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The hydrodynamics of quantum spacetime – The minimal essentials of a new quantum theory

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Abstract.

This is a somewhat long and extended abstract of a paper that presents a relatively short and concise review of a new quantum mechanics. This new theory is anchored in the hydrodynamical paradigm first introduced by L. Prandtl in his famous boundary layer theory. In addition the original ideas of L. Prandtl are expanded to encompass and combine with ideas from von Neumann-Connes' pointless noncommutative geometry, Penrose-like fractal tiling cosmology, E-infinity Cantorian theory and the platonic golden mean number system based transfinite set theory. Proceeding in this way it is reasoned that while the pre-quantum particle and the pre-quantum wave may be best described as a multi dimensional zero set and empty set respectively in stringent mathematical terms, in physical terms however the new picture of a bluff body modelling the quantum particle and a surrounding Prandtl boundary layer modelling the quantum wave is virtually a quantum jump in our understanding of quantum physics in general and quantum wave collapse in particular. In that respect the work has some resemblance to that pilot wave theory of de Broglie and Bohm but is by no means more than that. The work is naturally connected to very specialized hydrodynamics related fields apart of Batchelor's law and the important earthquake engineering subject of liquefaction which is of paramount importance for designing buildings with high resistance to earthquakes among other things. The concerted use of all these mathematical, experimental and number theoretical tools combine in the present paper to give a new synthesis for a deeper understanding of what we label the classical and quantum world predominantly for simplicity rather than logical, mandatory reasons.

Keywords: Hydrodynamics of quantum spacetime, new quantum theory.

1. Introduction

The connection between fractal Cantorian spacetime and hydrodynamics [1-20] is a profound subject [16-17] which was initiated sometime ago [21-27] in numerous but relatively unsung papers by E-infinity active researchers, notably Prof. M. Agop [24-25] and his schools in Germany and Romania [28-33] as well as the school of Prof. T. Kapitaniak [33-37] in Poland, Prof. Ji-Huan He and his team in China

[21],[38-39] all apart of the extensive work done on super fluids [40-43] and the contributions of the two pioneers of fractal spacetime, Prof. G. Ord in Canada [44-46] and Prof. L. Nottale in France [47] as well as the present author in the UK and Egypt [5],[16-17],[21-23],[26],[33-35],[37].

Very recently El Naschie expanded the paradigm of a Prandtl boundary layer [16-18],[48] far beyond its initial form and meaning to encompass quantum mechanics and in particular the supposedly mysterious metaphysical quantum wave collapse [16-17],[45], i.e. state vector reduction as well as the remarkable Batchelor's theory of turbulence [16-17],[45]. Even more down to earth we will touch upon the unexpected relation between all these aspects of quantum hydrodynamics and the very important real world engineering problems of liquefaction [16-17],[27]. It is also quite relevant to point out the resemblance of the present theory to the famous pilot wave theory of de Broglie and Bohm [44],[70] and stress that there are also fundamental differences between the two theories. The readers must be warned that the scope of the present paper requires the quintessential minimal review and some substantial reading around the subject which is not only recommended but is rather obligatory [1-70].

2. The ten commandments of the present theory

1. G. Cantor's set theory is both pure mathematics and down to earth reality [1-7],[9-15].
2. The universe is the set of all sets in the sense of the advanced form of transfinite set theory and its axioms [15-20].
3. In terms of M. Heidegger's "Sein und Zeit" [49], the universe is equivalent to "being" and the opposite of being is nothingness (Das Nichts) which is not a set at all so that in this case there is no set of all sets and there is no universe and there are no answers because there are no questions [49-52].
4. K. Menger-P. Urysohn's deductive dimensional theory [15],[50] is the beginning of the beginning to compute in our E-infinity Cantorian spacetime theory [1-26].
5. Alain Connes' dimensional function [4],[6],[15],[50]

$$D \equiv a + b\phi \tag{1}$$

where $a, b \in Z$ and $\phi = \left(\sqrt{5}-1\right)/2$ is the basis of all our E-infinity computation [4-5],[9],[15],[19-20].

6. El Naschie's bijection formula [19-20]

$$d_c^{(n)} = \left(1/\phi\right) \tag{2}$$

is completely equivalent to A. Connes' dimensional function where d_c a is Hausdorff dimension corresponding to a topological dimension n [15], [19,20].

