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A Grand Unification of the Sciences, Arts & Consciousness:
Rediscovering the Pythagorean Plato's Golden Mean Number System

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Abstract

In this condensed paper, by combining the insights from E-Infinity theory, along with Plato's initiatory insights into the golden section imbedded in his Principles of the One and Indefinite Dyad, David Bohm's ontological framework of the superimplicate, implicate and explicate orders, andthe pervasive presence throughout physics, chemistry, biology and cosmology of the golden ratio (often veiled in Fibonacci and Lucas numbers), a profound *golden mean number system* emerges underlying the cosmos, nature and consciousness. This ubiquitous presence is evident in quantum mechanics, including quark masses, the chaos border, fine structure constant and entanglement, entropy and thermodynamic equilibrium, the periodic table of elements, nanotechnology, crystallography, computing, digital information, cryptography, genetics, nucleotide arrangement, Homo sapiens and Neanderthal genomes, DNA structure, cardiac anatomy and physiology, biometric measurements of the human and mammalian skulls, weather turbulence, plant phyllotaxis, planetary orbits and sizes, black holes, dark energy, dark matter, and even cosmogenesis – the very origin and structure of the universe. This has been pragmatically extended through the most ingenious biomimicry, from robotics, artificial intelligence, engineering and urban design, to extensions throughout history in architecture, music and the arts. We propose herein a grand unification of the sciences, arts and consciousness, rooted in an ontological superstructure known to the ancients as the One and Indefinite Dyad, that gives rise to a *golden mean number system* which is the substructure of all existence.

Keywords

Cantorian-fractal spacetime; E-Infinity theory; golden mean number system; superstring theory; super space; exceptional Lie symmetry groups; electromagnetic fine structure constant; Witten's fractal M-theory; Vafa's F-theory; hidden extra spacetime dimensions; Dvoretzky's theorem; holographic boundary theory; Indian mathematics; Arab mathematics; golden mean Turing machine; Grosse's Heterotic string theory; pure dark energy; dark matter energy; ordinary energy; zero set pre-quantum particle; empty set pre-quantum wave; cobordism; Umov's mass energy equivalence; Poincare-Lorenz; Einstein's formula; A. Connes'noncommutative geometry; K. Menger; P. Uhryson's deductive dimensional theory; John von Neumann's continuous geometry; the ultimate theory of unification; theory of consciousness; the brain; Penrose and Hameroff's ORCH theory; microtubule; clathrin; Frolich coherence; synchrony; C.S. Peirce; abduction; entanglement; nonlocality; David Bohm; superimplicate order; implicate order; explicate order; divided line; Indefinite Dyad; Greater; Lesser; golden ratio; phi; stichometry; Pythagoras; Pythagorean cosmic music theory; Plato; entheogen; samadhi; phyllotaxis; Fibonacci numbers; Lucas numbers; panpsychism; cosmopsychism; unification of art and science.

E-Infinity Theory Contributors:

This paper has been inspired by and is dedicated to the many contributors of E-Infinity theory including but not limited to the following: Carlos Castro, Leila Marek-Crnjac, Mohamed S. El Naschie, Ervin Goldfain, Mohamed Habeeb, A. Harb, Ji-Huan He, Mohamed A. Helal, Mae-Wan Ho, G. Iovane, Alan L. Mackay, Ray Munroe, S. Nada, Laurent Nottale, Nader Okko, Scott Olsen, Garnet Ord, Hermann Otto, Otto E. Rössler, A. Mary Selvam, Alexey Stakhov, Susie Vrobel, Harald Weiss and Volkar Weiss.

1. Introduction and Philosophical Background

> "And all things that can be known contain number,
> without this nothing can be thought or known."
> (Philolaus, Fr. 4)

The present work sets out to do nothing less than suggest (in broad outline) a grand unification of the sciences, arts and consciousness by explicating the underlying reality of the universe's *golden mean number system*. Following the diligence, determination, and professionalism of this eminent group of scholars who together developed the E-Infinity theory, we will work closely with the actual scientific evidence. However, following the admonition of Gottfried Wilhelm Leibniz[1], we will have the courage to travel some pathways and routes less travelled and too often avoided by those limited to practicing what Thomas Kuhn referred to in *TheStructure of Scientific Revolutions* [87]as merely "normal science". Caveat emptor, reader please beware, our job here is not to cover up the deeper insights of the

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[1]Leibniz: "Two roads diverged in a wood and I – I took the one less traveled by, and that has made all the difference."

creative geniuses of humanity, but rather to "lift the carpet" and bring them forth into the light of day. We are proposing a radical paradigm shift, a Copernican revolution and recognition of a golden physics in which the universe functions as a golden supercomputer [41,86]. Along the way we will address a family of striking anomalies (section 19), which when taken together point to a radical paradigm shift, and the very foundation principles for a *golden mean number system*, alluded to in antiquity as the One and the Indefinite Dyad. And in so doing we will necessarily be called upon to venture an answer to the greatest philosophical question of all: "How does the One become the Many?"

It was David Bohm who maintained, "The essential features of [quantum interconnectedness] are that the whole universe is in some way enfolded in everything, and that each thing is enfolded in the whole." (*The Undivided Universe*) [88] Thus, like a hologram, each fractal part contains (in potential) the information of the whole. Not only will we establish by the end that consciousness and the associated resonant states of awareness are guided by the same *golden mean number system*, but that this potential can ultimately be fully realized through resonant unfoldment into a nonlocal state of cosmic consciousness, known as Samadhi. Witness as an example, the climactic cosmic conscious experience of Apollo 14 astronaut Edgar Mitchell during his return journey to earth after becoming the 6th person to walk on the moon [89].

A quick preliminary note regarding our usage of the symbolism for the golden ratio. There are two versions, one we will refer to as the Greater and symbolize by the upper case Greek letter $\Phi = \dfrac{\sqrt{5}+1}{2} \approx 1.6180339...$ and the other we will refer to as the Lesser and symbolize it by the lower case Greek letter $\phi = \dfrac{\sqrt{5}-1}{2} \approx 0.6180339...$. The numbers are identical but for the difference of exactly one. They are reciprocals of each other, so that $\dfrac{1}{\Phi} = \phi$ and $\dfrac{1}{\phi} = \Phi$. Typically, it is more common in the sciences to emphasize the use of the Lesser or ϕ. Whereas in art and aesthetics it is more common to emphasize the use of the Greater or Φ.

2. **Background Information- Cantorian Spacetime - Bijection Formula - Noncommutative Geometry and the Dimensional Function**

This story may have started with Pythagoras's extensive studyand initiatory experiences at the hands of the priests in ancient Egypt and Babylon, and maybe even before.At the heart of this is a theory of cosmic music, the music of the spheres, as encoded in their sacred tetraktys and quadrivium through number theory [8-15], andincludes the tremendous work which sprung from it [11-12] and influenced many modern research directions which are visible in our own present time[6,8,12,49-53].

The present work may be extensive [1-139] but it is really quite condensed and is a relatively short account of the long story of the impact of utilizing the *golden mean number system* [1-22,38-41,53] in quantum cosmology in particular [14-22] and in unifying art with science and the "missing science" of consciousness in general [6,8,10-12,45-53,79-80,99-100,110-111,116,118-120]. We draw attention to parallel developments coming from different directions of logical and historical motivation converging on the same critical point, namely number theory in general [13,23] and the *golden mean number system* in particular [19,21]. To put the preceding point in explicit terms, we just need to recall the fundamental fact that the spacetime manifold of our E-Infinity Cantorian theory is virtually made up of an infinite number of unions and intersectionsof random Cantor sets with the golden mean $\phi = \left(\sqrt{5}-1\right)/2$ to the power of n as Hausdorff dimension where n takes the value from one to infinity as well as n = o which gives us unity (i.e. $\phi^0 = 1$)as shown in numerous previous publications [1-9,13-22].

It is a well-known fact due to American mathematicians Maudlin and William's theorem, that a random Cantor set has the golden mean as its Hausdorff dimension and that is why we are using ϕ^n as statistical weight for the hierarchical E-Infinity Cantorian spacetime [1-3,19,24]. In this way, the deductive dimensional theory developed by K. Menger and P. Uhryson has enteredinto E-infinity and was integrated into its mathematics via the bijection formula[1,2,3,19,24]

$$d_c^{(n)} = \left(1/d_c^{(0)}\right)^{n-1}, \; d_c^{(0)} = \phi \tag{1}$$

which in turn servesthe same purpose and has the same meaning asvonNeumann-Connes' dimensional function of the Penrose fractal tiling universe [25-28]

$$D = a + b\phi, \; a,b \in z \; \text{and} \; \phi = \left(\sqrt{5}-1\right)/2. \tag{2}$$

Let us now show how equation (1) and (2) stated above deliver the same Hausdorff dimension of a spacetime which is topologically four dimensional.

