-\frac{1338568557014159 36}{50290065}k_{11}$$

$$+\frac{1188901328896}{50290065}k_{12}+\frac{4957570701412352}{1862595}k_{13}-\frac{267709950004127744}{50290065}k_{14}$$

$$+\frac{401565623561918464}{50290065}k_{15}-\frac{59491328693118976}{5587785}k_{16}+\frac{669280169813776384}{50290065}k_{17}$$

$$-\frac{803142861954443264}{50290065}k_{18}+\frac{34704275433736192}{1862595}k_{19}-\frac{1070908888383678464}{50290065}k_{20}$$

$$+\frac{172120743976364032}{7184295}k_{21}-\frac{16529225453225984}{620865}k_{22}+\frac{1606911370029942784}{50290065}k_{23}$$

$$-\frac{3213822739777323008}{50290065}k_{24},$$

$$r_{13}: \quad =-11403264k_0+9510912k_1-8527872k_2+7692288k_3$$

$$-6864896k_4+6029312k_5-5185536k_6+4333568k_7$$

$$-3473408k_8+2605056k_9-1728512k_{10}+843776k_{11}$$

$$+49152k_{12}-950272k_{13}+1859584k_{14}-2777088k_{15}$$
$$+3702784k_{16}-4636672k_{17}+5578752k_{18}-6529024k_{19}$$
$$+7487488k_{20}-8454144k_{21}+9437184k_{22}-11403264k_{23}$$
$$+22806528k_{24}$$

$$r_{14}: \quad =-\frac{1869871231808}{213525}k_0+\frac{190623641288}{23725}k_1-\frac{1561030669112}{213525}k_2$$
$$+\frac{1406251909912}{213525}k_3-\frac{139017335888}{23725}k_4+\frac{1095554299312}{213525}k_5$$
$$-\frac{939155300192}{213525}k_6+\frac{86835101888}{23725}k_7-\frac{621973972352}{213525}k_8$$
$$+\frac{459558595072}{213525}k_9-\frac{32541764608}{23725}k_{10}+\frac{119970107392}{213525}k_{11}$$
$$+\frac{618834805248}{213525}k_{12}-\frac{28455882752}{23725}k_{13}+\frac{467445587968}{213525}k_{14}$$
$$-\frac{701645201408}{213525}k_{15}+\frac{107299930112}{23725}k_{16}-\frac{1267694870528}{213525}k_{17}$$
$$+\frac{1616342007808}{213525}k_{18}-\frac{224449257472}{23725}k_{19}+\frac{2486585786368}{213525}k_{20}$$
$$-\frac{3031543390208}{213525}k_{21}+\frac{414814584832}{23725}k_{22}-\frac{5136907010048}{213525}k_{23}$$
$$+\frac{10273814020096}{213525}k_{24},$$

$$r_{15}: \quad =\frac{1481359611526592}{1921725}k_0-\frac{50214327864904}{71175}k_1+\frac{1232152828027688}{1921725}k_2$$
$$-\frac{11089116811260888}{1921725}k_3+\frac{36508614266704}{71175}k_4-\frac{8625539068062888}{1921725}k_5$$
$$+\frac{7393645400310008}{1921725}k_6-\frac{684623899541112}{213525}k_7+\frac{4929446157620048}{1921725}k_8$$
$$-\frac{3697145997537728}{1921725}k_9+\frac{30428725370888}{23725}k_{10}-\frac{1232204995502008}{1921725}k_{11}$$
$$-\frac{397410795552}{1921725}k_{12}+\frac{15222872350072}{23725}k_{13}-\frac{2465726914232232}{1921725}k_{14}$$
$$+\frac{3698381064273392}{1921725}k_{15}-\frac{547885872250888}{213525}k_{16}+\frac{6163461287198872}{1921725}k_{17}$$

$$-\frac{7395815714897 92}{1921725}k_{18}+\frac{31955681099776}{71175}k_{19}-\frac{9860170444447232}{1921725}k_{20}$$
$$+\frac{11092334579793 92}{1921725}k_{21}-\frac{45646495891456}{71175}k_{22}+\frac{14788992512491 52}{1921725}k_{23}$$
$$-\frac{2957798502498304}{1921725}k_{24},$$