7. The golden mean number system [10-20] is behind the fine tuning of the cosmos as well as the standard model of high energy elementary particles and is implicitly included in the Connes-El Naschie number system [10-20] as expressed by Connes' dimensional function and El Naschie's bijection formula [19].
8. The zero set is the set with only zeros in it [15],[50] and is fixed by the bi-dimension

$$D(\text{zero}) \equiv (0, \phi) \tag{3}$$

i.e. a topological dimension zero and a Hausdorff dimension corresponding to the zero, namely $f = \left(\sqrt{5} - 1\right)/2 = 0.61803398$

9. On the other hand the empty set is a set with no elements in it what so ever except the surfaces of the zeros (i.e. f^2) for which a random triadic Cantor set is the best example and for which we have [15],[50]

$$D(\text{empty} \equiv (-1, f^2) \quad (4)$$

Now we could go on making the empty set emptier and emptier (i.e. the Cantor set thinner and thinner) until the entire set totally disappears and that way we define mathematically Heidegger's nothingness (Das Nichts) as [49-52]

$$D(\text{nothingness}) \equiv (-\infty, f^{\infty} = 0)$$

$$\equiv (-\infty, 0) \quad (5)$$

$\equiv$ nothing not even a set is there

10. Plato-Pythagoras mathematical cosmic music [53-54] should be respected and taken seriously and this goes hand in hand with K. Gödel theorems, A. Turing thesis, L. Prandtl' boundary layer theory [16-18], Heisenberg central order [55] and a deep respect for all forms of life is fundamental [49-55].

3. The most important new results and insights

In general nothing is new under the sun. In fact the roots of the ideas presented here may be traced in principle to Leonardo da Vinci and even earlier to antiquity and the Pythagorean School in Alexandria's Egypt and earlier still in the Platonic philosophy. Never the less mathematical and experimental sophistication over the years has carried us quite substantially further still. As far as our present work is concerned, we could cite the following new results and facts as very recent advances in our understanding of nature in general and mechanics in particular:

a. Similar to the pure mathematical interpretation of quantum particles as a five dimensional zero set [2],[4],[15],[50] and quantum waves as the five dimensional empty set [15],[50] surface of this zero set we can now announce a corresponding new physical picture mirroring Bohm-de Broglie pilot wave picture [15-17],[50], namely that spacetime may be seen indeed as a plenum with quantum particles as quasi bluff body with a Prandtl-like spacetime boundary layer [16-17], [48] resembling a quantum wave [48].

b. The new Prandtl quantum layer picture leads to more hydrodynamical analogies encompassing the liquefaction analogy [27] all apart from the von Karman vortex street [16-17] as well as Batchelor's turbulence.

c. The points to be made do not stop here but reach further still to super fluids and quantum turbulence [43].

4. One plus one is frequently not equal two in advanced pure mathematics

The title of this paragraph is probably and intentionally provocative. Never the less, it is true and such mathematics are also quite useful in theoretical physics. As an example let us mention the following fact taken from text books on superstring theory and love it or loathe it, string theory may not be complete but it uncovered many useful facts and true and correct aspects of high energy particle physics. Let us just look at the following "true" equation [56-58]

$$\sum_{n=1}^{\infty} n = -\frac{1}{12} \tag{6}$$

This seemingly absurd infinite sum is of course not the straight forward summing of *n* from one to infinity but $-1/12$ is the Cesaro limit [56-58] which as well known, is used in deriving and explaining some subtle points in superstring theory.

In a somewhat similar vein the present author showed some time ago that applications of fractal counting of fractal logic to the number of elementary particles of the standard model leads to an outrageous but quite sound equation which states that the 12 messenger particles of the standard model

$$n_1 = \left| SU(3)\, SU(3)\, U(1) \right| = 12 \tag{7}$$

integers are really equal to fractal [19],[54]

$$\begin{aligned} n_2 &= \sqrt{\bar{a}_o} \\ &= 11.7082039325 \end{aligned} \tag{8}$$

where $\bar{a}_o = 137 + k_o$ is the inverse E-infinity electromagnetic fine structure constant, $k_o = f^5\left(1 - f^5\right)$, f^5 is Hardy's probability of quantum entanglement [2-3],[16-17] and $f = \left(\sqrt{5} - 1\right)/2$. As if this is not sufficiently weird for the one plus one equals two logic, this value corresponds to fourteen "fractal" elementary particles, [57-58] i.e.

$$n_3 = 14 \tag{9}$$

and consequently we have

$$n_1 = n_2 = n_3 \tag{10}$$

The author hopes equation (10) rivals that of equation (6) in weirdness.