In the case of equation (2) all we need is to set n =4 and $d_c^{(0)} = \phi$ to find immediately the famous Hausdorff dimensionality. Consequentlyfor Einstein's space-timewith $D(4) = n = 4$ one finds [1-4,25,26]

$$
\begin{aligned}
d_c^{(4)} &= \left(1/\phi\right)^{4-1} \\
&= \left(1/\phi\right)^3 \\
&= 4.2360679... \\
&= 4 + \phi^3 \\
&= 4 + \cfrac{1}{4 + \cfrac{1}{4 + \cfrac{1}{4 + ...}}}
\end{aligned}
\tag{3}
$$

The result we just found goesfar beyond Einstein D=4 because every single internal region is also four dimensional.And so everything which appears in this space to be a mere point, turns out on careful observation to be an entire Cantor set as is obvious from the continued fraction representation of $4 + \phi^3$ given inequation(3). Such a space is clearly scale invariant [6,15,19,21]. This scale invariance is in general an important and essential property of a truly consistent theory as discussed in many earlier publications [1,8,15,17,19,21,24].The preceding equations were derived based on a topological-geometrical picture of E-Infinity spacetime. Moreover, there is a second dynamical-mechanical picture of the same space which is frequently referred to as the nested oscillation picture of E-Infinity and was used in connection with the brain [51-53]. See section 26.

3. M-Theoryand Dark Energy

Continuing this way we derived equation (3) above and we have shown how in Wittens' M-theory D=11 can be converted to a general fractal and scale invariant M-Theory with a dimensionality equal to eleven plus the exact Hardy quantum entanglement probability $P\left(H\right) = \phi^5$ [2,3,29]. Consequently, we see that we have

$$
\begin{aligned}
D &= 11 + \phi^5 \\
&= 11 + \cfrac{1}{11 + \cfrac{1}{11 + ...}}
\end{aligned}
\tag{4}
$$

i.e. an eleven-dimensional spacetime inside a larger one and so on indefinitely [15,30].

In fact in the last twenty-five or so years we have deduced numerous similar relations which the reader may find mathematically derived and physically explained in the relevant voluminous literature on E-Infinity Cantorian spacetime theory [1-8,13-31].

However, in the present work we will concentrate on and recall only theoretical results which are of immediate relevance and direct use with respect to our concern here as stated at the very beginning with the connection between science, the arts and consciousness through the golden mean [5,6].This is testified to by our derivation of the energy density of the dark section of the cosmos i.e. pure dark energy which amounts to almost 73.5%of the total energy of the universe, as well as the dark matter energy density found using accurate cosmic measurement to be roughly 22% [3,6,16,17,18,32]. Consequently, the total dark energy sector is about 95.5% which harmonizeswellwith the result of the cosmic energy density which can be measured in a direct way rather than indirectly inferred, namely the ordinary cosmic energy of about 4.5%. In other words,the total energy density adds to the maximal energy density given by that most famous of all equations[33],namely equation (5) which will be discussed next.

4. E=mc² and Cosmic Dark Energy

The celebrated mass-energy equivalence equationstates that

$$E = \gamma mc^2$$
$$= mc^2 \tag{5}$$

where m is the mass and c is the speed of light.

Thus,we may conclude that the energy E, given by equation (5),where c is the velocity of light and m is the mass, is the maximal energy density possible [17] for $\gamma = 1$ which marksthe corresponding 100% cosmic energy density [17,31,33]. We note in passing that the remarkable form of equation (5) shows that only the value of γ changes but nothing else except the velocity $V \rightarrow C$ so that:

(a) $\gamma = \dfrac{1}{2}$ gives the kinetic energy for v<<c

$$E = \frac{1}{2}mv^2, \tag{6}$$

(b) $\gamma = 1$ gives the maximal energydensity possible [17]

$$E = mc^2, \tag{7}$$

(c) $\gamma \approx 0.735$ gives pure dark energy density [3,18]

$$E = 0.735mc^2, \text{ and} \tag{8}$$

(d)$\gamma \approx 0.22$ gives dark matter energy density[3,18,29]

$$E = 0.22mc^2.$$ (9)

Finally, for ordinary energy we have [33-36]

(e) $\gamma \approx 0.045$ and one finds

$$E = 0.045mc^2$$ (10)

so that the total adds up to maximal cosmic energy destiny [29-30,32-36]

$$
\begin{aligned}
E &= (0.735 + 0.22 + 0.045)mc^2 \\
&= (0.955 + 0.045)mc^2 \\
&= (1)mc^2 \\
&= E(\text{Einstein})
\end{aligned}
$$ (11)

as it should be [28-36]. It is relevant at this point that all the preceding results were anticipated indirectly by N. Umov's E=kmc² as discussed in [34]. Remarkably the work of Umov occurred long before the advent of the theory of relativity and quantum mechanics. Thus, it was arrived at more or less through free-spirited research somewhat similarto E-Infinity theory [34-45].

5. The Set Theoretical Foundation of Quantum Mechanics

Returning now to our results given by equation (3) namely $d_c^{(4)} = 4 + \phi^3$ we may ask two relevant questions. First how to get the same results using equation (2) rather than equation (1). That is easily answered by settinga = 3 and b = 2in equation (2) and finding that [16-19,34-36]

$$D = 3 + (2)(\phi) = 3 + 1 + \phi^3 = 4 + \phi^3$$ (12)

which is what we were expecting due to the equivalence of equation (1) and equation (2). Second we would like to decide which is the real dimension of this space? Is it the topological 4 or the Hausdorff$4 + \phi^3$? The answer is actually clear from our very mathematical derivation procedure and a little reflection teaches us that having both dimensions together is an optimal way of fixing this space. That way we are naturally led to the notion of the bi-dimensionality [2,19,35].

$$D \equiv (4, 4 + \phi^3)$$ (13)

which is vital in our computation of the dark energy density of the cosmos [3,19], as we will see momentarily. We mean by this that the two fundamental notions of the set theoretical quantum mechanics of E-Infinity theory, namely the Zero set and the Empty setcan only be of use when defined using the E-Infinity bi-dimension [2,19,31]. To show this point in a systematic way we recall one of the most important starting points of the E-Infinity set theoretical formulation enshrined in the basic tenant of real life and the potent philosophy of being and nothingness [35]. In short,the point is that all we need to assert is what every pure mathematician working on set theory has long known, namely that Zero, Empty and Nothing are entirely different concepts [19]. Zero in set theory is a set full of zeros i.e. it is not empty so it is a very respectable something to reckon with as it is full.And even when all the things which we call zero disappear, leaving the set functioning, we still refer to it as the empty set. It was the fundamental contribution of a great Australian mathematician K. Menger and probably a little earlier the achievement of a very young Russian mathematical genius P. Uhryson that they discovered the deductive topological dimensional system and gave the Zero set the obvious dimension zero and the Empty set the somewhat not as obvious topological dimension minus one [19]. It took them some time and the combined ingenuity of many mathematicians, particularly Johnvon Neumann, Sir Roger Penrose and the inventor of noncommutative geometry, A. Connes,to come up with equation (2) of the present work. Subsequently after some focused effort by the pioneers of E-Infinity, led by El Naschie (one of the present authors),it was found that the Zero set is uniquely and pragmatically fixed by the bi-dimension[15-22]

$$D(0) \equiv (0, \phi) \tag{14}$$

and was immediately recognized by El Naschie to be the best way to model the pre-quantum particle while the Empty set is given by [2,19]

$$D(-1) \equiv (-1, \phi^2) \tag{15}$$

and the best model possible for the illusive pre-quantum wave. As a consequence of the above, it follows then that there are infinitely many empty sets with increasing degree of emptiness, i.e. we have [2,19]

$$D(-2) \equiv (-2, \phi^3) \tag{16}$$

which is what we call fractal scale invariant Einstein space-time because $1/\phi^3 = 4 + \phi^3$, and [2,19]

$$D(-3) \equiv \left(-3, \phi^4\right)$$
$$D(-4) \equiv \left(-4, \phi^5\right) \tag{17}$$
$$D(-5) \equiv \left(-5, \phi^6\right)$$

and so on till we find [2,19]

$$D(-\infty) \equiv \left(-\infty, \phi^\infty\right)$$
$$\equiv \left(-\infty, 0\right) \tag{18}$$

which is the true mathematical definition of Nothingness. This nothingness has kept philosophers busy for a long time. Ancient philosophers,such as Thales, Parmenides, Pythagoras and Plato,and modern philosophers,such as Heisenberg, Husserl, Sartre and Camus [36]to mentiona few,have impacted our lives throughout history on both the artistic and political levels [37].