$$r_{16}:\quad =\frac{190425570 56}{1971}k_0-\frac{17417912272}{1971}k_1+\frac{15825766064}{1971}k_2$$
$$-\frac{14241484144}{1971}k_3+\frac{12658834208}{1971}k_4-\frac{11076452320}{1971}k_5$$
$$+\frac{9494101952}{1971}k_6-\frac{7911751552}{1971}k_7+\frac{6329401088}{1971}k_8$$
$$-\frac{4747050496}{1971}k_9+\frac{3164699648}{1971}k_{10}-\frac{1582348288}{1971}k_{11}$$
$$-\frac{4096}{1971}k_{12}+\frac{1582358528}{1971}k_{13}-\frac{3164717056}{1971}k_{14}$$
$$+\frac{4747083776}{1971}k_{15}-\frac{6329466880}{1971}k_{16}+\frac{7911882752}{1971}k_{17}$$
$$-\frac{9494364160}{1971}k_{18}+\frac{11076976640}{1971}k_{19}-\frac{12659851264}{1971}k_{20}$$
$$+\frac{14243250176}{1971}k_{21}-\frac{15827697664}{1971}k_{22}+\frac{18996592640}{1971}k_{23}$$
$$-\frac{37993185280}{1971}k_{24},$$

$$r_{17}:\quad =-\frac{675631595 52}{65}k_0+\frac{606633492 48}{65}k_1-\frac{54957293568}{65}k_2$$
$$+\frac{49453580288}{65}k_3-\frac{43970101248}{65}k_4+\frac{38484492288}{65}k_5$$
$$-\frac{32995688448}{65}k_6+\frac{27502624768}{65}k_7-\frac{22004236288}{65}k_8$$
$$+\frac{16499458048}{65}k_9-\frac{10987225088}{65}k_{10}+\frac{5466472448}{65}k_{11}$$
$$+\frac{63864832}{65}k_{12}-\frac{5604851712}{65}k_{13}+\frac{11157553152}{65}k_{14}$$

$$-\frac{16723034112}{65}k_{15}+\frac{22302359552}{65}k_{16}-\frac{27896594432}{65}k_{17}$$
$$+\frac{33506803712}{65}k_{18}-\frac{39134052352}{65}k_{19}+\frac{44779405312}{65}k_{20}$$
$$-\frac{50444992512}{65}k_{21}+\frac{56151048192}{65}k_{22}-\frac{67563159552}{65}k_{23}$$
$$+\frac{135126319104}{65}k_{24},$$

$$r_{18}: \quad =\frac{2259593385627712}{17295525}k_0-\frac{22563359060648}{213525}k_1+\frac{16242852832 44568}{17295525}k_2$$
$$-\frac{1459590726318968}{17295525}k_3+\frac{48102043415344}{640575}k_4-\frac{1137597798509168}{17295525}k_5$$
$$+\frac{975989657022688}{17295525}k_6-\frac{90417848708032}{1921725}k_7+\frac{650691310918528}{17295525}k_8$$
$$-\frac{486505133201408}{17295525}k_9+\frac{11883767803904}{640575}k_{10}-\frac{153351813951488}{17295525}k_{11}$$
$$-\frac{16505961807872}{17295525}k_{12}+\frac{7009858482176}{640575}k_{13}-\frac{365555846193152}{17295525}k_{14}$$
$$+\frac{546070928269312}{17295525}k_{15}-\frac{81285366611968}{1921725}k_{16}+\frac{922849280008192}{17295525}k_{17}$$
$$-\frac{1120726339469312}{17295525}k_{18}+\frac{49109500174336}{640575}k_{19}-\frac{1539185916772352}{17295525}k_{20}$$
$$+\frac{1761706857766912}{17295525}k_{21}-\frac{24726059958272}{213525}k_{22}+\frac{2485018854326272}{17295525}k_{23}$$
$$-\frac{4970037708652544}{17295525}k_{24},$$

$$r_{19}: \quad =\frac{8599370989568}{729}k_0-\frac{972983418 88}{9}k_1+\frac{7163915165696}{729}k_2$$
$$-\frac{6447141855232}{729}k_3+\frac{212244709376}{27}k_4-\frac{5014191751168}{729}k_5$$
$$+\frac{4297835995136}{729}k_6-\frac{397945561088}{81}k_7+\frac{2865198989312}{729}k_8$$
$$-\frac{2148895326208}{729}k_9+\frac{53059084288}{27}k_{10}-\frac{716296855552}{729}k_{11}$$

$$-\frac{1097728}{729}k_{12}+\frac{26529611776}{27}k_{13}-\frac{1432599568384}{729}k_{14}$$
$$+\frac{2148903231488}{729}k_{15}-\frac{318357143552}{81}k_{16}+\frac{3581540237312}{729}k_{17}$$
$$-\frac{4297895993344}{729}k_{18}+\frac{185715236864}{27}k_{19}-\frac{5730846097408}{729}k_{20}$$
$$+\frac{6447619407872}{729}k_{21}-\frac{88455184384}{9}k_{22}+\frac{8599370989568}{729}k_{23}$$
$$-\frac{17198741979136}{729}k_{24}.$$

