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# A New Technique to Solve the Instant Insanity Problem with 5 Cubes 

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#### Abstract

Instant Insanity consists of four cubes, each of whose six faces are colored with one of the four colors: red, blue, white, and green. The object is to stack the cubes in such a way that each of the four colors appears on each side of the resulting column.

Traditionally, this could be solved using graph theory. We solved this problem without using graph theory [1]. However, in this article, we introduce a new technique to solve the problem without using graph theory by adding another cube resulting in a puzzle with five cubes instead of four cubes.


Keywords: Mathematical Puzzles.

## Introduction



Fig 1. Visual representation of the original Instant Insanity Problem


Fig 2. Visual repersentation of the Modified Instant Insanity Problem

This problem is practically impossible to solve by means of trial and error, since there are 497,664 arrangements possible in total. Each cube has 24 arrangements possible therefore there will be $24^{5}$ possible arrangements (order of the cube does not matter). However, not all 24 arrangements will count because there are duplicate solutions. Any of the solution will appear 16 times. Therefore the total will be $24^{5} / 16=497664$


Fig 3. Opened \& Flattened Representation

## Method

In this example, to illustrate the method we have in mind, we first need to imagine that the cube has been opened and flattened. We have five different cubes colored as above (figure 3). There are five steps to complete the stack that satisfies the conditions for Instant Insanity.

## STEP 1

Count the total number of each color among the five cubes:
Red: 6

Blue: 6

Green: 6
White: 6
Yellow: 6

## STEP 2



Fig 4. The schematics for the complete stack and individual cube
Since we want to complete the stack while each of the four colors appear on each side of the resulting column, we can draw a conclusion that we need a total of:

4 reds, 4 blues, 4 greens, 4 whites, and 4 yellows on the column sides.
Now, we count the colors to be hidden by subtracting 4 from the total number of each color.
For instance, there are 6 reds in total. Among those 6 reds, we only need 4 of them to be placed on the column sides. Thus the remaining 2 reds must be hidden by placing them either on the top or the bottom of the cube.

Therefore, by applying the same logic to the rest, the top and the bottom must consist of:
2 Reds (6-4 = 3)
2 Blues $(6-4=1)$
2 Greens (6-4 = 2)
2 Whites $(6-4=2)$


Fig 5. The Top and Bottom Color Pairs For Cube 1

We can notice that if Green is designated as the top, then Yellow is assigned to be the bottom - Green and yellow are paired (G-Y).

Likewise, there are two other possible pairs that exist on the figure 4.
Red \& White (R-W), White \& Blue (W-B)
These top and bottom color pairs are our primary focus; once top and bottom pair is selected, the colors on the sides are automatically fixed.

For instance, if we decide $\mathrm{G}-\mathrm{Y}$ to be the top and bottom pair, the remaining $\mathrm{R}, \mathrm{W}, \mathrm{W}$, and B will be placed on the sides.

## STEP 3

Now we make a chart that consist of the top and the bottom pairs for each cube which fulfill the designated values produced in step 2

|  | T-B pairs | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |
| Cube 1 | G-Y | X | X |  |  |
|  | R-W |  |  | X |  |
|  | W-B |  |  |  | X |
|  |  |  |  |  |  |
| Cube 2 | W-R | X | X | X |  |
|  | Y-B |  |  |  | X |
|  | G-G |  |  |  |  |
|  |  |  |  |  |  |
| Cube 3 | Y-B |  |  |  |  |
|  | W-R | X |  |  | X |
|  | G-Y |  | X | X |  |
|  |  |  |  |  |  |
| Cube 4 | W-Y |  |  |  |  |
|  | B-B | X | X | X |  |
|  | G-R |  |  |  | X |
|  |  |  |  |  |  |
| Cube 5 | R-W |  | X |  |  |
|  | R-B |  |  |  |  |
|  | Y-G | X |  | X | X |

Note: each arrangementmust satisfy the conditions from STEP 2; hiding 2 reds, 2 blue, 2 green, 2 whites, and 2 yellow in total

## STEP 4

Using the top \& bottom arrangements from step 3, we construct the front-back-left-right color table.

