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Proposed Nonparametric Tests Using Moses Test For Location and Scale Testing

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ABSTRACT

Three nonparametric tests are proposed for the simple tree alternative to test for differences in location and/or scale. These tests are combinations of the Fligner-Wolfe test and a modified Moses test. A simulation study is conducted to determine how well the proposed tests maintain their significance levels. Powers are also estimated for the proposed tests under a variety of conditions for three and four populations. Three different types of variable parameters vectors are considered with each vector containing a location and a scale parameter. The first type of parameter vectors considered include different location parameters and equal scale parameters. The second type include different scale parameters and equal location parameters, and the third type include both different location parameters and different scale parameters. Results are given as far as which test does better under certain conditions.

Keywords: Completely Randomized Design, Location-Scale problem, Moses Test, Power.

1. INTRODUCTION

Researchers sometimes find themselves in a situation where they want to compare one or more treatments with a standard or control treatment. In these cases, the simple tree hypothesis may be the most appropriate hypothesis (Conroy, 2011). The simple tree alternative for location is given by:

$$H_0: \mu_1 = \mu_2 = \cdots = \mu_k \quad (1)$$

$$H_\alpha: \mu_1 \leq [\mu_2, \dots, \mu_k] \text{ (At least one strict inequality)}$$

where μ_i is the location parameter of population i.

There may be some situations in which a treatment not only may affect the location but also may affect the variance (or scale) of a distribution (Marozzi, 2013). The treatment may affect one or the other or both simultaneously.

The most common test for the two-sample location-scale problem is the Lepage test (Lepage, 1971). This test is based on a combination of the Mann-Whitney test (Mann and Whitney, 1947) and the Ansari-Bradley test (Ansari and Bradley, 1960). The null hypothesis and alternative hypothesis are given below:

$$H_0: \mu_1 = \mu_2 \text{ and } \sigma_1 = \sigma_2 \quad (2)$$
$$H_\alpha: \mu_1 \neq \mu_2 \text{ and / or } \sigma_1 \neq \sigma_2$$

where μ_i and σ_i are the location and scale parameters of population i, respectively.

Alsubie and Magel (2020) extended the hypotheses test given in (2) to the hypotheses test given in (3). This research was concerned with testing the hypothesis for the simple tree alternative:

$$H_0: \mu_1 = \mu_2 = \cdots = \mu_k, \quad (3)$$
$$H_0: \sigma_1 = \sigma_2 = \cdots = \sigma_k, \text{ versus}$$
$$H_\alpha: \mu_1 \leq [\mu_2, \ldots, \mu_k] \text{ and / or}$$
$$H_\alpha: \sigma_1 \leq [\sigma_2, \ldots, \sigma_k] \text{(At least one inequality is strict)}$$

where μ_i is a location parameter (the median or mean) for population$i$ and $\sigma_i$is a scale parameter with $i = 1,2, \ldots, k$ and k are the total number of populations. Population one ($i = 1$) is usually referred to the control population, while populations 2 through k are the treatment populations. The treatments may be increasing dosages of a drug.

In this paper, three new tests are proposedfor the simple tree alternative for location and scale testing.

1.1 BACKGROUND

1.1.1 Mann-Whitney

The Mann-Whitney test is a standard test statistic for examining the null hypothesis of equal population location parameters (Mann and Whitney, 1947). The null hypothesis and alternative hypothesis are given below:

$$H_0: \mu_1 = \mu_2 \quad (4)$$
$$H_{\alpha 1}: \mu_1 \neq \mu_2, H_{\alpha 2}: \mu_1 < \mu_2, H_{\alpha 3}: \mu_1 > \mu_2$$

In order to compute the test statisticMW, it will be assumed that there is a sample of size n_1 from the population 1 and a sample size n_2 of the population 2. The measurements of combined set of $n_1 + n_2 = N$, have been arranged in order from smallest to largest. Ranks are then assigned to the ordered measurements and S_j will be the rank of jth observation in sample 2, within the set of ranks. The test statistic MW is the sum of the ranks of all measurements in the sample 2.

$$MW = \sum S_j \quad (5)$$

The standardized version of Mann-Whitney test is given by:

$$MW^* = \frac{MW - E_0(MW)}{\sqrt{var_0(MW)}} \quad (6)$$

$$E_0(MW) = \frac{n_2(N+1)}{2} \quad (7)$$

$$var_0(MW) = \frac{n_1 n_2 (N+1)}{12} \quad (8)$$

When H_0 is true, the test statistic MW^* has approximately a standard normal distribution. H_0 will be rejected for the two sided alternative when$MW^* \geq Z_{\alpha/2}$ at the α level of significance where $Z_{\alpha/2}$ is the (1- $\alpha/2$) 100% percentile of the standard normal distribution.