Table 2

No	b_1	b_2	b_3	b_4	b_5	a_2	a_4	a_6	a_{12}	c_1	c_2	c_3	c_4	c_6	c_{12}
1	2	6	2	6	6	24	0	0	0	$\frac{1}{2268}$	$-\frac{683}{756}$	0	$\frac{512}{567}$	0	0
2	6	2	6	2	2	0	0	24	0	$\frac{1}{2268}$	0	$-\frac{683}{756}$	$\frac{512}{567}$	$\frac{5938768}{1701}$	$-\frac{73649827}{15309}$

References

[1] A.Alaca, S.Alaca and K. S. Williams, *On the two-dimensional theta functions of Borweins*, Acta Arith. 124 (2006) 177-195.

[2] _________, *Evaluation of the convolution sums* $\sum_{l+12m=n}\sigma(l)\sigma(m)$ *and* $\sum_{3l+4m=n}\sigma(l)\sigma(m)$, Adv. Theor. Appl. Math. 1(2006), 27-48.

[3] B.Gordon, *Some identities in combinatorial analysis*, Quart. J. Math. Oxford Ser.12 (1961), 285-290.

[4] B. Gordon and S. Robins, *Lacunarity of Dedekind* η *-products,* Glasgow Math. J. 37 (1995), 1-14.

[5] F. Diamond, J. Shurman, *A First Course in Modular Forms,*Springer Graduate Texts in Mathematics 228

[6] V. G. Kac, *Infinite-dimensional algebras, Dedekind's* η *-function, classical Möbius function and the very strange formula,* Adv. Math. 30 (1978) 85-136.

[7] B. Kendirli, "*Evaluation of Some Convolution Sums by Quasimodular Forms"*, European Journal of Pure and Applied Mathematics ISSN 13075543 Vol.8., No. 1, Jan. 2015, pp. 81-110

[8] B. Kendirli, "*Evaluation of Some Convolution Sums and Representation Numbers of Quadratic Forms of Discriminant 135*", British Journal of Mathematics and Computer Science, Vol6/6, Jan. 2015, pp. 494-531.

[9] B. Kendirli, *Evaluation of Some Convolution Sums and the Representation numbers,* ArsCombinatorica Volume CXVI, July, pp 65-91.

[10] B. Kendirli, *Cusp Forms in* $S_4(\Gamma_0(79))$ *and the number of representations of positive integers by some direct sum of binary quadratic forms with discriminant -79,* Bulletin of the Korean Mathematical Society Vol 49/3 2012

[11] B. Kendirli, *Cusp Forms in* $S_4(\Gamma_0(47))$ *and the number of representations of positive integers by some direct sum of binary quadratic forms with discriminant -47,*Hindawi , International Journal of Mathematics and Mathematical Sciences Vol 2012, 303492 10 pages

[12] B. Kendirli, *The Bases of* $M_4(\Gamma_0(71)), M_6(\Gamma_0(71))$ *and the Number of Representations of Integers,* Hindawi, Mathematical Problems in Engineering Vol 2013, 695265, 34 pages

[13] G. Köhler, *Eta Products and Theta Series Identities* (Springer-Verlag, Berlin, 2011).

[14] I. G. Macdonald, *Affine root systems and Dedekind's* η *-function,* Invent. Math. 15 (1972), 91-143.

[15] Olivia X. M. Yao, Ernest X. W. Xia and J. Jin, Explicit Formulas for the Fourier coefficients of a class of eta quotients, International Journal of Number Theory Vol. 9, No. 2 (2013) 487-503.

[16] I. J. Zucker, *A systematic way of converting infinite series into infinite products*, J. Phys. A 20 (1987) L13-L17.

[17] _________,*Further relations amongst infinite series and products:II. The evaluation of three-dimensional lattice sums*, J. Phys. A23 (1990) 117-132.

