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Pi from Probability Approach

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Abstract. In this paper I introduced a new Probability mass function (Pmf) that is named as Pavan’s Pmf, then used first and second raw moments of that distribution and De Moivre-Laplace theorem for large **n** later equated probability functions of binomial and normal distribution at model value to derive the formula for Pi.

Keywords: Combination, De Moivre-Laplace theorem, Mode, Pi, Probability density function, Probability mass function.

1. INTRODUCTION

Pi is one of the remarkable constants discovered by mankind. The quest of finding digits after a decimal point was continued from ancient times. Thousands of Pi generating formulas available dueto extensive research by many mathematicians. Everyone has approached in a different way to findthe value of π, here I discovered a formula through probability approach that generates value of π.

Value of π

$$\pi = \lim_{2k\to\infty} \frac{(k+1)^2}{k}\left(\frac{\mu_1'}{\sqrt{\mu_2'}}\right)^4$$

where, μ_1', μ_2' are first and second raw moments of Pavan's Pmf respectively.

$$\mu_1' = \frac{2^k}{k+1} \quad \mu_2' = \frac{\binom{2k}{k}}{k+1}$$

2. PROOF

Let us consider Pavan's probability mass function

$p(x_i) = \frac{2}{k+1}$ if $x_i = \binom{k}{0}, \binom{k}{1}, \binom{k}{i}$……..,$i = 0,1,2,\ldots, \frac{k}{2} - 1$, Where **k** is an even number

$= \frac{1}{k+1}$ if $x_i = \binom{k}{\frac{k}{2}}, i = \frac{k}{2}$

$$\sum_{i=0}^{\frac{k}{2}} p(x_i) = \sum_{i=0}^{\frac{k}{2}-1} p(x_i) + p\left(x_{\frac{k}{2}}\right) = \frac{2}{k+1}\binom{k}{2} + \frac{1}{k+1} = 1$$

Binomial expansion formula is

$(a+b)^k = \sum_{i=0}^{k}\binom{k}{i}a^{k-i}b^i$

if a=b=1

$2^k = \sum_{i=0}^{k}\binom{k}{i}$

$E(x_i) = \mu_1' = \sum_{i=0}^{\frac{k}{2}-1} x_i p(x_i) + x_{\frac{k}{2}} p\left(x_{\frac{k}{2}}\right) = \frac{2}{k+1}\sum_{i=0}^{\frac{k}{2}-1}\binom{k}{i} + \frac{1}{k+1}\binom{k}{\frac{k}{2}}$

take out common $\frac{1}{k+1}$

$\frac{1}{k+1}\left(2\sum_{i=0}^{\frac{k}{2}-1}\binom{k}{i} + \binom{k}{\frac{k}{2}}\right) = \frac{1}{k+1}\left(\sum_{i=0}^{\frac{k}{2}-1}\binom{k}{i} + \sum_{i=0}^{\frac{k}{2}-1}\binom{k}{i} + \binom{k}{\frac{k}{2}}\right)$

Since $\binom{k}{i} = \binom{k}{k-i}$, replace $\binom{k}{i}$ with $\binom{k}{k-i}$

Then

$\frac{1}{k+1}\left(\sum_{i=0}^{\frac{k}{2}-1}\binom{k}{i} + \sum_{i=0}^{\frac{k}{2}-1}\binom{k}{i} + \binom{k}{\frac{k}{2}}\right) = \frac{1}{k+1}\left(\sum_{i=0}^{\frac{k}{2}-1}\binom{k}{i} + \sum_{i=0}^{\frac{k}{2}-1}\binom{k}{k-i} + \binom{k}{\frac{k}{2}}\right) = \frac{2^k}{k+1}$

Since $\left(\sum_{i=0}^{\frac{k}{2}-1}\binom{k}{i} + \sum_{i=0}^{\frac{k}{2}-1}\binom{k}{k-i} + \binom{k}{\frac{k}{2}}\right) = \sum_{i=0}^{k}\binom{k}{i} = 2^k$

$$E(x_i) = \mu_1' = \frac{2^k}{k+1}$$

Central binomial coefficient $\binom{2k}{k}$ can be expressed as

$$\binom{2k}{k} = \sum_{i=0}^{k}\binom{k}{i}\binom{k}{k-i}$$

Since $\binom{k}{i} = \binom{k}{k-i}$

$$\binom{2k}{k} = \sum_{i=0}^{k}\binom{k}{i}^2$$

$$E(x_i^2) = \mu_2' = \sum_{i=0}^{\frac{k}{2}-1} x_i^2 p(x_i) + x_{\frac{k}{2}}^2 p\left(x_{\frac{k}{2}}\right) = \frac{2}{k+1}\sum_{i=0}^{\frac{k}{2}-1}\binom{k}{i}^2 + \frac{1}{k+1}\binom{k}{\frac{k}{2}}^2$$

take out common $\frac{1}{k+1}$

$\frac{1}{k+1}\left(2\sum_{i=0}^{\frac{k}{2}-1}\binom{k}{i}^2 + \binom{k}{\frac{k}{2}}^2\right) = \frac{1}{k+1}\left(\sum_{i=0}^{\frac{k}{2}-1}\binom{k}{i}^2 + \sum_{i=0}^{\frac{k}{2}-1}\binom{k}{i}^2 + \binom{k}{\frac{k}{2}}^2\right) = \frac{\binom{2k}{k}}{k+1}$