1.1.2. Fligner-Wolfe Test

Often in biological sciences it is necessary to investigate the response of treatments compared to a control. Situations in which this often occurs are clinical trials, pharmacology experiments and agricultural experiments (Olet, 2014). The Fligner-Wolfe test statistic is designed for use in this type of situation (Fligner and Wolfe, 1982).

The Fligner-Wolfe test statistic compares the median of the control group, to the medians of a number of other treatment groups simultaneously (Fligner and Wolfe, 1982). There are k samples with $i = 1$ denoting the control sample and the remaining $2 \leq i \leq k$indicating treatment samples. It is assumed that the means in the treatment populations are at least as large as the mean of the control population. The null hypothesis and alternative hypothesis are given below:

$$H_0: \mu_1 = \mu_2 = \cdots = \mu_k, \text{ versus} \tag{9}$$

$$H_\alpha: \mu_1 \leq [\mu_2, \ldots, \mu_k] \text{ with at least one strict inequality.}$$

In calculating the Fligner-Wolfe test statistic, it is useful to visualize two populations. One population is the control ($i = 1$) and the remaining $k - 1$ populations are the combined treatment population. It will be assumed that there is a sample of sizen_1 from the control population and a sample size n_2 of the combined treatment population. In both the control sample and treatment sample, all of the observations will be merged together and subsequently ranked from smallest to largest. Let the rank r_{ij} with $i = 1{,}2$ and $j = 1{,}2{,}3, \ldots, n_i$ indicate the rank of the j^{th} observation in the i^{th} sample with i equal to 1 for the control sample and i equal to 2 for the combined treatment sample.

$$T_1 = FW = \sum_{\substack{2 \leq i \leq k \\ 1 < j < n_i}} r_{ij} \tag{10}$$

k is the number of treatments, $n_i$the number of observations in treatment i and r_{ij} the rank of the observation in the jthgroup subjected to the ith treatment. The expected value and variance of FW under the null distribution are outlined below:

$$E(T_1) = E_0(FW) = \frac{n_2(N+1)}{2} \text{and} var(T_1) = var_0(FW) = \left\{\frac{n_1 n_2(N+1)}{12}\right\} \tag{11}$$

where, n_1 is the size of the control population, and n_2 the number of observations in the remaining $k - 1$ treatment populations$n_2 = N - n_1$. The standardized version of Fligner-Wolfe testFW^* is stated below.

$$FW^* = \frac{FW - E_0(FW)}{\sqrt{var_0(FW)}} \tag{12}$$

The null hypothesis is rejected when$FW^* \geq Z_\alpha$ at the α level of significance where Z_α is the (1- α) 100% percentile of the standard normal distribution.

1.1.3. Ansari-Bradley Test

The Ansari-Bradley test is a nonparametric test designed to test for equality of variances based on independent samples from 2 populations (Ansari and Bradley, 1960). The null hypothesis and alternative hypothesis are given below:

$$H_0: \sigma_1 = \sigma_2 \tag{13}$$

$$H_{\alpha1}: \sigma_1 \neq \sigma_2,\ H_{\alpha2}: \sigma_1 < \sigma_2,\ H_{\alpha3}: \sigma_1 > \sigma_2$$

In calculating the Ansari-Bradley test, all the observations from the two samples will be combined together. The combined set of $n_1 + n_2 = N$observations will be arrangedin order from smallest to largest. The ranks will be assigned to the ordered observations as follows:

- The smallest observation and the largest observation will each be given a rank of 1
- The second smallest observation and the second largest observation will each be given a rank of 2