[18] K. S. Williams, Fourier series of a class of eta quotients, Int. J. Number Theory 8 (2012), 993-1004.

Table 3A

No	b_1	b_2	b_3	b_4	b_5	a_2	a_4	a_6	a_{12}	c_1	c_2	c_3	c_4	c_6
1	0	0	0	0	24	−36	72	12	−24	$\frac{1079167}{68024320}$	$-\frac{30308739}{931840}$	$-\frac{3163327}{68024320}$	$\frac{552832439}{17006080}$	$\frac{6466241427}{68024320}$
2	0	0	1	0	22	−35	67	17	−25	$\frac{9446783}{612218880}$	$-\frac{88396417}{2795520}$	$-\frac{142785817}{3061094400}$	$\frac{4837051831}{153054720}$	$\frac{97275474551938768}{1020364800}$
3	0	0	2	0	20	−34	62	22	−26	$\frac{161321}{10761660}$	$-\frac{1509439}{49140}$	$-\frac{717389}{15373800}$	$\frac{82596352}{2690415}$	$\frac{488724967}{5124600}$
4	0	0	3	0	18	−33	57	27	−27	$\frac{1864}{128115}$	$-\frac{17441}{585}$	$-\frac{717389}{15373800}$	$\frac{3817472}{128115}$	$\frac{488724967}{5124600}$
5	0	0	4	0	16	−32	52	32	−28	$\frac{151831}{10761660}$	$-\frac{1420649}{49140}$	$-\frac{717389}{15373800}$	$\frac{77737472}{2690415}$	$\frac{4887249667}{5124600}$
6	0	0	5	0	14	−31	47	37	−29	$\frac{73543}{5380830}$	$-\frac{688127}{24570}$	$-\frac{717389}{15373800}$	$\frac{75308032}{2690415}$	$\frac{488724967}{5124600}$
7	0	0	6	0	12	−30	42	42	−30	$\frac{47447}{3587220}$	$-\frac{443953}{16380}$	$-\frac{717389}{15373800}$	$\frac{24292864}{896805}$	$\frac{488724967}{5124600}$
8	0	0	7	0	10	−29	37	47	−31	$\frac{34399}{2690415}$	$-\frac{321866}{12285}$	$-\frac{717389}{15373800}$	$\frac{70449152}{2690415}$	$\frac{488724967}{5124600}$
9	0	0	8	0	8	−28	32	52	−32	$\frac{132851}{10761660}$	$-\frac{1243069}{49140}$	$-\frac{717389}{15373800}$	$\frac{68019712}{2690415}$	$\frac{488724967}{5124600}$
10	0	0	9	0	6	−27	27	57	−33	$\frac{7117}{597870}$	$-\frac{66593}{2730}$	$-\frac{717389}{15373800}$	$\frac{7287808}{298935}$	$\frac{488724967}{5124600}$
11	0	0	10	0	4	−26	22	62	−34	$\frac{17623}{1537380}$	$-\frac{164897}{7020}$	$-\frac{11300063}{242137350}$	$\frac{9022976}{384345}$	$\frac{7698228439}{80712450}$
12	0	0	11	0	2	−25	17	67	−35	$\frac{29654}{2690415}$	$-\frac{277471}{12285}$	$-\frac{50876381}{1089618075}$	$\frac{60731392}{2690415}$	$\frac{34659852568}{363206025}$
13	0	0	12	0	0	−24	12	72	−36	$\frac{177376}{16763355}$	$-\frac{1659704}{76545}$	$-\frac{50944616}{1089618075}$	$\frac{363266048}{16763355}$	$\frac{34706456128}{363206025}$
14	0	1	0	1	22	−27	63	9	−21	$\frac{813447}{68024320}$	$-\frac{22719339}{931840}$	$-\frac{16252737}{340121600}$	$\frac{414392319}{17006080}$	$\frac{33176766573}{340121600}$
15	0	1	1	1	20	−26	58	14	−22	$\frac{97529}{8503040}$	$-\frac{2735893}{116480}$	$-\frac{5969773}{127545600}$	$\frac{49901633}{2125760}$	$\frac{4066306019}{42515200}$
16	0	1	2	1	18	−25	53	19	−23	$\frac{29654}{2690415}$	$-\frac{277471}{12285}$	$-\frac{717389}{15373800}$	$\frac{60731392}{2690415}$	$\frac{488724967}{5124600}$
17	0	1	3	1	16	−24	48	24	−24	$\frac{37957}{3587220}$	$-\frac{355163}{16380}$	$-\frac{717389}{15373800}$	$\frac{19433984}{896805}$	$\frac{488724967}{5124600}$
18	0	1	4	1	14	−23	43	29	−25	$\frac{54563}{5380830}$	$-\frac{510547}{24570}$	$-\frac{717389}{15373800}$	$\frac{55872512}{2690415}$	$\frac{488724967}{5124600}$
19	0	1	5	1	12	−22	38	34	−26	$\frac{104381}{10761660}$	$-\frac{976699}{49140}$	$-\frac{717389}{15373800}$	$\frac{53443072}{2690415}$	$\frac{488724967}{5124600}$
