



Calibration Estimation for Ratio Estimators In Stratified Sampling for Proportion Allocation

El-Sheikh A.A

Prof. of Applied Statistics and Econometrics, I.S.S.R, Cairo University
E-Mail: aham103@yahoo.co

El-Kossaly H.A

Lecturer of Applied Statistics and Mathematic
High Institute of Specific Study, Giza, High Minister of Education-Egypt
E-Mail: hrahman2004@yahoo.co

Abstract.

Calibration has established itself as an important methodological instrument in large scale production of statistics. In this paper, we propose calibration estimation for ratio estimator in stratified sampling and derive the estimator of the variance of the calibration estimation ratio estimator in stratified sampling in case proportion allocation.

Keywords: Auxiliary Information; Calibration Approach; Estimation of Variance; Proportion Allocation; Ratio Estimator; Stratified Sampling.

1. Introduction

Calibration is commonly used in survey sampling to include auxiliary information to increase the precision of the estimates of population parameter. Advantages, comments and references to earlier literature: Calibration as a linear weighting method, Calibration as a systematic way to use auxiliary information, Calibration can deal effectively with surveys where auxiliary information exists at different levels. In two-stage sampling information may exist for the first stage sampling units (the clusters) and in surveys with nonresponse.

In this paper, the main aim of this paper is propose new calibration approach estimation for ratio estimators too improve the variance estimator in stratified random sampling for case proportion allocation.

A review for ratio estimator in sampling survey has been introduced in Section (2) general settings for stratified sampling has been introduced in Section (3) we presented a new calibration approach ratio estimator of ratio estimator in stratified sampling for proportion allocation and the estimator of variance of calibration estimation for combined ratio estimator has been introduced in Section (4). Various calibration approach ratio estimators introduced in Section (5) Finally, we present the conclusion remarks in Section (6).

2. A Review for Ratio Estimator in Sampling Survey

Okafor and Lee (2000), presented some double (two phase) sampling ratio and regression estimators for the case where the population mean of the auxiliary character is not known in advance,. The relative performances of the proposed estimators are compared with the estimator proposed by Hansen and Hurwitz (1943).

Rodriguez, et al (2001), presented the problem of estimation of the population ratio for the current occasion based on the samples selected over two occasions. Expressions for optimum estimator and its variance have been derived. The proposed strategy has been compared with other sampling strategies and an empirical study is made to study the performance of the proposed strategy.

Kadilar and Cingi(2003), suggested a ratio estimator in stratified random sampling based on the Prasad (1989) estimator. Theoretically, they obtained the mean square error (MSE) for this estimator and compare it with the MSE of traditional combined ratio estimate. By this comparison, they demonstrated that proposed estimator is more efficient than combined ratio estimate in all conditions.

Yanzaizai (2006),presented results that pertaining to concerning randomized response sampling. He considered a ratio estimation procedure that is a development for some existing investigations, and then provided the suitable ratio estimator for estimating an unknown proportion of people bearing a sensitive characteristic in a given community.

Housila and Gajendra(2007), presented exponential ratio and product estimators for estimating finite population mean using auxiliary information in double sampling and analyzes their properties. These estimators are compared for their precision with simple mean per unit, usual double sampling ratio and product estimators.

Singh, et al (2008), proposed some ratio estimators for estimating the population mean of the variable under study, which make use of information regarding the population proportion possessing certain attribute. Under simple random sampling without replacement (SRSWOR) scheme, the expressions of bias and mean-squared error (MSE) up to the first order of approximation are derived.

Kadilar,et al(2009), adapted the ratio estimation using ranked set sampling, suggested by Samawi and Muttlak (1996), to the ratio estimator for the population mean, based on Prasad (1989), in simple random sampling. Theoretically, they showed that the proposed ratio estimator for the population mean is more efficient than the ratio estimator, in Prasad (1989), in all conditions.

Chao,et al (2010), proposed two ratio estimators under adaptive cluster sampling, one of which is unbiased for adaptive cluster sampling designs. The efficiencies of the estimators to existing unbiased estimators, which do not utilize the auxiliary information, for adaptive cluster sampling and the conventional ratio estimation under simple random sampling without replacement, are compared in this article.