The ordered observations will continue to be ranked in this manner until all observations have been assigned a rank.At this pointR_i will be the rank of i^{th}observation in the first sample in the set of ranks. The test statistic Ansari-Bradley (*AB)* is the sum of the ranks of all observations in the first sample:

$$\mathrm{AB} = \sum R_i \tag{14}$$

The standardized version of Ansari-Bradley test is:

$$AB^* = \frac{AB - E_0(AB)}{\sqrt{var_0(AB)}} \tag{15}$$

If $N = n_1 + n_2$ is an even number :

$$E_0(AB) = \frac{n_1(N+2)}{4} \tag{16}$$

$$var_0(AB) = \left\{\frac{n_1 n_2 (N+2)(N-2)}{48(N-1)}\right\} \tag{17}$$

If $N = n_1 + n_2$ is an odd integer:

$$E_0(AB) = \frac{n_1(N+1)^2}{4N} \tag{18}$$

$$var_0(AB) = \left\{\frac{n_1 n_2 (N+1)(3+N^2)}{48N^2}\right\} \tag{19}$$

The asymptotic null distribution of AB^* is the standard normal distribution.

1.1.4. Lepage's Test

A nonparametric test for the two-sample location-scale problem is the test of Lepage (Lepage, 1971). The purpose of the Lepage test is to determine whether there are differences (between 2 populations) in either location parameters μ_1 and $\mu_2$or scale parameters σ_1 and σ_2. The Lepage's test is an amalgamation of the Mann-Whitney test for detecting location changes and the Ansari-Bradley test for detecting scale changes. The null hypothesis and alternative hypothesis are given below:

$$H_0: \mu_1 = \mu_2 \text{ and } \sigma_1 = \sigma_2 \tag{20}$$

$$H_\alpha: \mu_1 \neq \mu_2 \text{ and/ or } \sigma_1 \neq \sigma_2$$

The Lepage test statistics is given by:

$$Lepage = \frac{[(MW - E_0(MW)]^2}{var_0(MW)} + \frac{[(AB - E_0(AB)]^2}{var_0(AB)} = (MW^*)^2 + (AB^*)^2 \tag{21}$$

The Lepage test has a chi-square distribution with two degrees of freedom when the null hypothesis is true. H_0 is rejected when Lepage $\geq \chi^2_{2,\alpha}$ where $\chi^2_{2,\alpha}$ is upper a percentile point of the chi-square distribution with two degrees of freedom.

1.1.5. Modified Ansari-Bradley Test

Alsubie and Magel (2020) proposed a modified version of the Ansari-Bradley test. A modified version of the Ansari-Bradley test for simple tree alternative is stated below

$$H_0: \sigma_1 = \sigma_2 = \cdots = \sigma_k, \tag{22}$$
$$H_\alpha: \sigma_1 \leq [\sigma_2, \ldots, \sigma_k] \text{where at least one inequality is strict.}$$

In calculating the modified Ansari-Bradley test, it is helpful to consider a situation with two populations. One population is the control (i=1) and the remaining $k-1$ populations is the combined treatment population. It will be assumed that there is a sample of size n_c from the control population and a sample size n_t of the combined treatment population. The combined set of $n_c + n_t = N$observations will be arrangedin order from smallest to largest. The ranks will be assigned to the ordered observations as follows:

- The smallest observation and the largest observation will each be given a rank of 1
- The second smallest observation and the second largest observation will each be given a rank of 2

The ordered observations will continue to be ranked in this manner until all observations have been assigned a rank. At this pointR_i will be the rank of i^{th} observation in the control sample in the combined set of ranks. The test statistic *AB* is the sum of the ranks of all observations in the control sample,

$$\text{AB} = \sum R_i \tag{23}$$

The standardized version of Ansari-Bradley test is given by:

$$AB^* = \frac{AB - E_0(AB)}{\sqrt{var_0(AB)}} \tag{24}$$

If N= $n_c + n_t$ is an even number :

$$E_0(AB) = \frac{n_c(N+2)}{4} \tag{25}$$
$$var_0(AB) = \left\{\frac{n_c n_t (N+2)(N-2)}{48(N-1)}\right\} \tag{26}$$

If $N = n_c + n_t$ is an odd integer:

$$E_0(AB) = \frac{n_c(N+1)^2}{4N} \tag{27}$$

$$var_0(AB) = \left\{\frac{n_c n_t (N+1)(3+N^2)}{48N^2}\right\} \tag{28}$$

The asymptotic null distribution of AB^* is the standard normal distribution (Ansari and Bradley, 1960).