20	0	1	6	1	10	−21	33	39	−27	$\frac{8303}{896805}$	$-\frac{77692}{4095}$	$-\frac{717389}{15373800}$	$\frac{17004544}{896805}$	$\frac{488724967}{5124600}$
21	0	1	7	1	8	−20	28	44	−28	$\frac{94891}{10761660}$	$-\frac{887909}{49140}$	$-\frac{717389}{15373800}$	$\frac{48584192}{2690415}$	$\frac{488724967}{5124600}$
22	0	1	8	1	6	−19	23	49	−29	$\frac{6439}{768690}$	$-\frac{60251}{3510}$	$-\frac{717389}{15373800}$	$\frac{6593536}{384345}$	$\frac{488724967}{5124600}$
23	0	1	9	1	4	−18	18	54	−30	$\frac{3163}{398580}$	$-\frac{29597}{1820}$	$-\frac{1256617}{26904150}$	$\frac{1619456}{99645}$	$\frac{856078901}{8968050}$
24	0	1	10	1	2	−17	1	59	−31	$\frac{20164}{2690415}$	$-\frac{188681}{12285}$	$-\frac{210227}{4484025}$	$\frac{41295872}{2690415}$	$\frac{143219956}{1494675}$
25	0	1	11	1	0	−16	8	64	−32	$\frac{4880}{691821}$	$-\frac{45664}{3159}$	$-\frac{51490496}{1089618075}$	$\frac{9994240}{691821}$	$\frac{35079284608}{363206025}$
26	0	2	0	2	20	−18	54	6	−18	$\frac{32157}{4251520}$	$-\frac{951849}{58240}$	$-\frac{828291}{21257600}$	$\frac{17361189}{1062880}$	$\frac{1709993799}{21257600}$
27	0	2	1	2	18	−17	49	11	−19	$\frac{71167}{9565920}$	$-\frac{671393}{43680}$	$-\frac{2177573}{47829600}$	$\frac{36736439}{2391480}$	$\frac{1485381019}{15943200}$
28	0	2	2	2	16	−16	44	16	−20	$\frac{75911}{10761660}$	$-\frac{710329}{49140}$	$-\frac{717389}{15373800}$	$\frac{38866432}{2690415}$	$\frac{488724967}{5124600}$
29	0	2	3	2	14	−15	39	21	−21	$\frac{11861}{1793610}$	$-\frac{110989}{8190}$	$-\frac{717389}{15373800}$	$\frac{12145664}{896805}$	$\frac{488724967}{5124600}$
30	0	2	4	2	12	*14	34	26	−22	$\frac{66421}{10761660}$	$-\frac{621539}{49140}$	$-\frac{717389}{15373800}$	$\frac{34007552}{2690415}$	$\frac{488724967}{5124600}$
31	0	2	5	2	10	−13	29	31	−23	$\frac{15419}{2690415}$	$-\frac{144286}{12285}$	$-\frac{717389}{15373800}$	$\frac{31578112}{2690415}$	$\frac{488724967}{5124600}$
32	0	2	6	2	8	−12	24	36	−24	$\frac{2711}{512460}$	$-\frac{25369}{2340}$	$-\frac{717389}{15373800}$	$\frac{1388032}{128115}$	$\frac{488724967}{5124600}$
33	0	2	7	2	6	−11	19	41	−25	$\frac{26093}{5380830}$	$-\frac{244177}{24570}$	$-\frac{717389}{15373800}$	$\frac{26719232}{2690415}$	$\frac{488724967}{5124600}$
34	0	2	8	2	4	−10	14	46	−26	$\frac{47441}{10761660}$	$-\frac{443959}{49140}$	$-\frac{1266107}{26904150}$	$\frac{24289792}{2690415}$	$\frac{862560571}{8968050}$
35	0	2	9	2	2	−9	9	51	−27	$\frac{1186}{298935}$	$-\frac{11099}{1365}$	$-\frac{649661}{13452075}$	$\frac{2428928}{298935}$	$\frac{442623208}{4484025}$
36	0	2	10	2	0	−8	4	56	−28	$\frac{9488}{2690415}$	$-\frac{88792}{12285}$	$-\frac{6081464}{121068675}$	$\frac{19431424}{2690415}$	$\frac{4143774112}{40356225}$
37	0	3	0	3	18	−9	45	3	−15	$\frac{979}{151840}$	$-\frac{15783}{2080}$	$-\frac{520991}{5314400}$	$\frac{287723}{37960}$	$\frac{1036155459}{5314400}$
38	0	3	1	3	16	−8	40	8	−16	$\frac{1186}{298935}$	$-\frac{9734}{1365}$	$-\frac{666577}{11957400}$	$\frac{2129993}{298935}$	$\frac{450312731}{3985800}$
39	0	3	2	3	14	−7	35	13	−17	$\frac{16603}{5380830}$	$-\frac{155387}{24570}$	$-\frac{717389}{15373800}$	$\frac{17001472}{2690415}$	$\frac{488724967}{5124600}$
40	0	3	3	3	12	−6	30	18	−18	$\frac{9487}{3587220}$	$-\frac{88793}{16380}$	$-\frac{717389}{15373800}$	$\frac{4857344}{896805}$	$\frac{488724967}{5124600}$