|  | Arrangement 1 |  |  |  |  |  | Arrangement2 |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Front | Back | Left | Right |  |  | Front | Back | Left | Right |  |  |
| Cube 1 | W | B | R | W |  | Cube 1 | W | B | R | W |  |  |
| Cube 2 | G | G | Y | B |  | Cube 2 | G | G | Y | B |  |  |
| Cube 3 | G | Y | Y | B |  | Cube 3 | R | W | Y | B |  |  |
| Cube 4 | G | R | W | Y |  | Cube 4 | G | R | W | Y |  |  |
| Cube 5 | B | R | R | W |  | Cube 5 | Y | G | R | B |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | Front | Back | Left | Right |  |  | Front | Back | Left | Right |  |  |
| Cube 1 | W | B | G | Y |  | Cube 1 | W | R | Y | G |  |  |
| Cube 2 | G | G | Y | B |  | Cube 2 | G | G | R | W |  |  |
| Cube 3 | R | W | Y | B |  | Cube 3 | G | Y | B | Y |  |  |
| Cube 4 | G | R | W | Y |  | Cube 4 | B | B | W | Y |  |  |
| Cube 5 | B | R | R | W |  | Cube 5 | B | R | R | W |  |  |

## STEP 5

Now we verify each arrangement one by one.
Initially, none of the arrangements above seem to work, however, we must consider the fact that each cube can rotate in multiple directions while keeping the top and bottom pair unchanged.

For instance, we can simply switch the color between front \& back; left \& right by rotating the cube by 180 degrees. Also, if we rotate the cube 90 degrees, we can switch the color between front $\&$ left; back \& right. We can also perform the combination of both 90 and 180 degree rotation.

After the verification process, we can relocate the arrangement \#4 as below, which is the solution for the problem.

|  | Arrangement4 |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Front | Back | Left | Right |
| Cube 1 | W | R | Y | G |
| Cube 2 | G | G | R | W |
| Cube 3 | G | Y | B | Y |
| Cube 4 | B | B | W | Y |
| Cube 5 | B | R | R | W |


|  | Arrangement4 |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Front | Back | Left | Right |
| Cube 1 | Y | G | W | R |
| Cube 2 | R | W | G | G |
| Cube 3 | G | Y | B | Y |
| Cube 4 | B | B | Y | W |
| Cube 5 | W | R | R | B |

We can easily eliminate non-working arrangement by looking at only 2 columns at a time (either front \& back or left \& right).

First, try to switch the colors in such a way that there are 2 reds, 2 blues, 2 whites, and 2 greens within 2 columns because that is what we are trying to achieve eventually. After a few switch-overs, we can easily notice that certain arrangements are impossible to achieve that allocation, meaning they are not the solution. Then simply move on to the next arrangement and continue the verification process

Unlike the original four-cube variant of Instant Insanity, the five-cube Instant Insanity can have more than one solutions possible. Therefore, one must complete the verification process on every arrangement thoroughly. Nevertheless, in this particular example, we were able to find the only one arrangement as a solution.

The Perl programming language was used to implement the new approach for instant insanity with 5 cubes.

```
STC149-C5:Desktop jgeorge$ perl instant_instanity_5_cubes.pl
```



```
When entering the colors enter only the first letter corresponding to each color and in the order of left, front, right, back, top, bottom.
R/r: red
\(B / b\) : blue
G/g: green
W/w: white
Y/y: Yellow
Enter in the colors for cube 1: wyrgwb
Enter in the colors for cube 2
brywgg
Enter in the colors for cube 3:
rbwygy
Enter in the colors for cube 4:
bybwgr
Enter in the colors for cube 5:
bwrryg
The top and bottom must consist of this many Reds: 2
The top and bottom must consist of this many Greens: 2
The top and bottom must consist of this many Whites: 2
The top and bottom must consist of this many Blues: 2
The top and bottom must consist of this many Yellow: 2
Found 224 solutions (28 unique solutions)
```


## Program Output:

Print out of 1 of the 28
solutions
**********
wyrg
grgw
bgyy
ybwb
rwbr
**********

## References

[1] A New Technique to Solve the Instant Insanity Problem, Hwee Jung Kim, Julie George \& Salar Alsardary, Journal of Advances in Mathemartics, Vol. 11, No. 10 (2016).
[2] How to play Instant Insanity [Instant Insanity math forum]. (2004, January 15). Retrieved March 28, 2012, from Department of Math, Cornell University website: http://www.math.cornell.edu/~mec/ 2003-2004/graphtheory/ii/howtoplayinstantinsanity.html
[3] Puzzle Locurainstantánea (Instant Insanity)(2011, December 13),Garrido, E. P. B. G. Retrieved from
http://divulgamat2.ehu.es/divulgamat15/index.php?option=com_content\&view=article\&id=1 3498:40-puzzle-locura-instantanea-instant-insanity-de-p apiroflexia\&catid=65:papiroflexia-ymatemcas\&directory $=6$
[4] Instant Insanity Instructor's Guid. (2006, January 1). Retrieved March 28, 2012, from http://campus.houghton.edu/webs/employees/kcamenga/Teacher\ Resources/Instant\ In sanity\%20teachers\%20guide.pdf.