Koyuncu and Kadilar (2010), showed that a general family of estimators for estimating population mean using known value of some population parameter(s) ,these estimators are more efficient than the classical ratio estimator and that the minimum value of the mean square error (MSE) of this family is equal to the value of MSE of regression estimator. They also proposed a new family of estimators for the stratified random sampling.

Jozani, and Johnson(2011), considered design-based estimation using ranked set sampling (RSS) in finite populations. They derived the first and second-order inclusion probabilities for an RSS design and presented two Horvitz–Thompson type estimators using these inclusion probabilities. They also developed an alternate Hansen–Hurwitz type estimator and investigate its properties.

Housila, and Ramkrishna(2011), introduced some generalized ratio and product methods of estimation for estimating the population total Y . In addition to many Sriven and Tracy (1979) estimators are shown as members of the proposed estimators. The properties of the suggested estimators have been studied and their merits are examined through numerical illustration.

[Al-Omari](#) (2012), suggested two modified ratio estimators of the population mean provided that the first or third quartiles of the auxiliary variable can be established when the mean of the auxiliary variable is known. He used the double-sampling method to estimate the mean of the auxiliary variable if it is unknown. The suggested estimators are investigated under simple random sampling (SRS) and median ranked set sampling (MRSS) schemes. The estimators using MRSS are compared to their counterparts under SRS.

Singh,et al(2013), proposed the ratio estimator for the estimation of population mean in the stratified random sampling by using the estimators in Bahl and Tuteja (1991) and Kadilar and Cingi (2003). The mean square error (MSE) equations of the proposed estimators are obtained. Also, theoretical conditions that the proposed estimators are more efficient than the other estimators have been given.

Swain (2013),considered different modified ratio and product type estimators. Their biases and mean square errors are compared and necessary conditions are derived Further, numerical illustrations are provided with the help of some natural population to compare their biases and efficiencies.

Amelia (2013), attempted the problem of estimation of the population ratio of mean in mail surveys. This problem is conducted for current occasion in the context of sampling on two occasions when there is non-response (i) on both occasions, (ii) only on the first occasion and (iii) only on the second occasion. He obtained

the loss in precision of all the estimators with respect to the estimator of the population ratio of mean when there is no non-response.

Luengo(2016),considered the problem of estimating the finite population mean on the samples selected over two occasions, when there is non-response (i) on both the occasions, (ii) only on the second occasion for both matched and unmatched portions of the sample, and (iii) only on the second occasion for unmatched portion of the sample. For the case when two auxiliary variables are positively and negatively correlated with the study variable, a double sampling ratio-cum-product estimate from the matched portion of the sample is presented. Expressions for optimum matching fraction and of the combined estimate have been derived.

Shahzad,et al(2019), proposed some estimators based on an adaptation of the estimators developed by Bahl and Tuteja(1991), and others utilizing available supplementary attributes. They have been developed a new family of ratio estimators under stratified sampling scheme alongside the non response issue, the expressions for the mean square errors (MSEs) of the adapted and proposed estimators have been determined.

3-General Settings for Stratified Sampling

Suppose the population consists of H strata with N_h units in the h^{th} stratum from which a Simple Random Sample (SRS) of size n_h is taken without replacement. Let total population size be $N = \sum_{h=1}^H N_h$ and sample size be $n = \sum_{h=1}^H n_h$, respectively. Associated with the i^{th} unit of the h^{th} stratum there are two values y_{hi} and x_{hi} with $x_{hi} > 0$ being the covariate which the i^{th} unit selected from the h^{th} stratum, where $i = 1, 2, \dots, n_h$, y_{hi} is the study variable is observed for every unit in the sample, y_{hi} is known for all $i \in s$ and the values, Assume X_{hi} denote the value of the i^{th} unit of the auxiliary variable in the h^{th} stratum about which information known at the unit level or at the stratum level X is total auxiliary variable.