Table 3B

No	c_{12}	r_{13}	r_{14}	r_{15}	r_{16}	r_{17}	r_{18}	r_{19}
1	$\frac{15526391371}{17006080}$	$-\frac{9437391}{64}$	$\frac{17700849}{18688}$	$\frac{2560137273}{18688}$	$\frac{13995585}{9344}$	$-\frac{94351311}{32}$	$\frac{396771455507}{18688}$	$\frac{242901}{6}$
2	$\frac{698476330861}{765273600}$	$-\frac{9175059}{64}$	$\frac{5769580671}{6073600}$	$\frac{806242411719}{6073600}$	$\frac{13443705}{9344}$	$-\frac{11923533261}{4160}$	$\frac{12536216937901}{6073600}$	$\frac{82123}{8}$
3	$\frac{1753920256}{1921725}$	$-\frac{8912895}{64}$	$\frac{45091197}{47450}$	$\frac{195468057331}{1518400}$	$\frac{100986}{73}$	$-\frac{5791286613}{2080}$	$\frac{856233992563}{427050}$	$\frac{5045113}{576}$
4	$\frac{1753920256}{1921725}$	-135168	$\frac{45091197}{47450}$	$\frac{5920428083}{47450}$	$\frac{97044}{73}$	$-\frac{1405222977}{520}$	$\frac{831017117113}{427050}$	$\frac{564209}{72}$
5	$\frac{1753920256}{1921725}$	-131072	$\frac{45091197}{47450}$	$\frac{5733427633}{47450}$	$\frac{93175}{73}$	$-\frac{170328064}{65}$	$\frac{805800241663}{427050}$	$\frac{63145}{9}$
6	$\frac{1753920256}{1921725}$	-126976	$\frac{45091197}{47450}$	$\frac{5547328733}{47450}$	$\frac{89379}{73}$	$-\frac{165003264}{65}$	$\frac{780583366213}{427050}$	$\frac{56584}{9}$
7	$\frac{1753920256}{1921725}$	-122880	$\frac{45091197}{47450}$	$\frac{5362083933}{47450}$	$\frac{85656}{73}$	$-\frac{159678464}{65}$	$\frac{755366490763}{427050}$	$\frac{50752}{9}$
8	$\frac{1753920256}{1921725}$	-118784	$\frac{45091197}{47450}$	$\frac{5177645783}{47450}$	$\frac{82006}{73}$	$-\frac{154353664}{65}$	$\frac{730149615313}{427050}$	$\frac{45568}{9}$
9	$\frac{1753920256}{1921725}$	-114688	$\frac{45091197}{47450}$	$\frac{4993966833}{47450}$	$\frac{78429}{73}$	$-\frac{149028864}{65}$	$\frac{704932739863}{427050}$	$\frac{40960}{9}$
10	$\frac{1753920256}{1921725}$	-110592	$\frac{45091197}{47450}$	$\frac{4810999633}{47450}$	$\frac{74925}{73}$	$-\frac{143704064}{65}$	$\frac{75523990207}{47450}$	4096
11	$\frac{110495761408}{121068675}$	-106496	$\frac{22547971}{23725}$	$\frac{20829137671}{213525}$	$\frac{71494}{73}$	$-\frac{138379264}{65}$	$\frac{2945246472881}{1921725}$	$\frac{299008}{81}$
12	$\frac{994408404992}{1089618075}$	-102400	$\frac{203012404}{213525}$	$\frac{180104058404}{1921725}$	$\frac{68136}{73}$	$-\frac{133054464}{65}$	$\frac{25485952034044}{17295525}$	$\frac{2428928}{729}$
13	$\frac{994268659712}{1089618075}$	-98304	$\frac{203119264}{213525}$	$\frac{172769029664}{1921725}$	$\frac{1750976}{1971}$	$-\frac{127729664}{65}$	$\frac{24464705262304}{17295525}$	$\frac{2195456}{729}$