6. Hardy's Quantum Entanglement, Unruh Temperature and the Immirzi Parameter ϕ^6

In 1993, Lucien Hardy wrote a paper titled, "Nonlocality for two particles without inequalities." [55]. This was a masterful piece of work, a thought experiment, in which Hardy demonstrated that entanglement occurs with the probability of roughly 9.017%. Unfortunately, Hardy had rounded off the actual calculation, and as a resultothersmissed seeing the connection to the golden ratio. The result suggests that in roughly 1 out of 11, or 9 out of 100 trials, entanglement of two particles will be observed. This also is tantamount to 9 instances in 100 trials where a quantum particle will be in two different locations at exactly the same time!

Eventually El Naschie realized that Hardy had rounded off his result. The actual calculation was 0.090169945… or written as a percentage, should be 9.0169945…%. Hardy apparently did not initially realize it, but he had discovered that entanglement occurs at the "Lesser" golden ratio, ϕ, raised to the 5th power, i.e. $\phi^5 \approx (0.6180339...)^5 \approx 0.090169945...$. El Naschie found that Hardy's result was perfectly consistent with what he derived through E-Infinity theory employing "fractal" Cantor sets.

As we definednothingness by equation (18), it is now important to define infinity using equation (2) to be [2,19,35-36]

$$D\left(+\infty\right) \equiv \left(+\infty, \phi^{-\infty}\right)$$
$$\equiv \left(+\infty, \left(1/\phi\right)^{\infty}\right) \qquad (19)$$
$$\equiv \left(+\infty, +\infty\right)$$

which is in fact not only infinite but incommensurately infinite i.e. it is an infinity which cannot be even counted making it an infinity which is larger than ordinary infinity [19].

Building on the above we have shown on many previous occasions that there are at least three fundamental physical phenomena of predominant theoreticalnotions which may be topologized using the E-Infinity golden mean system [19,35,38].

Here we are referring to the topological Unruh temperature given by $\left(-3, \phi^4\right)$, the topological Hardy quantum entanglement given by $\left(-4, \phi^5\right)$ and the topological Barbiro-Immirzi parameter given by $\left(-5, \phi^6\right)$ and which links loop quantum gravity to superstring theory in order to produce the correct black hole results [14].

7. The Degree of Emptiness and an Empty Set

The following empty setswith increasing degree of emptiness [2,19,35,38]:

$$D\left(-2\right) = \left(-2, \phi^3\right),$$
$$D\left(-3\right) = \left(-3, \phi^4\right),$$
$$D\left(-4\right) = \left(-4, \phi^5\right), \qquad (20)$$
$$\text{and}$$
$$D\left(-5\right) = \left(-5, \phi^6\right)$$

will play a pivotal role and it is relatively easy to show that they are a sequence of cobordism meaning that the empty set is the surface of the zero set and that [2,19,35,38]

$$D\left(-2\right) = \left(-2, \phi^3\right) \qquad (21)$$

the spacetime set is thus the surface of the empty set. Consequently, the topological Unruh temperature set [2,19]

$$D(-3) = (-3, \phi^4) \tag{22}$$

is the surface of the spacetime set and [2,19,35,38]

$$D(-4) = (-4, \phi^5) \tag{23}$$

of the Hardy quantum entanglement set is the surface of the previous set D (-3) and so on indefinitely [19]. This particular feature is behind the existence of the hidden dark energy sector of the cosmos as we will show in this work, which in turn means that the non-classical character of the quantum universe is nothing more or less than the role which an infinite dimensional hierarchical topology of our cosmos displays to three dimensional time-bound creatures which are known as humans.

8. Ordinary Energy Density and Dark Energy Density of the Universe

Now our next task is to draw on the Zero set and the empty set as well as the fact known to E-Infinity researchers for quite some time that Kaluza-Klein's five dimensionality is the exact mean dimension of a Cantorian spacetime consistent with the phenomenon of the hidden dark energy of the cosmos [2,19,35,38]. This is all that we need to perform the elementary calculation apart from realizing that while the ordinary energy of the cosmos is correlated, the dark energy is uncorrelated as is evident from the Rindler wedge space theory [43]. Consequently, applying this to the ordinary energy found from a five-dimensional intersection of the Zero set will lead us to the value ϕ^5 which is the magnitude of Hardy's quantum entangled probability.Therefore letting v go to c in our classical formula of the kinetic energy will capture the exact solution with deceptive ease leading to [3,16,19,38]

$$E(0) = \frac{1}{2}\phi^5 mc^2$$
$$= (\phi^5/2)(mc^2). \tag{24}$$

Within approximaterational terms this may be written as[2,3,19]

$$E(0) \approx mc^2/22. \tag{25}$$

In other words, the ordinary energy density of the universe is only $\phi^5/2 \approx 1/22$ percent,i.e. 4.5%, of the maximum energy,which is in complete agreement with our empirical measurements and observations. In analogy with the above, however, using the empty five-dimensional set, we find the total dark energy density of the cosmos as the uncorrelated union rather than intersection of five empty sets with the topological volume $5\phi^2$ so that one finds the total uncorrelated dark energy density of the universe to be [2,3,19,38]

$$E(D) = \frac{1}{2}\gamma(D)(m)(v \to c)^2$$

$$= \frac{1}{2}(5\phi^2)mc^2 = (5\phi^2/2)mc^2. \tag{26}$$

Ignoring irrationality and writing the above in a rational form, we find our accurate approximation for E(D) to be given by [2,3,19]:

$$E(D) \approx \left(\frac{21}{22}\right)mc^2 \tag{27}$$

which is about 95.5% of the total maximum energy of $E = mc^2$ and this again is consistent with all accurate empirical measurements and observations [3,17,29].

9. Dark Matter Energy and Pure Dark Energy

Armed with the above, we are now ready to tackle the new main points of the present work which is to accurately discriminate between pure dark energy and dark matter energy and give it a mathematical and physical meaning. First, and to avoid considerable misunderstanding, we repeat what we now know for sure, namely that ordinary energy is directly proportionate to a five-dimensional zero set pre-quantum particle of a topological value ϕ^5 which is Hardy's quantum probability of two entangled quantum particles. The total dark energy density is similarly proportionate to the uncorrelated five-dimensional topological volume of an empty set pre-quantum wave i.e. $5\phi^2$. Thus, we are truly content now to have derived the astonishing fact that the pure dark energy is proportionate to a ten-dimensional Unruh temperature uncorrelated set given by $(10)(\phi^4)$ and leading to

$$(PD) = \frac{1}{2}(10\phi^4). \tag{28}$$

This is thus an energy density equal to

$$E = (5\phi^4)mc^2$$

$$= 0.7294901699mc^2. \tag{29}$$

On the other hand, and quite similarly, the dark matter energy is proportionate to the topological Barbiro-Immirzi parameter

$$\gamma(DM) = (8)(\phi^6) \tag{30}$$

where ϕ^6 is the Barbiro-Immirzi parameter [14]leading to the corresponding energy density in eight-dimensional super space [1-3,18]

$$
\begin{aligned}
E(DM) &= \left(\frac{1}{2}\right)\left(8\phi^6\right)mc^2 \\
&= \left(4\phi^6\right)\left(mc^2\right) \\
&= \left(0.2229123605\right)mc^2.
\end{aligned}
\tag{31}
$$

This means the dark matter energy of the universe is about 22% of the maximal energy $E=mc^2$ which is in very reasonable agreement with the most accurate measurement [3,32].

Alas, the preceding result, although qualitatively very accurate, is quantitivelyno more than a very good approximation.This becomes obvious when we add the dark energy sector as given by equations(23) and (30)and find that

$$
mc^2\left(5\phi^4 + 4\phi^6\right) = \left(0.9524025\right)mc^2
\tag{32}
$$

while the exact sum should be

$$
\left(mc^2\right)\left(\frac{5\phi^2}{2}\right) = \left(0.954915\right)mc^2.
\tag{33}
$$

As we said this result of $0.9524mc^2$ is a very good approximation to $(0.954915)mc^2$ but is inexact. The reason is that both pure dark energy and dark matter energy are weakly coupledas shown in previous publications using a different method which we will discuss next [18].