5. Searching for meaning in number theoretical results

In E-infinity theory we never use the phrase "this is a numerical coincidence" early and even when we do not immediately find a simple rationale for why we obtained the numerical result we got, we exercise modesty, patience, gained experience and deep seated belief in the basic soundness of the old Plato-Pythagoras school. Here we give a neat example for how we arrive at the dimensionality of the by now famous fractal version of Kaluza-Klein spacetime [15],[59-61], i.e. $5+f^3$ which is basically nothing but the scale invariant Einstein space $4+f^3$ plus the extra Kaluza-Klein dimension, i.e. $\left(4+f^3\right)+1=5+f^3$. To illustrate our above point however we start here from the Gaussian integral [62]

$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi} \tag{11}$$

Since Gaussian distribution and fractal randomness of a Mauldin-William random Cantor set [2],[4],[19], seems related, albeit only from a "social" distance we may intuitively feel that the transfinite correction technique inspired by F. John operators methods used in the simplification of elastic shells equations may lead to an interesting result. With this in the back of our minds, we proceed in our by now familiar way of harmonizing the value of the integral as follows [2],[4],[19]

$$p \to 3+f^3 \tag{12}$$

which corresponds to a classical space fractal $\left(4+f^3\right)- 1=3+f^3$. The second step is neutralizing the geometrical process of taking the square root by replacing it with the much simpler middle point value, i.e. dividing by 2. This leads to a new value that is identical to $5+f^3$ as we hoped based on intuition and extensive experience with arithmetic manipulation with the help of the golden mean number system of E-infinity theory [1-20]. That way one finds that

$$\frac{1}{2}\left(\int_{-\infty}^{\infty} e^{-x^2} dx\right)^4 = \frac{1}{2}(\sqrt{\pi})^4$$

$$=5+f^3 \tag{13}$$

We are well aware that the preceding analysis will be faced with a partially justified scepticism however we decided never the less to go ahead and stick our necks out for it because mathematics applied to theoretical physics is an adventure which is risky but it is worthwhile to accept such peaceful risks where the harm is really minimal and the possible gains could be maximal and this in marked contrast to dangerous world economics and political decisions taken recklessly in peace and war [15-50].

6. The ten categorical imperatives of E-infinity Cantorian spacetime theory

The author is confident that this paragraph will be understood by the reader to have been titled more to less with what the British call 'a tongue in cheek'. With that said, let us list our "Kantian" "imperatives" [63]

1. Do not say simply $D=4$ is Einstein's space-time. Say instead $D=4+f^3=4.23606...$ is Einstein's scale invariant space-time dimension [1-4],[19].

2. Do not say simply quantum particle. Say the zero set given by El Naschie-Connes bi-dimension $(0,f)$ modelling the pre-quantum particle [15],[50].

3. Do not say the quantum wave and say instead the boundary layer of the pre-quantum particle, i.e. the surface (i.e. cobordism) of the zero set $(0,f)$ and is given by the bi-dimensions of the empty set given by $(-1,f^2)$ where -1 is the deductive Menger-Urysohn empty set topological dimension and f^2 is Connes' Hausdorff dimension of the empty set corresponding to -1. This is easily found from El Naschie's bijection formulas as follows[15],[19],[50]. Since

$$d_c^{(n)}=\left(1/f\right)^{n-1} \tag{14}$$

then

$$\begin{aligned} d_c^{(-1)} &= \left(1/f\right)^{-1-1} \\ &= \left(1/f\right)^{-2} \\ &= f^2 \end{aligned} \tag{15}$$

4. Do not say wave collapse nor state vector reduction and say instead the transition from a zero set to the first empty set by transmutation [1-4],[15],[50]

$$(0,f)\rightarrow(-1,f^2) \tag{16}$$

5. Relativity has a very long history preceding the work of A. Einstein. Consequently $E=mc^2$ also has a long history [1-4]. It is imperative for E- infinity researchers to have a close look at N. Umov $E=kmc^2$ [64-65]. This formula was derived without relativity or quantum mechanics.

6. We cannot stress sufficiently that number theory and the golden mean number system is of paramount importance for physics [53-54].

7. There is far more behind $\bar{a}_o = 137 + k_o$ than meets the eye and it is probably the most important coupling not only in E-infinity theory but in the entirety of physics and consequently also chemistry and biology [66].

8. There is an important reconstruction equation for $\bar{a}_o$ which must be noted and never forgotten. This is [1-4],[15],[19],[50]

$$\bar{a}_o = (1/f)\bar{a}_1 + (\bar{a}_2 = \bar{a}_1/2) + \bar{a}_3 + \bar{a}_4 \tag{17}$$

where $\bar{a}_1 = 60,\ \bar{a}_2 = 30,\ \bar{a}_3 = (8+1) = 9,\ \bar{a}_4 = 1$. It is equally vital that [15],[50]

$$60 + 30 + 9 + 1 = 100 \tag{18}$$

i.e.

$$\sum_{n=1}^{4} \bar{\alpha}_i = 100 \tag{19}$$

should be noted as being the core degrees of freedom of our theory of dark energy [2],[15],[50].