14	$\frac{5955436617}{6540800}$	$-\frac{7076403}{64}$	$\frac{5904018639}{6073600}$	$\frac{599824795671}{6073600}$	$\frac{9028665}{9344}$	$-\frac{9186435909}{4160}$	$\frac{9665374122909}{6073600}$	$-\frac{232497}{8}$
15	$\frac{29093314609}{31886400}$	$-\frac{6815583}{64}$	$\frac{723551697}{759200}$	$\frac{146727692541}{1518400}$	$\frac{1098279}{1168}$	$-\frac{4427446473}{2080}$	$\frac{1163526943107}{759200}$	$-\frac{210759}{64}$
16	$\frac{1753920256}{1921725}$	$-\frac{819201}{8}$	$\frac{45091197}{47450}$	$\frac{17667353657}{189800}$	$\frac{65508}{73}$	$-\frac{1064433567}{520}$	$\frac{629282113513}{427050}$	$\frac{4096}{9}$
17	$\frac{1753920256}{1921725}$	-98304	$\frac{45091197}{47450}$	$\frac{4237424033}{47450}$	$\frac{62223}{73}$	$-\frac{127729599}{65}$	$\frac{604065238063}{427050}$	$\frac{4096}{9}$
18	$\frac{1753920256}{1921725}$	-94208	$\frac{45091197}{47450}$	$\frac{4058537533}{47450}$	$\frac{59011}{73}$	$-\frac{122404864}{65}$	$\frac{578848362613}{427050}$	$\frac{4096}{9}$
19	$\frac{1753920256}{1921725}$	-90112	$\frac{45091197}{47450}$	$\frac{3880125533}{47450}$	$\frac{55872}{73}$	$-\frac{117080064}{65}$	$\frac{553631487163}{427050}$	$\frac{4096}{9}$
20	$\frac{1753920256}{1921725}$	-86016	$\frac{45091197}{47450}$	$\frac{3702140583}{47450}$	$\frac{52806}{73}$	$-\frac{111755264}{65}$	$\frac{528414611713}{427050}$	$\frac{4096}{9}$
21	$\frac{1753920256}{1921725}$	-81920	$\frac{45091197}{47450}$	$\frac{3524535233}{47450}$	$\frac{49813}{73}$	$-\frac{106430464}{65}$	$\frac{503197736263}{427050}$	$\frac{4096}{9}$
22	$\frac{1753920256}{1921725}$	-77824	$\frac{45091197}{47450}$	$\frac{3347262033}{47450}$	$\frac{46893}{73}$	$-\frac{101105664}{65}$	$\frac{477981287863}{427050}$	$\frac{4096}{9}$
23	$\frac{12276227072}{13452075}$	-73728	$\frac{22566951}{23725}$	$\frac{1585139139}{23725}$	$\frac{44046}{73}$	$-\frac{95780864}{65}$	$\frac{226382825429}{213525}$	$\frac{4096}{9}$
24	$\frac{4090456064}{4484025}$	-69632	$\frac{22628636}{23725}$	$\frac{1496773004}{23725}$	$\frac{41272}{73}$	$-\frac{90456064}{65}$	$\frac{213775583444}{213525}$	$\frac{4096}{9}$
25	$\frac{993150697472}{1089618075}$	-65536	$\frac{203974144}{213525}$	$\frac{114088799744}{1921725}$	$\frac{115712}{219}$	$-\frac{85131264}{65}$	$\frac{16294731088384}{17295525}$	$\frac{327680}{729}$
26	$\frac{4929633903}{5314400}$	$-\frac{4729023}{64}$	$\frac{303973317}{379600}$	$\frac{120899641527}{1518400}$	$\frac{427923}{584}$	$-\frac{3101274621}{2080}$	$\frac{403184182527}{379600}$	$\frac{12982977}{64}$