10. Cosmic Energy Density from D. Gross, et al. and Heterotic String Theory

As is well known from earlier derivations using the heterotic string theory,this theory is defined dimensionally by [18]:

$$
26\text{-}10 = 16
\tag{34}
$$

where 26 is the dimensionality of the old bosonic string theory and 10 is the superstring's dimensionality while 16 is the David Grosse, et al. extra bosonic dimensionality. Utilizing this theory by subtracting Einstein's $D = 4$ from $D = 26$ we are left with a reference dimension equal to $D = 26 - 4 = 22$ and find in the first crude approximation the following equation [18]:

$$E = \left(\frac{22}{22}\right)mc^2$$

$$= \left(\frac{1+5+16}{22}\right)mc^2$$

$$= \left(\frac{1}{22} + \frac{5}{22} + \frac{16}{22}\right)mc^2 \tag{35}$$

$$= \left(\gamma_0 + \gamma_{DM} + \gamma_{PE}\right)mc^2$$

where $\gamma_0, \gamma_{DM}, \gamma_{PD}$ are the following: γ_0 is the cosmic energy density of the ordinary energy, γ_{DM} is the cosmic energy density of dark matter and γ_{PE} is the cosmic energy density of pure dark energy [18]. The corresponding equations are thus: [2,3,18-19]

$$E(0) = mc^2 / 22 = (0.04545)mc^2 \tag{36}$$

for ordinary energy

$$E(DM) = (5/22)mc^2 = (0.22727)mc^2 \tag{37}$$

for dark matter energyand

$$E(PD) = (16/22)mc^2 = (0.7272)mc^2 \tag{38}$$

for pure dark energy [18].

The above result can be refined by using the exact transfinitely corrected Heterotic dimension [18]26 + k, 10, 16 + k and consequently 22 + k where k is 'tHooft's renormalization equal to [16]

$$K = \phi^3\left(1-\phi^3\right) = 0.18033989. \tag{39}$$

Consequently, we have [3,16]

$$E(0) = \frac{mc^2}{22 + k}$$

$$= (0.0450849)mc^2 \tag{40}$$

which is the exact value. As for the dark section, we obtain in this case [18]

$$\mathrm{E(DM)} = (5/22 + \mathrm{k})\mathrm{mc}^2$$
$$= (0.225424)\mathrm{mc}^2 \tag{41}$$

and [18]

$$\mathrm{E(PD)} = \left[(16+\mathrm{k})/(22+\mathrm{k})\right]\mathrm{mc}^2$$
$$= 0.7294901688\mathrm{mc}^2. \tag{42}$$

This last value is exactly equal to the approximation given by equation (28)which is an improvement compared to (16/22). However, the first result $(5/22+\mathrm{k}) = 0.225424$ is an improvement on $5/22=0.227$ but is not equal to $4\phi^6 = 0.2229123$. As we said earlier, the reason for all these small discrepancies is that E(DM) and E(PD) are weakly coupled and this coupling can be easily determined and is equal to $\Delta = 0.080325$. That way the exact expression is given by [18]

$$\gamma(\mathrm{DM}) = (5-\Delta)/(22+\mathrm{k}) = 22.1803\% \tag{43}$$

which is the exact value and gives dark matter a convincing picture,that is dueto the compactified $22 + \mathrm{k}$ dimension of the old superstring theory [2,3,19]. Similarly, the exact value for the pure dark energy density is found to be

$$E(PD) = mc^2 \left(\frac{16+k+\Delta}{22+k}\right)$$
$$= \frac{16.26066429}{22+k}mc^2$$
$$= (0.733111619)mc^2. \tag{44}$$

Again, this is the exact value as discussed in previous publications [2,3,18].

One maybe interested in seeing how the Heterotic theory could give the same approximation found using equation (30). This is the case indeed as shown below

$$\mathrm{E(PD)} = \frac{5-\phi^6}{22+\mathrm{k}}\mathrm{mc}^2$$
$$= \frac{4.94427191}{22.18033989}\mathrm{mc}^2 \tag{45}$$
$$= 0.22291236\mathrm{mc}^2$$

which is exactly equal to the result of said equation (30), namely

$$E(PD) = 4\phi^6 mc^2$$
$$= 0.22291236mc^2.$$

(46)

11.Hidden Extra Dimension, Measure Concentration of Dvoretzky's Theory and Related Issues

Having reached this stage, it is not irrelevant to contemplate the geometrical topological picture which relates the total energy to ten dimensions that surround an eight-dimensional super space which in turn surrounds five-dimensional space while the entire structure is embedded and paradoxically so in five-dimensional space. So how could this be consistent? To understand this, we have to understand the nature of infinite dimensional topology in general and E-Infinity as well as hierarchal topology in particular. E-infinity has formally infinite dimensions but its mean dimension is equal to 4.23606797 as an average Hausdorff dimension as well as an equal average topological dimension $\langle d_c \rangle$ and $\langle n \rangle$ respectively as discussed at length in the relevant literature [3,16,20,39]. That is how we come to dimensions like $5 + \phi^3$ fractal Kaluza-Klein space, $11 + \phi^5$ fractal Witten space [15]and fractal Vafa's twelve-dimensional space [6]. Then and even more importantly we have the measure concentration phenomena as per Dvortesky's theorems which apply where dimensionality exceeds four when it is basically fractal as in 'tHooft-Weltman-Wilson's space 4-k compared to Einstein's 4 from which we could deduct the total cosmic dark energy density by simply finding [16]:

$$\gamma(D) = \frac{4-k}{4}$$
$$= 95.49150275\%$$

(47)

while ordinary energy is given by:

$$\gamma(0) = \frac{k}{4}$$
$$= 4.50849725\%.$$

(48)

And this is not all or the end of the story as we have the possibility of dividing space into bulk and holographic boundaries implied by the fundamental equation relating the dimension of E8E8 Lie Symmetry group of superstring and the Heterotic superstringi.e. 496 with the dimensionality of the boundary governed by the symmetry group SL(2,7) i.e. 336 dimensions as given by

$$|\,E_8 E_8\,|=|\,SL(2.7)\,|+\overline{\alpha}_0 + G + SU(2)$$
$$= 336 + 137 + 20 + 3 \qquad\qquad (49)$$
$$= 496,$$

where $\overline{\alpha}_0$ is the inverse electro-magnetic fine structure constant, G is the number of the components of the Riemennian tensor in four dimensions and SU(2) the weak force compactification of SL(2,7)[1,3,44,45].

12. Art and Science - Two Sides of the Same Coin

There is another important issue which the present paper will be elucidating and which is immensely important for understanding the enormous power of unification dormant inside number theory as applied to physics in general and the *golden mean number system* of E-Infinity in particular. This not only unifies all the fundamental interactions, but upon deep consideration unifies Art and Science in the same spirit of the Pythagorean mathematical cosmic music of numbers from which the universe emerges [6,8-13,131,134]. Thus, these esoteric ideas become far less puzzling and far more real when we look deeply and realize how the golden mean is present in works of art as a harmonic principle of composition of painting and music as well as the experimentally tested Hardy quantum entanglement ϕ^5,to mention only one example of many which we have discussed in various old and recent publications [3,6-8,15,18-19,21-22,30,38-45].

13. Anomalies and Abduction – C.S. Peirce

"It [abduction] is the only logical operation which introduces any new ideas."
[97: 5.171]

It was Charles Sanders Peirce [85,97], the founder of Pragmatism, who rediscovered the central role of abductive reasoning (novel hypothesis formation) in Plato's Academy [134]. It is a powerful method when facing anomalies, puzzles or problems that tend to be inexplicable under a prevailing paradigm. Employing the technique, one creatively hypothesizes a new paradigm which will explain and predict what was previously anomalous under the old paradigm (theory). Though conjectural, it is recognized as the only method of inference that results in something new. Deduction and induction are limited to inferring consequences of what is already contained in the premises. The greatest theoretical thinkers have used abduction throughout history to forge radical paradigm changes (Copernican revolutions) in the various disciplines. For example, Albert Einstein abducted the theory of relativity when confronting the anomalies of blackbody radiation, photoelectric effect and specific heat - which were not fully explicable under classical Newtonian physics. Because the intuition of the non-dominant right hemisphere appears to be involved, these are typically the "eurekas" and "aha" moments of creative inspiration that we see throughout history with a Pythagoras, Archimedes, Kepler, Einstein, Bohm or a Sherlock Holmes. And it is why the great

theoreticians will focus on what William James called the "wild facts" or "novel observations" in an attempt to abduct a sufficient explanatory and predictive hypothesis. Interestingly, today abductive reasoning is the central problem encountered in artificial intelligence in an effort to "biomimic" actual creative thought under novel circumstances.