Since $\left(\sum_{i=0}^{\frac{k}{2}-1}\binom{k}{i}^2 + \sum_{i=0}^{\frac{k}{2}-1}\binom{k}{k-i}^2 + \binom{k}{\frac{k}{2}}^2\right) = \sum_{i=0}^{k}\binom{k}{i}^2 = \binom{2k}{k}$

$E(x_i^2) = \mu_2' = \frac{\binom{2k}{k}}{k+1}$

$Var(x_i) = E(x_i^2) - \left(E(x_i)\right)^2 = \frac{\binom{2k}{k}}{k+1} - \frac{2^{2k}}{(k+1)^2}$

take the ratio of terms

$$R = \frac{\mu_2'}{(\mu_1')^2(k+1)} = \frac{\binom{2k}{k}}{2^{2k}} \quad (1)$$

Here $\binom{2k}{k}\frac{1}{2^{2k}}$ looks similar to binomial distribution with n=2k, calculating probability at x= k and $p = \frac{1}{2}$.

we can write like $p(x = k) = \binom{2k}{k}\frac{1}{2^{2k}}$

Probability mass function of Binomial Distribution of a random variable **X** is

$$p(x) = \binom{n}{x} p^x (1-p)^{n-x}$$

where **p** is probability of success of Bernoulli's trail and **n** is number of Bernoulli's trails.

$0 < p < 1, p + q = 1$, x = 0,1, 2,….,n. Take $p = \frac{1}{2}$, for n=2k the range of x is 0,1, 2,….,2k.

$$E(x) = np = k, \quad Var(x) = \sqrt{npq} = \sqrt{\frac{k}{2}}$$

For $p = \frac{1}{2}$ the skewness of binomial distribution is $\frac{q-p}{\sqrt{npq}} = 0$ hence the distribution is symmetric.

Since n = 2k it is an even number, then the value of $(2k+1)p = \frac{2k+1}{2} = k + \frac{1}{2}$ is a non-integer value. So, the model value Gupta (2000) is integral part of $k + \frac{1}{2}$, it means **k** is modal value.

probability of **x** at Model value is $p(x = k) = \binom{2k}{k}\left(\frac{1}{2}\right)^{2k}$

Probability density function of Normal Distribution of a random variable **X** is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$-\infty < x < \infty, -\infty < \mu < \infty, \sigma^2 > 0$$

where, μ is mean and σ^2 is variance.

De Moivre–Laplace theorem Papoulis (2002) States that as **n** grows large, for **x** in the neighbourhood of **np** we can approximate

$$\binom{n}{x} p^x (1-p)^{n-x} \approx \frac{1}{\sqrt{npq\,2\pi}} e^{-\frac{1}{2}\left(\frac{x-np}{\sqrt{npq}}\right)^2}$$

in the sense that the ratio of the left-hand side to the right-hand side converges to **1** as $n \to \infty$

for n=2k

$$\binom{2k}{x} p^x (1-p)^{2k-x} \approx \frac{1}{\sqrt{2kpq\,2\pi}} e^{-\frac{1}{2}\left(\frac{x-2kp}{\sqrt{2kpq}}\right)^2}$$

Substitute $p = \frac{1}{2}$

$$\binom{2k}{x}\frac{1}{2^{2k}} \approx \frac{1}{\sqrt{k\pi}} e^{-\frac{1}{2}\left(\frac{x-k}{\sqrt{\frac{k}{2}}}\right)^2}$$

at modal value **k**

$$\binom{2k}{k}\frac{1}{2^{2k}} \approx \frac{1}{\sqrt{k\pi}}$$

$$\sqrt{k}\binom{2k}{k}\frac{1}{2^{2k}} \approx \frac{1}{\sqrt{\pi}} \quad (2)$$

Substitute (1) in (2) it results into

$$\sqrt{k}\frac{\mu_2'}{(\mu_1')^2(k+1)} \approx \frac{1}{\sqrt{\pi}}$$

$$\sqrt{\pi} \approx \frac{k+1}{\sqrt{k}}\frac{(\mu_1')^2}{\mu_2'}$$

Squaring on both the sides

$$\pi \approx \frac{(k+1)^2}{k}\left(\frac{\mu_1'}{\sqrt{\mu_2'}}\right)^4$$

$$\pi = \lim_{2k\to\infty}\frac{(k+1)^2}{k}\left(\frac{\mu_1'}{\sqrt{\mu_2'}}\right)^4$$

RESULT

Value of π

$$\pi = \lim_{2k\to\infty}\frac{(k+1)^2}{k}\left(\frac{\mu_1'}{\sqrt{\mu_2'}}\right)^4$$

where, μ_1', μ_2' are first and second raw moments of Pavan's Pmf respectively.

$$\mu_1' = \frac{2^k}{k+1} \qquad \mu_2' = \frac{\binom{2k}{k}}{k+1}$$

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