9. The second most fundamental equation of E-infinity Cantorian spacetime theory that must be remembered is that while the bulk of E-infinity space is modeled by $|E8E8| = 496$ exceptional Lie symmetry groups [1-6], the holographic boundary of the theory is given by the Penrose fractal tiling universe [1-6], i.e.

$$|SL(2,7)| = 336 \tag{20}$$

in the integer form which must be made symplectic by compactification via the addition of 16 'tHooft renormalon $k = f^3(1 - f^3)$. Thus the exact expression is [1-6],[15-17],[50]

$$\begin{aligned} 336 + 16k &= 336 + 2.88543824 \\ &= 338.88543824 \\ &\simeq 339 \end{aligned} \tag{21}$$

10. Similar to the above, the exact bulk is not simply

$$|E8E8| = 496 \tag{22}$$

but a slightly corrected value by subtracting k^2 from 496 and finding the exact transfinite fractal value 495.9674775 [1-8],[19].

7. On a host of mostly pre-E-infinity theory amazing papers constituting a precourse to the theme of fluid mechanics-quantum mechanics and visa versa

As indicated in the title of this section we will review in a random and in a quick walk some truly unexpected well known classical papers and many nearly unknown older papers with seriously surprising and unexpected deep relations with the main subject of the present paper.

1. In 2006, i.e. fourteen years ago Prof. Ji-Huan He in Shanghai, China wrote a forward looking paper entitled 'Applications of E-infinity (i.e. a basically quantum theory) to turbulence [6],[9],[21].

The paper was published in Chaos, Solitons & Fractals and anticipated much of the present conclusions and it is one of the classics on the subject and an essential reading which we recommend [21].

2. Even earlier than Prof. He's paper above, there was a remarkable paper by Dr. A. Mary elvam [67-69] which the present author also happily accepted to publish in the journal he founded (Chaos, Solitons & Fractals) [67-69] at the time thinking that it was an interesting application of E-infinity and meteorology and did not notice the deep profound relation of our present work. It was entitled 'Cantorian fractal spacetime quantum-like chaos and scale relativity in atmospheric flow [69]. This paper was concerned with hydrodynamics and fluid turbulence and was published around 1999 [69]. Thus even though it is about a quarter of a century ago, we must belatedly thank Dr. Selvam and recommend that every E-infinity researcher read her work [67-69]. Incidentally we now realize that Dr. Selvam was the first to notice that $\bar{a}_o$ can be written in terms of the golden mean leading to the famous formula [67-69]

$$\bar{a}_o = (20)(1/f)^4 \tag{23}$$

which has proven to be so fundamental for E-infinity theory where the present author discovers that it is equal to the inverse Unruh topological temperature $T = f^4$ multiplied with the number of the Riemannian tensor components in D = 4 Einstein spacetime (i.e. 20) so that we can write [15-50].

$$\begin{aligned} \bar{a}_o &= \left(R^{(4)}\right)(1/T) \\ &= (20)(1/f)^4 \end{aligned} \tag{24}$$

Equation (24) is indeed amazing and shows that a theory can sometimes be more intelligent than the creator of the theory.

3. Space limitation precludes giving the many names that paved the road to the present work by connecting fluid mechanics and turbulence to quantum mechanics so we are forced to lump the names of these true pioneers of our science and be content to give their respective contributions in the list of references. In the very front of these names are the researchers in quantum hydrodynamics and super fluids, notably I. Procaccia in Israel [40-48], L. Nottale in France [74], G. Ord in Canada [44-46] and the very strong groups of T. Kapitaniak in Poland [[33-37] and M. Agop in Romania [28-32]. In addition we must mention a relatively newer 2017 very relevant paper by D. Wang entitled 'Wave-particle liquefaction in geotechnical engineering' published in IOP Conference Series [27].

8. Conclusion

An extreme view of the results of the present work is to say there is no classical nor quantum mechanics but there are historical developments and accidents which made us unable to see easily that both subjects are just special cases of the most general mechanics, namely that of fluids. This, as we said, may be taking things to the extreme end. However to find all these unexpected connections between turbulence, liquefaction and Prandtl's boundary layer theory on the one hand and quantum mechanics on the other is at a minimum a remarkable aspect of nature which may have been suspected by ancient philosophers of antiquity and artists of the renaissance but for sure it was never part of the classical education of modern time scientists. It is therefore understandable that we suspect that things are in a phase of momentous change that will affect all conceivable branches of knowledge.

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