14. Plato and the Academy

> "To the man who pursues his studies in the proper way, all geometric constructions, all systems of numbers, all duly constituted melodic progressions, the single ordered scheme of all celestial revolutions, should disclose themselves, and disclose themselves they will, if, as I say, a man pursues his studies aright with his mind fixed on their single end. As such a manreflects hewill receive the revelation of *asingle bond of natural interconnection between these problems*."**-**Plato,*Epinomis* 991e

As a proponent of Pythagorean doctrines, and an Egyptian and Eleusinian initiate, Plato had to be extremely careful. The golden sectioning of his divided line in the *Republic* expeditiously solves many of the platonic puzzles and riddles. His method in the written dialogues appears to be similar to his reported approach in the Academy where he acted as a kind of midwife. He would present the problem, puzzle, anomaly, apparent contradiction or incomplete result, intending that the attentive and resourceful student would abduct (give birth to) an explanatory hypothesis overcoming the apparent contradiction or incomplete result. Carefully and subtly Plato provided clues which can assist the observer in this act of abductive discovery, or creative solution. For example, in the *Timaeus* after proposing the √2 and √3 triangles as the fundamental structural components of nature, he writes:

> "These then… we assume to be the original elements of fire and other bodies but the principles which are prior to these Deity only knows, and he of men who is a friend of Deity…. Anyone who can point out a more beautiful form than ours for the construction of these bodies shall carry off the palm, not as an enemy, but as a friend…. He who disproves what we are saying, and shows that we are mistaken, may claim a friendly victory."
>
> -Plato, *Timaeus* 53a – 54b.

We know from Proclus in the *Eudemian* summary that Plato proposed problems regarding "The Section" to which Eudoxus and Leodamas made many discoveries applying abduction. In fact, Eudoxus "multiplied the number of propositions concerning 'The Section', applying the method of analysis [ἀπαγωγή (apagoge) or abduction] to them." [84]. Sir Thomas Heath noted this must have referred to the golden section which appears in Euclid [83]: "namely the division of a straight line in extreme and mean ratio which appears in Euclid II.11 and is *therefore most probably Pythagorean*."

When several of the Platonic puzzles and unsolved issues are viewed in conjunction, they help direct one to the required solution. The *Timaeus* and the *Republic* together point to the golden section and its relation to Plato's two ontological principles, the One and the Indefinite Dyad. The *Timaeus* does so in two ways. First, by its puzzling silence regarding an appropriate triangle for the construction of the Dodecahedron, while hinting, as just observed, that there may be a more beautiful triangle (ratio) than the √2 and √3 triangles that were the basis of the other four geometric solids. Second, by stressing the importance of the geometric mean in providing the crucial bond of nature. And in the *Republic*, Plato subtly and with great economy embeds the golden section into the ontology of his Sun and Divided
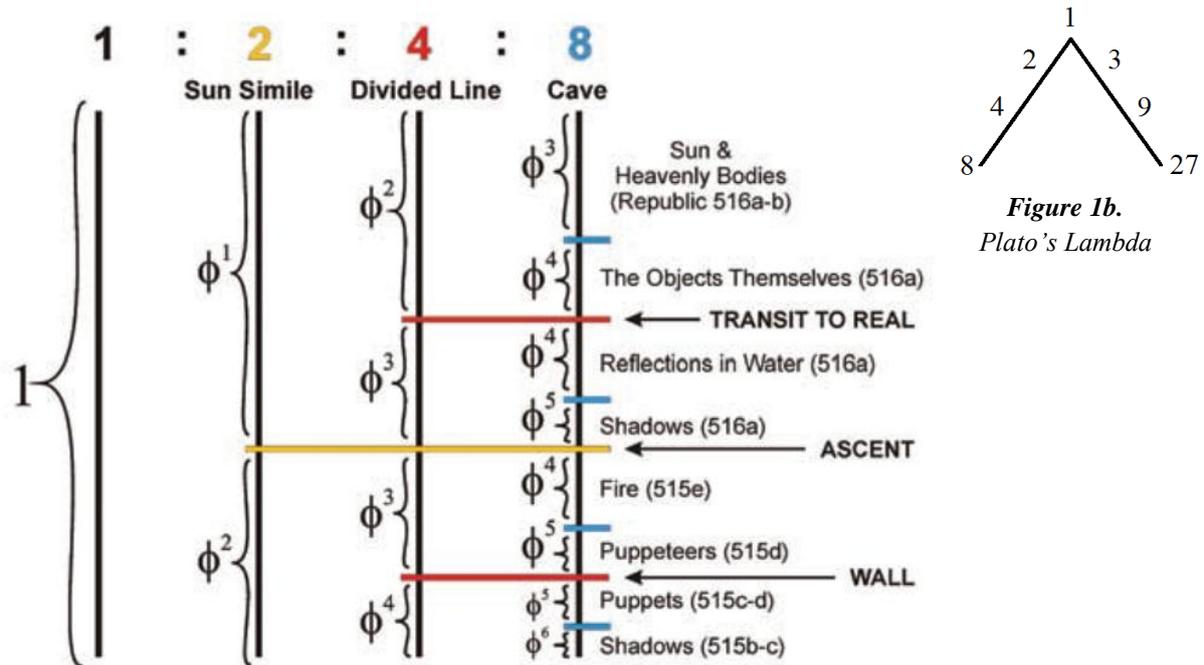

Figure 1b.
Plato's Lambda

Figure 1a. Plato's Republic *similes.*

Line analogies (see figure 1a and stichometry, section 15,below). Together the *Timaeus*, *Republic*, and *Parmenides* 133a-134e with the "worst difficulty argument" concerning bridging the immaterial and material, or mind and matter, point to continuous geometric proportion of the *golden mean number system* as that which binds together Plato's realms of Being and Becoming.

Plato inserts the initial golden ratio of the Sun Simile at the exact golden cut (61.8%) of the Republic scroll. [90] This is repeated in the Divided Line with the 1st cut separating his Immaterial World of Being from the Material World of Becoming. The two segments are then cut in the same golden ratio, and the two levels are now interwoven together through continuous geometric proportion of the golden mean number system. The Divided Line now reveals the One and Indefinite Dyad. Since the two intermediate sections are both equal, Plato has masterfully secreted the great secret of the Mystery schools (Egyptian, Eleusinian, Pythagorean, etc.) of the superimplicate order, what we may call the underlying superstructure principles of the One and Indefinite Dyad into his Divided Line, $[\Phi:1::1:\phi] \equiv [\phi^2:\phi^3::\phi^3:\phi^4]$. It is placed there for those capable of abducting the solution.

Thus, the great secret of the golden mean number system is concealed and yet revealed by Plato without overtly breaking the Sodalian oath of silence. Plato has taken the initial One and is essentially showing how it becomes Many, and is simultaneously hinting at the key of continuous geometric proportion, by going from the One (1 whole) to two (2 segments) in the initial cut or ratio, and thence to four (4 segments) in the proportion in the Divided Line, and finally to eight (8 segments) in the Cave. The progression of 1:2:4:8 is the even numbered arm of Plato's lambda (Figure 1b).

15. Plato, Stichometry and the Golden Section

The ancient Greeks measured the length of their texts in lines on a scroll. Based upon Homer's epics, the *Iliad* and *Odyssey*, the standard line was an epic hexameter of about 15 syllables or 35 Greek letters. In *The Musical Structure of Plato's Dialogues,* Jay B. Kennedy discovered through a careful designed computer process of getting as close as possible to Plato's original texts (the autographs), that Plato had employed stichometry, or the counting of hexameter lines to carefully divide up his dialogues into segments based on a 12-note Pythagorean musical scale. The 12-note Pythagorean temperament is based on a stack of intervals called perfect fifths, each tuned in the ratio 3:2, the next simplest ratio after
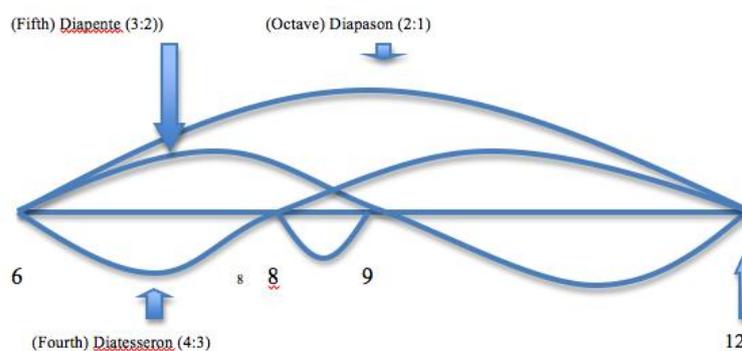
2:1. Notes of consonance, 3, 4, 6, 8, and 9, in relation to the entire 12 notes, were linked to The Good, virtue, friendship, laughter, joy, love, peace and truth. Notes or areas of dissonance, 5, 7, 10, and 11, were linked to evil, vice, conflict, war, death, disease, dishonesty, and Hades. In the very center, i.e. 6/12 or 1/2 , in an octave ratio of $12:6 \equiv 2:1$, he would often discuss divine wisdom, justice, the ideal philosopher, harmonic proportion, and Pythagorean theology. Thus, in the center of the *Republic* he discusses the philosopher king and the ideal just man.