1.1.6. Alsubie and Magel

Alsubie and Magel (2020) proposed two testsL_1 and $L_2$for the simple tree alternative for location and scale testing. These tests are a combinationof the Fligner-Wolfe test for detecting location changes and the modified Ansari-Bradley test for detecting scale changes. The hypotheses are stated below:

$$H_0: \mu_1 = \mu_2 = \cdots = \mu_k, \tag{29}$$

$$H_0: \sigma_1 = \sigma_2 = \cdots = \sigma_k, \text{ versus}$$
$$H_\alpha: \mu_1 \leq [\mu_2, \ldots, \mu_k] \text{ and / or}$$
$$H_\alpha: \sigma_1 \leq [\sigma_2, \ldots, \sigma_k] \text{(At least one inequality is strict)}$$

The L_1 test is the sum of standardized test statistic for two tests. The first test being Fligner-Wolfe test statistic FW, obtained using Equation (10) and the second being the modified Ansari-Bradley test statistic AB, obtained using Equation (23). The mean and variance for the Fligner-Wolfe test statistic are given by $E_0(FW)$ and $var_0(FW)$and obtained using Equation (11). Similarly, the mean and variance for modified Ansari-Bradley test statistics are given by $E_0(AB)$and $var_0(AB)$, and obtained using Equation (25) and (26).

$$L_1 = \frac{FW^* + AB^*}{\sqrt{2}} \tag{30}$$

where the FW^* represents the standardized test statistic for Fligner-Wolfe test statistic and AB^*represents thestandardized test statistic for Ansari-Bradley test statistics.

The second test is given by:

$$L_2 = \frac{FW + AB - E(FW + AB)}{\sqrt{var\,(FW) + var\,(AB)}} \tag{31}$$

The sum of the null distribution of the mean is given by $E(FW + AB)$= $E(FW)$ + $E(AB)$ and the null standard deviation is$\sqrt{var(FW) + var(AB)}$. When the null hypothesis is true, the asymptotic distribution of L_2 is also a standard normal distribution. For more details about these tests, see Alsubie and Magel (2020).

2. MATERIALS AND METHODS

2.1 PROPOSED TESTS

2.1.1 Modified Moses Test

A modified version of the Moses test will be proposed. The modified version of the Moses test for simple tree alternative is outlined below:

$$H_0: \sigma_1 = \sigma_2 = \cdots = \sigma_k, \tag{32}$$
$$H_a: \sigma_1 \leq [\sigma_2, \ldots, \sigma_k] \text{where at least one inequality is strict.}$$

In calculating the modified Moses test, it is useful to visualize two populations. One population is the control (i=1) and the remaining $k-1$ populations is the combined treatment samples.

In order to calculate the modified test statistic for the Moses test, first, the control and the combined treatment samples will be divided up into m_1 andm_2 subsamples of equal size q. For each of the first m_1 subsets, the sample mean will be calculated, the distance between the sample mean and each observation is found and then squared. These squared values will then added up. The valuesC_1, C_2 , …, C_{m1}will be used to denote these sum of squared values for each of the $m_1$subsets in the control sample. The values D_1, D_2 , …, D_{m2}denote these sum of squared values for each of the m_2 subsets in the combined treatment samples.

Next, Mann-Whitney test (Mann and Whitney, 1947) will be applied. This means, the m_1 subsamples of C's and m_2 subsamples of D's will be combined. Following this, all observations in the combined set will be ranked from smallest to largest. The ranks of the observations from m_2 subsamples (which is, the D's) will then be added together. The Moses test statistic (M), is then the sum of the ranks assigned to the sums of squares $\sum S_i$, computed from the subsamples of combined treatment samples, which is the sum of the ranks that is assigned to the D's,

$$T_2: \text{M} = \sum S_i \tag{33}$$

The standardized version of Moses test is given by:

$$M^* = \frac{M - E_0(M)}{\sqrt{var_0(M)}} \tag{34}$$

$$E(T_2): E_0(M) = m_2(m_1 + m_2 \ + 1)/2 \tag{35}$$

$$var(T_2): var_0(M) = \ m_1 m_2(m_1 + m_2 \ + 1)/12 \tag{36}$$

The asymptotic null distribution of $M^*$is the standard normal distribution (Moses, 1963).