27	$\frac{10939590509}{11957400}$	$-\frac{278613}{4}$	$\frac{88089849}{94900}$	$\frac{3069622593}{47450}$	$\frac{80865}{146}$	$-\frac{725009157}{520}$	$\frac{95037283919}{94900}$	$\frac{243527}{8}$
28	$\frac{1753920256}{1921725}$	-65535	$\frac{45091197}{47450}$	$\frac{2802108983}{47450}$	$\frac{35943}{73}$	$-\frac{85132824}{65}$	$\frac{402330234463}{427050}$	$\frac{4096}{9}$
29	$\frac{1753920256}{1921725}$	-61440	$\frac{45091197}{47450}$	$\frac{2627445533}{47450}$	$\frac{33315}{73}$	$-\frac{79806984}{65}$	$\frac{377113359013}{427050}$	$\frac{4096}{9}$
30	$\frac{1753920256}{1921725}$	-57344	$\frac{45091197}{47450}$	$\frac{2452829533}{47450}$	$\frac{30760}{73}$	$-\frac{74481664}{65}$	$\frac{351896483563}{427050}$	$\frac{4096}{9}$
31	$\frac{1753920256}{1921725}$	-53248	$\frac{45091197}{47450}$	$\frac{2278260983}{47450}$	$\frac{28278}{73}$	$-\frac{69156864}{65}$	$\frac{326679608113}{427050}$	$\frac{4096}{9}$
32	$\frac{1753920256}{1921725}$	-49152	$\frac{45091197}{47450}$	$\frac{2103692433}{47450}$	$\frac{25869}{73}$	$-\frac{63832064}{65}$	$\frac{301462732663}{427050}$	$\frac{4096}{9}$
33	$\frac{1753920256}{1921725}$	-45056	$\frac{45091197}{47450}$	$\frac{1929076433}{47450}$	$\frac{23533}{73}$	$-\frac{58507264}{65}$	$\frac{276249700663}{427050}$	$\frac{4096}{9}$
34	$\frac{12266509312}{13452075}$	-40960	$\frac{22737771}{23725}$	$\frac{877204119}{23725}$	$\frac{21270}{73}$	$-\frac{53182464}{65}$	$\frac{125520277409}{213525}$	$\frac{4096}{9}$
35	$\frac{12232497152}{13452075}$	-36864	$\frac{23122116}{23725}$	$\frac{789843924}{23725}$	$\frac{19080}{73}$	$-\frac{47857664}{65}$	$\frac{112917647564}{213525}$	$\frac{4096}{9}$
36	$\frac{109612187648}{121068675}$	-32768	$\frac{22945056}{23725}$	$\frac{6321143456}{213525}$	$\frac{16960}{73}$	$-\frac{42532864}{65}$	$\frac{902889080416}{1921725}$	$\frac{32768}{81}$
37	$\frac{1080322703}{1328600}$	$-\frac{286011}{8}$	$\frac{184862007}{94900}$	$-\frac{10393228179}{189800}$	$-\frac{128061}{146}$	$-\frac{322533171}{520}$	$\frac{48967190217}{94900}$	$-\frac{5395617}{4}$
38	$\frac{1337851283}{1494675}$	-32589	$\frac{53461377}{47450}$	$\frac{662000103}{47450}$	$\frac{27}{73}$	$-\frac{41186259}{65}$	$\frac{22181098787}{47450}$	-239431
39	$\frac{1753920256}{1921725}$	-28680	$\frac{45091197}{47450}$	$\frac{1230137933}{47450}$	$\frac{12291}{73}$	$-\frac{37200264}{65}$	$\frac{175378355413}{427050}$	$\frac{4096}{9}$
40	$\frac{1753920256}{1921725}$	-24576	$\frac{45091197}{47450}$	$\frac{1055901533}{47450}$	$\frac{10320}{73}$	$-\frac{31879104}{65}$	$\frac{150161479963}{427050}$	$\frac{4096}{9}$