Most importantly he would represent the primordial action of generating the ontological principles by imbedding references to the golden section, means and middles, at the actual golden cut (roughly 61.8%) of the dialogue text**.** In the *Parmenides* at 61.7% through 61.8% of the scroll, Plato writes "the One is equal and greater and less than itself … And if greater and less than equal, it would be of equal measures and more and less than itself … and in number less and more…." This recalls Euclid's definition of the golden section in his *Elements*. And in the *Republic* he places the original golden ratio in the Sun simile leading into the Divided Line at 61.7% of the scroll. And there are similar golden mean allusions near 61.8% in the *Symposium* 203e5, 204b1-5, *Philebus* 45e1-7, and *Phaedrus* 259a2, 6 and d8.

To accommodate the 12-note scale, Plato organizes his dialogs in exact multiples of 12. The *Apology* contains 1,200 lines, or 100 per twelfth. The *Protagoras*, *Cratylus*, *Philebus*, and

Symposium are each 2,400 lines, or 200 per twelfth. The *Gorgias* is 3,600 lines or 300 per twelfth. The *Republic* is 12,000 or 1,000 per twelfth. And the Laws is 14,400 lines in length, or 1,200 per twelfth[90,106].

16. Plato, Initiation and the Pythagorean One & Indefinite Dyad

The deep mathematical mystery of the *golden mean number system* was revealed to Pythagoras and Plato through their entheogenically induced initiations into the mysteries in Egypt (and also Eleusis for Plato) [121,122,135]. As Blavatsky says, "…all the rules of proportion are those taught anciently at Initiations, [and one should] acquaint [oneself] with the truly divine art, and understand the deep esoteric significance hidden in every rule and law of proportion…." [91]. "In India the initiated received the 'Soma' a sacred drink which helped to liberate one's soul from the body; and in the Eleusinian mysteries it was the sacred drink [Kykeon] offered at the Epopteia."[92]. The Pythagoreans maintained an oral tradition in which their major tenets were guarded with great secrecy. Aristoxenus of Tarentum, a student of Aristotle relates: "The strictness of their secrecy is astonishing; for in so many generations evidently nobody ever encountered any Pythagorean notes before the time of Philolaus." [98]. The initiate receiving the revelation was subject to the Sodalian oath of strict silence (the violation of which led to the death of Hippasus for disclosing the dodecahedron's golden construction)[136]. The epiphany at the summit of the Cave is that of Truth (the Greater golden ratio), the Good (the One), and Beauty (the Lesser golden ratio). The One is the product, difference, and geometric mean of the Greater and the Lesser. The Indefinite Dyad [132] of the Greater (1.6180339…) and the Lesser (0.6180339…) also emerge from E-Infinity theory in the eigenvalue solutions to the coupled oscillator equation modeling the brain, quark masses and double-slit experiment. [51,52] Also, technically, the One and Indefinite Dyad (Greater and Lesser) are *ontologicalprenumber principles*(see section 18) from which the other root ratios, e.g.√2 and √3, are geometrically derivable as seen in the following figure 2.
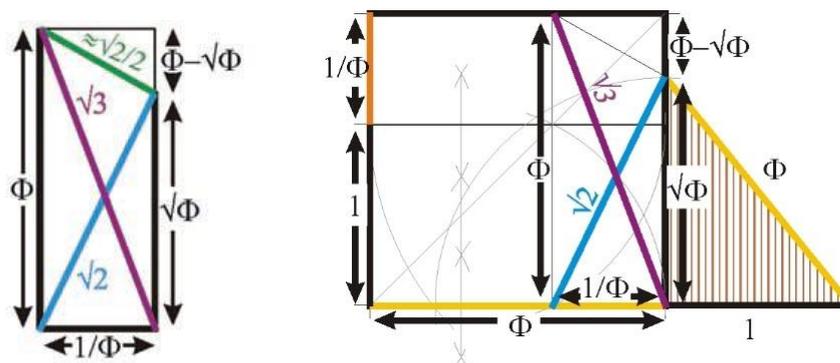
Figure 2. *Golden Chalice of Orion deriving √2 and √3 from the Greater (Φ) and Lesser (φ) Golden Ratios; and its method of construction with derivation of the Great Pyramid's semi-elevation (shaded area on right), Olsen © 2018 [96].*

17. The Penrose Anomaly and Rediscovering the One and Indefinite Dyad

Based upon his anesthesia research, Stuart Hameroff suggested that microtubules were involved in the emergence of consciousness at the level of quantum computation. Microtubules are self-assembling polymers (i.e. large scale, macro-molecules composed of repeating structural units) located in the cytoskeleton within neurons (and many other cells, both mammal and plant). In his 1994 book, *Shadows of the Mind, a Search for the Missing Science of Consciousness,*[53] Sir Roger Penrose posed the important "puzzling question," or anomaly: "Why do Fibonacci numbers appear in microtubules?" Microtubules have Fibonacci structure with 13 protofilaments that align side-by-side, forming spiral patterns that are similar to a pinecone or

Figure 3. *Microtubule*

sunflower, with 8 spirals winding in one direction and 5 spirals in the opposite direction (see Figure 3) [72]. Occasionally so-called double-microtubules form with (not 26) but 21 protofilaments, 13 spiraling in one direction and 8 in the other! When looking more closely for an abductive answer or explanation, we notice that the ratios of adjacent Fibonaccis (i.e. 21:13, 13:8 and 8:5) are each approximating the underlying golden ratio, and therefore, are explicate order expressions of the enfolded implicate order's *golden mean number system.* And in fact, the proportional "geometric mean" sequences of 13:8:5 and 21:13:8 are unfolded numerical approximations of the golden mean proportion itself, namely,Φ:1:ϕ.We argue that this golden mean proportion *emerges at the outset* in the Primordial First Act – the Ontological Asymmetric Cut that leads to the inherent golden proportional symmetry, Φ:1::1:ϕ.
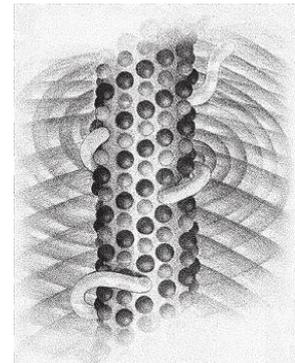
In their 1995 paper, "Quantum computing in microtubules: self-collapse as a possible mechanism for consciousness," [79] Penrose and Hameroff proposed that consciousness can be explained as quantum computations orchestrated through groups of microtubules in the neurons of the brain. They argued that the microtubules in a quantum coherent state could go through *objective reduction* or what in quantum physics is called, the collapse of the wave function. When observation occurs, the wave can collapse into an objective actuality. This is consistent with the present authors' position in E-Infinity theory that it is the ϕ^2 pre-quantum wave (empty set) that collapses under observation or measurement into its corresponding ϕ pre-quantum particle (zero set)[5,54,95].

Provocative research in 2003 began to demonstrate that quantum coherence occurs even in warm biological systems, including "bird brain" navigation, DNA, protein folding, biological water and microtubules. But, Max Tegmark objected claiming that microtubules would be

incapable of doing quantum computing because decoherence over time in the warm, wet environment of the brain would quickly set in to prevent it. Thus, the microtubule quantum coherence debate between Hameroff and Penrose on the one side, and Tegmark on the other had begun.

In 2011 Penrose and Hameroff wrote that the "multiple winding patterns… matching the Fibonacci series found widely in nature and possessing a helical symmetry, [are] suggestively sympathetic to large-scale quantum processes."[80] As part of the cytoskeleton, the microtubules establish cell shape, direct growth, and organize cellular functions, "defining cell architecture like girders and beams in a building." (Ibid.) But their lattice structure can be compared to computational systems. Penrose and Hameroff see them as *biomolecular quantum computers*. This of course means there can be entanglement, superposition (being in two places at once), and immediate "nonlocal connection" even when appearing to be "locally" separated. The present authors' position, simply stated, is that this is a reflection of the inherent *golden mean number system* infused into nature that gives rise to life and the resonant expansion of states of consciousness.