For the h^{th} stratum, let $d_h = N_h / N$ be the stratum weights, $X = \sum_{h=1}^H d_h X_h$ and $Y = \sum_{h=1}^H d_h Y_h$, $f = n_h / N_h$ the sample fraction, S_h^2 the variance between stratum. $\bar{x}, \bar{y}, \bar{X}, \bar{Y}$ the x -sample, the y sample, and population mean respectively. The samples are selected independently across the strata. A sample S is drawn from stratum h of size n_h is drawn without replacement according to a probabilistic sampling plan P with inclusion probabilities $\pi_{hi} = pr(h_i \in s)$. $d_{ih} = 1/\pi_{ih}$ denote the basic design weights. Deville and Särndal (1992).

4-A New Calibration Approach of Ratio Estimator in Stratified Sampling for Proportion Allocation

El-Sheikh and Mohamed (2013)suggested the calibration approach combined ratio estimator using auxiliary information in stratified sampling for proportion allocation.

Lemma: Calibration ratio estimator under the stratified sampling for generalized distance and proportion allocation is given by:

$$\hat{y}_{CST.PA} = \sum_{h=1}^H \frac{N_{hi}}{N} \sum_{i=1}^{n_h} d_{hi} y_{hi} + \frac{\sum_{h=1}^H \frac{N_h}{N} \sum_{i=1}^{n_h} d_{hi} q_{hi} x_{hi} y_{hi}}{\sum_{h=1}^H \frac{N_h}{N} \sum_{i=1}^{n_h} d_{hi} q_{hi} x'_{hi} x_{hi}} (\bar{X} - \sum_{h=1}^H \frac{N_h}{N} \sum_{i=1}^{n_h} d_{hi} x_{hi})$$

Where: $\hat{y}_{CST.PA}$ is the calibration estimator (CAL) for stratified sampling(STS) in case of Proportion Allocation(PA).

Proof:

Consider that, $\hat{y}_{CST.PA} = \sum_{h=1}^H \frac{N_h}{N} \sum_{i=1}^{n_h} w_{hi} y_{hi}$

w_{hi} are the calibration weights for i^{th} unit in the h^{th} stratum, which are chosen to minimize Φ_s and q_{hi} 's are known positive weights unrelated to d_{hi} .

Note that $w_{hi} = d_{hi} = 0$ for $i \notin S$. The uniform weights $q_{hi} = 1$ are used in most applications, but unequal weights can also be motivated.

Chi Square distance Φ_s has the form:

$$\Phi_s = \sum_{h=1}^H \frac{N_h}{N} \sum_{i=1}^{n_h} \frac{(w_{hi} - d_{hi})^2}{d_{hi} q_{hi}}$$

is minimum subject to calibration condition

$$\sum_{h=1}^H \frac{N_h}{N} \sum_{i=1}^{n_h} w_{hi} x_{hi} = X$$

CAL using Chi Square distance Φ_s will be derived as follow:

Consider the Lagrange function

$$L(\Phi_s; w_{hi}) = \sum_{h=1}^H \frac{N_h}{N} \sum_{i=1}^{n_h} \frac{(w_{hi} - d_{hi})^2}{d_{hi} q_{hi}} - 2\lambda \left(\sum_{h=1}^H \frac{N_h}{N} \sum_{i=1}^{n_h} w_{hi} x_{hi} - X \right)$$

$$\frac{\partial L}{\partial w_{hi}} = \frac{2N_h}{N} (w_{hi} - d_{hi}) / d_{hi} q_{hi} - 2\lambda \frac{N_h}{N} x_{hi} \tag{1}$$

By equating eq(1) to zero yields,

$$w_{hi} = d_{hi} (1 + \lambda x_{hi} q_{hi}) \tag{2}$$

By substituting from (2) in (1) to get

$$\lambda = (X - \sum_{h=1}^H \frac{N_h}{N} \sum_{i=1}^{n_h} d_{hi} x_{hi}) \left(\sum_{h=1}^H \frac{N_h}{N} \sum_{i=1}^{n_h} d_{hi} q_{hi} x_{hi}' x_{hi} \right)^{-1} \tag{3}$$