2.1.2. Proposed Test One

The first proposed test M_1 is the sum of standardized test statistic for two tests Fligner-Wolfe test statistic (T_1) obtained using (10) Equation and the modified Moses test statistics (T_2) obtained using Equation (33). The mean and variance for Fligner-Wolfe test statistic are given by $E(T_1)$ and $var(T_1)$ and obtained using Equation (11).

The standardized Fligner-Wolfe test statistics is given by:

$$Z_1 = \frac{T_1 - E(T_1)}{\sqrt{var\,(T_1)}} \tag{37}$$

Similarly, the mean and variance for the modified Moses test statistics are given by $E(T_2)$ and $var(\mathrm{T}_2)$ obtained using Equation (35), (36).

The standardized modified Moses test statistics is given by:

$$Z_2 = \frac{T_2 - E(T_2)}{\sqrt{var\,(T_2)}} \tag{38}$$

Both Z_1 and Z_2 have an asymptotic standard normal distribution under H_0 as given in (Fligner and Wolfe, 1982) and (Moses, 1963). When H_0 is true, the asymptotic distribution of Z_1 + Z_2 should be a normal with mean zero (0) and variance (2).

$$M_1 = \frac{Z_1 + Z_2}{\sqrt{2}} \tag{39}$$

2.1.3. Proposed Test Two

The second proposed test is given by:

$$M_2 = \frac{T_1 + T_2 - E(T_1 + T_2)}{\sqrt{var\,(T_1) + var\,(T_2)}} \tag{40}$$

The sum of the null distribution of the mean is given by $E(T_1 + T_2)$= $E(T_1)$ + $E(T_2)$ and the null standard deviation is$\sqrt{var(T_1) + var(T_2)}$.

2.1.4. Proposed Test Three

The third proposed test is given by:

$$M_3 = \frac{T_1 + 3T_2 - E(T_1 + 3T_2)}{\sqrt{var\,(T_1 + 3T_2)}} \tag{41}$$

The idea behind proposing this test is that the sample size of the Moses test is smaller than the sample size of the Fligner-Wolfe test and therefore, more weight is applied to the Moses test. In order to find Moses test, the original sample must be divided into subsamples and the sum of the squared deviations found within each subsample, The sum of the squares of each subsample areranked to find Moses test. Since subsamples of size 3 were used in this study, the sample size used for the Moses test was only 1/3 the sample size used for the Fligner-Wolfe test. Hence, a weight of 3 was applied to the Moses test.

The asymptotic distribution of each test is used and H_0 is rejected for a large value. Since each of the test statistics have an asymptotic standard normal distribution, the null hypothesis will be rejected when $M_1 \geq Z_\alpha$, $M_2 \geq Z_\alpha$, and $M_3 \geq Z_\alpha$ at the α level of significance where Z_α is the (1- α) 100% of the standard normal distribution.

3. SIMULATION STUDY

A simulation study is conducted to compare the three new proposed tests. The simulation study is implemented in SAS version 9.4. The properties of the proposed test statistics are compared assuming random samples followed normal distribution, t-distribution with 3 degrees of freedom and exponential distribution. In order to generate random samples from a specific distribution, the functions RAND are used in SAS. This requires the user to state the starting point "seed". This can be done using the Call streaminit function before using the RAND function. The syntax for this function is

Call streaminit (seed)

In this research, seed = 0 is used that instructs RAND to use the system clock. This means each run of the code will produce a different set of data (Bailer, 2010). The call function for the normal distribution is *RAND ('Normal', μ, σ)* where μ is the mean and σ is the standard deviation. The call function for the t-distribution is *RAND ('T',* 3*)* where T is the name of the distribution and 3 is the degrees of freedom. The call function for the exponential distribution is

RAND ('Exponential'). This function generates a random number from an exponential distribution with a mean and variance of one.