Penrose and Hameroff had responded to the earlier Tegmark decoherence objection, arguing that Frohlich coherence in pumped dissipative systems could provide a physical basis for warm quantum systems like microtubules. Frohlich coherence in microtubules has now been effectively established by Anirban Bandyopadhyay and his team, thus rebutting the Tegmark objection. The group demonstrated that microtubules stabilize as non-equilibrium Frohlich coherent systems when certain external frequencies are pumped into them, resulting in long range collective phenomena. In the abstract of their paper they explain what they observed: "As we bring tubulin protein molecules one by one into the vicinity, they self-assemble and the entire event we capture live via quantum tunneling. We observe how these molecules form a linear chain and then chains self-assemble into 2D sheets, an essential for microtubules, fundamental [for] nano-tubes in a cellular form…." [99]

In addition, clathrins, located at the tips of microtubules in the axon's synaptic boutons, are buckyball shaped proteins that selectively sort cargo at the cell membranes. As truncated icosahedra, they have internal rectangles constructed in the ratio 3Φ:1. During mitosis the clathrins bind directly to the microtubules (or microtubule-associated proteins called MAPS). But most importantly, together with microtubules the clathrins regulate synaptic activity. Our suggestion is that their "golden in-phase resonance" and attendant aesthetics hold a central key to the mystery of consciousness. Hence, beauty, function and illumination appear to be intimately tied together [5]!

18. Fibonacci and Lucas Numbers

> "The numbers are by him [Plato] expressly identified with the Forms themselves or principles, and are formed out of the Elements (i.e., Principles of the One and Indefinite Dyad)."
> -Aristotle, *De Anima* 404b24.

Nature widely expresses the golden ratio through two very simple series of whole numbers. The Fibonacci number series: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377... and the Lucas number series: 2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199 … are both additive, as each number is the sum of the previous two, and multiplicative, as each number approximates the previous number multiplied by the golden ratio. The ratio between any two adjacent numbers approximates the Greater golden ratio, Φ, when dividing the larger number by the smaller, asymptotically becoming more accurate as the numbers increase. Inversely, any number divided by its larger neighbor approximates the Lesser golden ratio, ϕ, alternating as more or less than ϕ, asymptotically becoming more accurate as the numbers increase. Each Fibonacci or Lucas number is the approximate geometric mean of its two adjacent numbers. For Fibonacci numbers one has to alternately add or subtract 1. For example, 3, 5, 8: $5 = \sqrt{[(3 \times 8)+1]}$; 5,8,13: $8 = \sqrt{[(5 \times 13)-1]}$. For Lucas numbers one has to alternately add or subtract 5. For example, 3, 4, 7: $4 = \sqrt{[(3 \times 7)-5]}$; 4, 7, 11: $7 = \sqrt{[(4 \times 11)+5]}$.

From an ontological (top down) perspective, the Greater and Lesser are *ontological prenumber principles* that give rise to all integers, through addition, subtraction, multiplication or division. Fibonacci and Lucas numbers have special relations to the Greater and Lesser as shown in the table below. Lucas numbers have an extremely interesting formula: they alternate between adding and subtracting the successive exponential golden powers of the Greater and Lesser. All non-Lucas and non-Fibonacci integers are easily seen to be composed of the powers of the Greater and Lesser. For example, one can easily see several ways in which the powers could be combined to give rise to numeral 6 [e.g. $(G^2 + L^2) + (G^2 + L^2) = 3 + 3 = 6$]. Likewise, the number 12 could arise through several combinations of the powers [e.g. $(G^5 - L^5) + (G^1 - L^1) = 11 + 1 = 12$], etc.

To the objection that an exponent, say 5 in a formula that includes Lesser[5] (i.e. L^5 in a formula like $G^5 - L^5 = 11$) already requires the integer 5, we reply that it is simply the fivefold iteration of the Lesser, i.e. L x L x L x L x L. It is equally legitimate to point out that the exponent 5 is simply the sum of two Lucas formulas of $(G^3 - L^3) + (G^1 - L^1) = 4 + 1 = 5$. Or we could use the Fibonacci formula for 5, $G^3 + 2L^2 = 5$. Here again one might object that the coefficient 2 is involved in the formula. But again, $2L^2$ is simply $L^2 + L^2$, or we can use the Lucas formula for 2, $G^0 + L^0 = 2$, or the Fibonacci formula for 2, $G^1 + L^2 = 2$. In the end, all integers are reducible to sums, differences, multiples and divisions of the Greater and Lesser in their capacity as *prenumber principles*. It is of course true that from an epistemological (bottom up) perspective we can calculate the numerical value of the Greater and Lesser with the formulas $\Phi = \dfrac{\sqrt{5}+1}{2} = 1.6180339...$ and $\phi = \dfrac{\sqrt{5}-1}{2} = 0.6180339....$ So, it is important that we clarify this distinction between perceiving the Indefinite Dyad as inherent powers of construction resulting from the initial ontological asymmetric cut in the beginning as seen from a macroscopic, holistic (top down) view, and the relative numerical values of these powers looked at and calculated epistemologically from a more piecemeal, fragmented (bottom up) view.

Fibonacci Numbers:

$$G^0\text{-}L^0 = \quad \Phi^0 - \frac{1}{\Phi^0} = \qquad 0 \qquad = 1 - 1$$

$$G^1\text{-}L^1 = \quad \Phi^1 - \frac{1}{\Phi^1} = \qquad 1 \qquad = 1.61803398\ldots - 0.61803398\ldots$$

$$G^1\text{-}L^1 = \quad \Phi^1 - \frac{1}{\Phi^1} = \qquad 1 \qquad = 1.61803398\ldots - 0.61803398\ldots$$

$$G^1\text{+}L^2 = \quad \Phi^1 + \frac{1}{\Phi^2} = \qquad 2 \qquad = 1.61803398\ldots + 0.38196601\ldots$$

$$G^2\text{+}L^2 = \quad \Phi^2 + \frac{1}{\Phi^2} = \qquad 3 \qquad = 2.61803398\ldots + 0.38196601\ldots$$

$$G^3\text{+}2L^2 = \quad \Phi^3 + \frac{2}{\Phi^2} = \qquad 5 \qquad = 4.2306797\ldots + 0.76393202\ldots$$

$$G^4\text{+}3L^2 = \quad \Phi^4 + \frac{3}{\Phi^2} = \qquad 8 \qquad = 6.85410196\ldots + 1.14589803\ldots$$

$$G^5\text{+}5L^2 = \quad \Phi^5 + \frac{5}{\Phi^2} = \qquad 13 \qquad = 11.09016994\ldots + 1.90983005\ldots$$

$$G^6\text{+}8L^2 = \quad \Phi^6 + \frac{8}{\Phi^2} = \qquad 21 \qquad = 17.94427191\ldots + 3.05572808\ldots$$

Lucas Numbers:

$$G^0\text{+}L^0 = \quad \Phi^0 + \frac{1}{\Phi^0} = \qquad 2 \qquad = 1 + 1$$

$$G^1\text{-}L^1 = \quad \Phi^1 - \frac{1}{\Phi^1} = \qquad 1 \qquad = 1.61803398\ldots - 0.61803398\ldots$$

$$G^2\text{+}L^2 = \quad \Phi^2 + \frac{1}{\Phi^2} = \qquad 3 \qquad = 2.61803398\ldots + 0.3819660\ldots$$

$$G^3\text{-}L^3 = \quad \Phi^3 - \frac{1}{\Phi^3} = \qquad 4 \qquad = 4.23606797\ldots - 0.23606797\ldots$$

$$G^4\text{+}L^4 = \quad \Phi^4 + \frac{1}{\Phi^4} = \qquad 7 \qquad = 6.85410196\ldots + 0.1458980\ldots$$

$$G^5\text{-}L^5 = \quad \Phi^5 - \frac{1}{\Phi^5} = \qquad 11 \qquad = 11.0901699\ldots - 0.0901699\ldots$$

$$G^6\text{+}L^6 = \quad \Phi^6 + \frac{1}{\Phi^6} = \qquad 18 \qquad = 17.9442719\ldots + 0.0557280\ldots$$

$$G^7\text{-}L^7 = \quad \Phi^7 - \frac{1}{\Phi^7} = \qquad 29 \qquad = 29.0344418\ldots - 0.0344418\ldots$$

$$G^8 + L^8 = \quad \Phi^8 + \frac{1}{\Phi^8} = \quad 47 \quad = 46.9787137\ldots + 0.0212862\ldots$$

$$G^9 - L^9 = \quad \Phi^9 - \frac{1}{\Phi^9} = \quad 76 \quad = 76.0131556\ldots - 0.0131556\ldots$$

$$G^{10} + L^{10} = \quad \Phi^{10} + \frac{1}{\Phi^{10}} = \quad 123 \quad = 122.9918693\ldots + 0.0081306\ldots$$

$$G^{11} - L^{11} = \quad \Phi^{11} - \frac{1}{\Phi^{11}} = \quad 199 \quad = 199.0050249\ldots - 0.0050249\ldots$$