So, eq(2) can be rewritten as:

$$w_{hi} = d_{hi} + \frac{d_{hi} q_{hi} x_{hi}}{\sum_{h=1}^H \frac{N_h}{N} \sum_{i=1}^{n_h} d_{hi} q_{hi} x_{hi}' x_{hi}} \left(X - \sum_{h=1}^H \frac{N_h}{N} \sum_{i=1}^{n_h} d_{hi} x_{hi} \right) \tag{4}$$

Since, $\hat{y}_{CST.PA} = \sum_{h=1}^H \frac{N_h}{N} \sum_{i=1}^{n_h} w_{hi} y_{hi}$, then

$$\hat{y}_{CST.PA} = \sum_{h=1}^H \frac{N_h}{N} \sum_{i=1}^{n_h} d_{hi} y_{hi} + \frac{\sum_{h=1}^H \frac{N_h}{N} \sum_{i=1}^{n_h} d_{hi} q_{hi} x_{hi} y_{hi}}{\sum_{h=1}^H \frac{N_h}{N} \sum_{i=1}^{n_h} d_{hi} q_{hi} x'_{hi} x_{hi}} \left(X - \sum_{h=1}^H \frac{N_h}{N} \sum_{i=1}^{n_h} d_{hi} x_{hi} \right) \quad (5)$$

Special case:

If $q_{hi} = x_{hi}^{-1}$, then eq(5) reduces to the well-known combined ratio estimator in stratified for proportion allocation

$$\hat{y}_{CST.PA} = \frac{\sum_{h=1}^H \frac{N_h}{N} \sum_{i=1}^{n_h} d_{hi} y_{hi}}{\sum_{h=1}^H \frac{N_h}{N} \sum_{i=1}^{n_h} d_{hi} x_{hi}} X$$

5-Various Calibration Approach Ratio Estimators

In this section, we suggest a calibration approach for various ratio estimators in stratified sampling for proportion allocation. Kadilar and Cingi (2003) proposed various ratio estimators in stratified sampling. Based on this work, we derived calibration approach for ratio estimators for proportion allocation for Sisodia–Dwivedi estimator (SD) (Sisodia and Dwivedi, 1981), Singh–Kakran ratio-type estimator (SK) (Singh and Kakran, 1993), Upadhyaya and Singh (1999) estimator 1 (US1) and Upadhyaya and Singh (1999) estimator 1 (US2). we proposed and summarized four ratio estimators and calibration approach ratio estimators in stratified sampling for proportion allocation in Table 1.

Table(1):Ratio Estimator and Calibration Approach Ratio Estimator for (Proportion Allocation)

Method	Ratio estimator	Calibration approach ratio estimator for proportion allocation
SD	$\bar{y}_{SD} = \bar{y} \left(\frac{\bar{X} + C_x}{\bar{x} + c_x} \right)$	$\bar{y}^*_{SD} = \left(\frac{\sum_{h=1}^H \frac{N_h}{N} \sum_{i=1}^{n_h} d_{hi} \bar{y}_h}{\sum_{h=1}^H \frac{N_h}{N} \sum_{i=1}^{n_h} d_{hi} (\bar{x}_h + C_{xh})} \right) \sum_{h=1}^H \frac{N_h}{N} \sum_{i=1}^{n_h} d_{hi} (\bar{X}_h + C_{xh})$
SK	$\bar{y}_{SK} = \bar{y} \left(\frac{\bar{X} + B_2 C_x}{\bar{x} + B_2 c_x} \right)$	$\bar{y}^*_{SK} = \left(\frac{\sum_{h=1}^H \frac{N_h}{N} \sum_{i=1}^{n_h} d_{hi} \bar{y}_h}{\sum_{h=1}^H \frac{N_h}{N} \sum_{i=1}^{n_h} d_{hi} (\bar{x}_h + B_{2h}(x))} \right) \sum_{h=1}^H \frac{N_h}{N} \sum_{i=1}^{n_h} d_{hi} (\bar{X}_h + B_{2h}(x))$
US ₁	$\bar{y}_{US_1} = \bar{y} \left(\frac{\bar{X} B_2(x) + C_x}{\bar{x} B_2(x) + C_x} \right)$	$\bar{y}^*_{US_1} = \left(\frac{\sum_{h=1}^H \frac{N_h}{N} \sum_{i=1}^{n_h} d_{hi} \bar{y}_h}{\sum_{h=1}^H \frac{N_h}{N} \sum_{i=1}^{n_h} d_{hi} (\bar{x}_h B_{2h}(x) + C_{xh})} \right) \sum_{h=1}^H \frac{N_h}{N} \sum_{i=1}^{n_h} d_{hi} (\bar{X}_h B_{2h}(x) + C_{xh})$
US ₂	$\bar{y}_{US_2} = \bar{y} \left(\frac{\bar{X}_2 C_x + B_2(x)}{\bar{x} C_x + B_2(x)} \right)$	$\bar{y}^*_{US_2} = \left(\frac{\sum_{h=1}^H \frac{N_h}{N} \sum_{i=1}^{n_h} d_{hi} \bar{y}_h}{\sum_{h=1}^H \frac{N_h}{N} \sum_{i=1}^{n_h} d_{hi} (\bar{x}_h C_{xh} + B_{2h}(x))} \right) \sum_{h=1}^H \frac{N_h}{N} \sum_{i=1}^{n_h} d_{hi} (\bar{X}_h C_{xh} + B_{2h}(x))$