For all simulations, replications of 10,000 samples are used. The proposed tests are compared in two parts. The first part of the simulation is to get the estimates of the alpha values of the proposed test statistics. The stated alpha values for the proposed test statistics are all 0.05. The alpha values are estimated by counting the number of times the null hypothesis was rejected and then dividing by 10,000. This is done when the null hypothesis is true, and all distributions are the same; namely all location parameters are equal, and all scale parameters are equal.

The second part of the simulation study is to compare powers of the test statistics under various conditions. Powers are estimated by counting the number of times the proposed tests are rejected divided by 10,000.

3.1. Simulation Outline

The following outline summarizes what is done in the simulations.

1. The alpha values of each test statistic are estimated and compared to the stated alpha values for each simulation conducted. The proposed test statistics are examined in the case of k=2, k=3, and k=4 populations.
2. Powers are estimated for three conditions. Under the first condition, the location parameters are different, while the scale parameters are equal. The second condition assumes that the location parameters are equal, while the scale parameters are different. The final condition assumes, both the location parameters and the scale parameters are different.
3. Equal samples of sizes 9, 18, 30 are used for all populations. In order to calculate Moses test, each of the original samples are randomly divided into subsamples of size 3.
4. A variety of situations where sample of sizes are unequal are considered.
5. Three underlying distributions are considered.

4. RESULTS AND DISCUSSION

Tables 1-3 outline the results of simulation study for three treatments under the normal distribution. The results for four treatments are similar. The estimated alpha values are around 0.05 (see first entry in Table 1). The results here are consisted with the results from the t-distribution with 3 degrees of freedom and for all the sample sizes included in the study. When the populations have unequal location parameters and equal scale parameters, M_2 has the highest estimated powers (Table 1). When the populations have equal location parameters and unequal scale parameters, L_1 has the highest estimated powers (Table 2). When the populations have unequal location parameters and unequal scale parameters, L_1 has the higher estimated powers (Table 3).

Table 1. Percentage of Rejection for k=3 Populations; Normal Distribution with different means and equal variances, $n_1 = n_2 = n_3 = 30$

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	M_1	M_2	M_3
0	1	0	1	0	1	0.0502	0.0492	0.0504	0.0513	0.0505
0	1	0.25	1	0.5	1	0.2787	0.4114	0.3102	0.4790	0.4010
0	1	0.5	1	0.75	1	0.5232	0.7540	0.5767	0.8406	0.7360
0	1	0.75	1	1	1	0.7487	0.9494	0.8151	0.9802	0.9387
0	1	1	1	1.25	1	0.8939	0.9941	0.9415	0.9985	0.9918
0	1	1.25	1	1.5	1	0.9659	0.9996	0.9860	1	0.9991
0	1	1.5	1	1.75	1	0.9924	1	0.9972	1	1

Table 2. Percentage of Rejection for k=3 Populations; Normal Distribution with same means and different variances, $n_1 = n_2 = n_3 = 30$

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	M_1	M_2	M_3
0	1	0	1.5	0	2	0.5704	0.2948	0.4554	0.0896	0.2753
0	1	0	1.75	0	2.25	0.7362	0.4009	0.5905	0.1114	0.3519
0	1	0	2	0	2.5	0.8293	0.4814	0.7039	0.1166	0.4104
0	1	0	2.25	0	2.75	0.8934	0.5578	0.7780	0.1280	0.4809
0	1	0	2.5	0	3	0.9348	0.6250	0.8333	0.1307	0.5373
0	1	0	2.75	0	3.25	0.9521	0.6618	0.8701	0.1350	0.5753

Table 3. Percentage of Rejection for k=3 Populations; Normal Distribution with different means and different variances, $n_1 = n_2 = n_3 = 30$

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	M_1	M_2	M_3
0	1	0.25	1.5	0.5	2	0.8472	0.6995	0.7896	0.4392	0.7201
0	1	0.25	1.75	0.5	2.25	0.9207	0.7637	0.8618	0.4274	0.7665
0	1	0.25	2	0.5	2.5	0.9601	0.8131	0.9061	0.4281	0.7735
0	1	0.25	2.25	0.5	2.75	0.9766	0.8402	0.9368	0.4027	0.8004
0	1	0.25	2.5	0.5	3	0.9843	0.8599	0.9528	0.3948	0.8169
0	1	0.25	2.75	0.5	3.25	0.9898	0.8787	0.9605	0.3868	0.8264

Tables 4-6 show the results of simulation study for four treatments under the exponential distribution. The results for 3 treatments were similar. The estimated alpha values for L_1 and L_2 are around 0.05 while the new proposed tests didn't maintain their alpha values very wellexcept M_2(see first entry in Table 4). Therefore, in this case, the comparison only has been made between L_1, L_2 and M_2. The results are consistent across all sample sizes included in the study. When the populations have unequal location parameters and equal scale parameters, M_2 has higher estimated powers than thecompeting tests(Table 4). When the populations have equal location parameters and unequal scale parameters, L_1 has the highest estimated power (Table 5). When the populations have unequal location parameters and unequal scale parameters, M_2 has the higher estimated powers (Table 6).

Table 4. Percentage of Rejection for k=4 Populations; Exponential Distribution with different means and equal variances, $n_1 = n_2 = n_3 = n_4 = 18$

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	M_1	M_2	M_3
1	1	1	1	1	1	1	1	0.0461	0.0478	0.0908	0.0628	0.0840
1	1	1.25	1	1.5	1	1.75	1	0.0609	0.3698		0.7407	
1	1	1.5	1	1.75	1	2	1	0.0657	0.5833		0.9210	
1	1	1.75	1	2	1	2.25	1	0.0840	0.7540		0.9802	
1	1	2	1	2.25	1	2.5	1	0.0967	0.8640		0.9956	
1	1	2.25	1	2.5	1	2.75	1	0.1122	0.9337		0.9991	
1	1	2.5	1	2.75	1	3	1	0.1192	0.9681		0.9997	

Table 5. Percentage of Rejection for k=4 Populations; Exponential Distribution with same means and different variances, $n_1 = n_2 = n_3 = n_4 = 18$

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	M_2
1	1	1	1.5^2	1	2^2	1	2.5^2	0.3698	0.0726	0.0032
1	1	1	2^2	1	2.5^2	1	3^2	0.4671	0.0850	0.0017
1	1	1	2.5^2	1	3^2	1	3.5^2	0.5325	0.0952	0.0013
1	1	1	3^2	1	3.5^2	1	4^2	0.5663	0.1067	0.0009
1	1	1	3.5^2	1	4^2	1	4.5^2	0.5818	0.1143	0.0003
1	1	1	4^2	1	4.5^2	1	5^2	0.6082	0.1298	0.0003

Table 6. Percentage of Rejection for k=4 Populations; Exponential Distribution with different means and different variances, $n_1 = n_2 = n_3 = n_4 = 18$

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	M_2
1	1	1.25	1.25^2	1.5	1.5^2	1.75	1.75^2	0.2858	0.3431	0.4072
1	1	1.5	1.5^2	1.75	1.75^2	2	2^2	0.4365	0.5461	0.6147
1	1	1.75	1.75^2	2	2^2	2.25	2.25^2	0.5722	0.7088	0.7734
1	1	2	2^2	2.25	2.25^2	2.5	2.5^2	0.6948	0.8365	0.8838
1	1	2.25	2.25^2	2.5	2.5^2	2.75	2.75^2	0.7820	0.9109	0.9413
1	1	2.5	2.5^2	2.75	2.75^2	3	3^2	0.8466	0.9571	0.9702

5. CONCLUSION

The overall conclusion is that M_2 has the highest powers when the change is only in location parameters. When the change is only in scale parameters, L_1 has the highest powers. When both the location and scale parameters are different, the test statistic that has higher powers changes depending on the underlying distribution. For both the normal distribution and the t-distribution with 3 degrees of freedom (symmetric distributions), L_1 has higher powers while M_2 has higher powers for the exponential distribution (skewed).

If the distribution that one is sampling from is assumed to be approximately symmetric, L_1 is recommended to test for both an increasing change in the location and/or scale when treatments are applied. L_1 did have lower powers if only the locations (means) were different,but did have higher powers in the other two cases. If one expects the underlying distribution to be relatively skewed, then M_2 is the recommended test statistic to test for both increasing changes in the location and scale when treatments are applied.

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