We also derived and summarized estimators of variance of calibration approach combined ratio estimator for proportion allocation in Table 2.

Table (2):Estimator of variance of calibration approach combined ratio estimator in stratified sampling For proportion allocation

Method	q_{hi}	Calibration approach ratio estimator for proportion allocation
SD	$(\bar{x}_h + C_{xh})^{-1}$	$Var_c(\bar{y}_{sd}^*) = \left(\frac{\bar{X} + C_x}{\bar{x}_{st}} \right) \sum_{h=1}^H \frac{N_h}{N} \sum_{i=1}^{n_h} \frac{d^2_h(1-f_h)}{n_h} S_{eh}^2$
SK	$(\bar{x}_h + B_{2h}(x))^{-1}$	$Var_c(\bar{y}_{sk}^*) = \left(\frac{\bar{X} + B_2(x)}{\bar{x}_{st}} \right)^2 \sum_{h=1}^H \frac{N_h}{N} \sum_{i=1}^{n_h} \frac{d^2_h(1-f_h)}{n_h} S_{eh}^2$
US ₁	$(\bar{x}_h B_{2h}(x) + C_{xh})^{-1}$	$Var_c(\bar{y}_{us1}^*) = \left(\frac{\bar{X} B_2(x) + C_x}{\bar{x}_{st}} \right)^2 \sum_{h=1}^H \frac{N_h}{N} \sum_{i=1}^{n_h} \frac{d^2_h(1-f_h)}{n_h} S_{eh}^2$
US ₂	$(\bar{x}_h C_{xh} + B_{2h}(x))^{-1}$	$Var_c(\bar{y}_{us2}^*) = \left(\frac{\bar{X} C_x + B_2(x)}{\bar{x}_{st}} \right)^2 \sum_{h=1}^H \frac{N_h}{N} \sum_{i=1}^{n_h} \frac{d^2_h(1-f_h)}{n_h} S_{eh}^2$

Under condition $q_{hi} = x_{hi}^{-1}$, we claim that the Singh et al. (1998) calibration estimator of variance of the combined ratio estimator can be found by the combinations of the estimator of variance of combined ratio estimator.

From Wu (1985, p. 151) paper, we found that the MSE of the estimator of variance of the calibration approach combined ratio estimator is more efficient than the MSE of the estimator of variance of combined ratio estimator in stratified sampling. Therefore, in general, calibration approach ratio estimator is more efficient than ratio estimator in stratified sampling.

6-Concluding remarks

This paper applied calibration estimation to ratio-type estimators in stratified sampling in case proportion allocation. We proposed and studied calibration approach in four estimators to the use of complete auxiliary information to estimate ratio-type estimator in stratified sampling in case proportion allocation and derived the estimator of variance of calibration approach ratio estimators in case proportion allocation. We also showed that the estimator of variance of the combined ratio estimator in stratified sampling using the calibration approach is more efficient than the standard estimator of variance of combined ratio estimator in stratified sampling in case proportion allocation.

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