19. A Family of Biometric Anomalies and the Golden Ratio as the Abductive Inference to the Best Explanation; How the One Becomes Many

1. Quark masses and the Chaos Border are governed by the golden ratio [72].
2. Entanglement (and therefore, nonlocality) of two particles is exactly equal to the Lesser golden ratio to the fifth power (see Lucien Hardy) [55].
3. MIT physicists, led by Pablo Jarillo-Herroro discovered that two-sheets of graphene twisted by about 1.1 degrees allow electrons to become easily entangled and superconductive. We (the authors) predict they will eventually discover this angle is precisely ϕ^{11} x $360^0 = 1.118023205\ldots$.
4. The biometric measurements of the human skull, in "Mammalian Skull Dimensions and the Golden Ratio (Φ)" by Tamargo and Pindrik, are ratios of adjacent Lucas numbers, approximating the golden ratio [18:11 and 11:7] see [56].
5. The biometric measurements of the lion and tiger skulls are ratios of adjacent Lucas numbers [7:4 and 4:3] see [56]. Of interest to those on a shamanic path where identification with the big cat is quite common.
6. Perez noted in 1991 that single-stranded whole human genome DNA gene-coding region sequences and codon populations were fractal, finely tuned to the golden ratio (1.618…) and related to Fibonacci and Lucas numbers [57].
7. Perez also found two binary code attractors. The top state matches the Lesser = 0.618..., and the bottom state matches ½ the Lesser = 0.309.... These two states create a self-organizing bi-stable binary code, that is in a perfect octave of one another [58].
8. Persaud-Sharma and O'Leary report in "Fibonacci Series, Golden Proportions and the Human Biology": in 2009, Perez found the widespread occurrence of Phi (Φ) and Fibonacci series throughout various species. Like human and chimpanzee genomes, the codon populations of 20 various species including eukaryotes, bacteria and viruses showed that 3 parameters [1, 2, and Phi (Φ)] define these populations to a precision of 99% and often 99.999% [58, 59].
9. The genomes of both Homo sapiens and Neanderthals exhibit Fibonacci and Lucas resonances (see Jean-Claude Perez) [57].
10. Petoukhov confirmed Perez's SUPRA-code of DNA [123]; and then went on to discover a law of golden genomatrices underlying the genetic code which is based upon two values only: the Greater, Φ, and the Lesser, ϕ [124].

11. Systolic and diastolic blood pressure follows the golden ratio[60].
12. The Greater golden ratio is present in the human heart beat's cardiac cycle [61].
13. Clathrins, closely associated with microtubules, are truncated isocahedra with $\Phi^3:1$ geometric resonance, and are involved in synaptic neurotransmitter release[75].
14. The divergence angle of plant phyllotaxis equals 137.5 degrees, the golden cutting of a circle[70].
15. In 2010 the golden ratio was discovered in QM phase transitions [67].
16. The orbits of the planets of the Trappist 1 solar system are in Fibonacci resonance [69].
17. The relative mean orbits and mean diameters of Earth and Mercury are close toΦ^2:1[73].
18. Paul Davies discovered that rotating black holes flip from a negative to a positive specific heat when the ratio of the square of the mass to the square of the spin parameter (rotation speed) equalsΦ[72].
19. The cosmic background radiation anomalies abductively suggest a dodecahedral universe (see Luminet) [63]. And the recent reassessment of the Planck satellite data now makes it immensely more probable that the universe is spherical-like than flat[64].
20. Measurement (i.e. observation) collapses a pre-quantum wave (Lesser squared 0.3819) into a pre-quantum particle (Lesser 0.618) [5,54,95].
21. Grzedzielsik proposed the role of the golden ratio in entropy as the governing proportion underlying thermodynamic equilibrium in self-organizing systems [125].
22. "What is it that gives life its distinctive oomph?" (see Paul Davies) [65].
23. Buddha's discovery of the middle path is not in the center, but at the golden cut[66].
24. Ideal human biomechanics and gait are governed by the golden ratio[62].
25. Golden ratio observed in magnetic field[68].
26. Ancient Egyptian Khesi-Ra panels from Saqqara depicts an architect holding two staffs and they are in 1:√5 relation[71].
27. The Great Pyramid Semi-elevation (the half meridian triangle) is a right-angled triangle composed of legs of √Φ (height) and 1 (half base), and a hypotenuse of Φ (apothem) see figure 4 [96, Olsen, 2018].
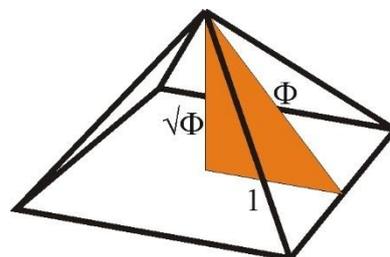
***Figure 4**. Great Pyramid Semi-elevation (the half meridian triangle).*

Taken together, this family of biometric puzzles lead abductively towards an inevitable "inference to the best explanation" of the deep significance of a golden ratio substructure enfolded into nature and the cosmos. The *golden mean number system* underpins nature's

optimal form, function and even consciousness. The overall result is that the Pythagorean Principles of the One and Indefinite Dyad, the golden ratio and its reciprocal (Greater = 1.6180339…, and Lesser = 0.6180339…), and the One as their geometric mean, i.e. Φ:1:ϕ, provides a proportional *golden mean number system* that is enfolded, infused into nature and the cosmos as it very substructure. Life (hylozoism) and consciousness (panpsychism) pour in from the very beginning riding these golden numbers from the superstructure that appears with the initial ontological act, Big Cut or Big Bang. This is also pervasive evidence of how the One, the Source, becomes the Many – infusing itself into each and every part through an initial Asymmetric Golden Cut that perpetuates itself holographically through a golden number fractal symphony. This is the grand unification of the sciences, arts and consciousness manifested through a *golden mean number system*. As Bohm would suggest, each part has the entire universe enfolded into it – with all the infinite potential and possibility implicit within that can be unfolded along the way.

20. Phyllotaxis and the Golden Rhythm of Nature

The unfoldment of plants moves in synchrony or consonance with the silent pulse or rhythm of nature – which is one with the cosmos. The sunflower's daily movement is attuned to the position of the sun relative to the movement of the earth. In the foreword to their monumental treatise, *Symmetry in Plants,* Jean and Barabe place the golden section front and center of the mystery when they write: "Daisies and sunflowers are the emblems of phyllotaxis: all the problems of phyllotaxis are summarized therein. The presence of particular numbers, e.g. Fibonacci numbers, an angle of 137.5 degrees, the golden number $\Phi = \dfrac{(\sqrt{5}+1)}{2} \simeq 1.618$, and forms (e.g. logarithmic spirals) in their capituli, and shoot apices, demands an explanation, and has served as a spur to the human intellect…. It is in phyllotaxis that symmetry in plants is most striking and puzzling." [70]

It turns out that there is an ideal angle for phyllotaxis or leaf arrangement in the plant kingdom. It is referred to as the "ideal divergence angle" or "genetic angle," and governs the tendency of buds, leaves, and limbs to emerge "ideally" at or near 137.5 degrees from the previous one. If you take a circle and golden section its 360 degrees, you will have two segments of 222.5 degrees and 137.5 degrees. It is a well-known fact that flowers tend to have a Fibonacci number of petals, and therefore are golden. And it was Johannes Kepler (1571-1630) who observed that the majority of flowers are pentagonal, i.e. have five petals [72, Olsen, 2006].

21. Solving Quantum Paradoxes

The realization of El Naschie was that fractal Cantor sets [1,95] (driven by the golden ratio) allow one to not only capture the fractal nature of quantum theory, but resolve its difficult paradoxes, including the wave-particle duality paradox. When using Cantor sets, the quantum particle state is a "zero measure Cantor set" of (0;ϕ) equal to 0.6180339… and the quantum wave state is the "empty measure Cantor set" of $(-1;\phi^2)$ equal to 0.3819660…. The wave and

particle are separated by (i.e. related through) what is beginning to appear to be the modular of nature itself: $\Phi \approx 1.6180339\ldots$. Hence, multiply a wave by Φ and it transforms into a particle. Divide a particle by Φ and it transforms into a wave. This in fact mimics (or is mimicked by as pointed out above) the interplay of adjacent Fibonacci numbers in nature. Multiply a Fibonacci number by the modular Φ and you will get an approximation to the next Fibonacci number. Of course, divide a Fibonacci number by the modular Φ and you will get an approximation to the previous Fibonacci number.

What is lurking behind the scenes here, and most importantly for our deeper interests, is the fact that these "fractal Cantor sets" – guided by the golden ratio – allow for nonlocality. When there is no spatial or temporal separation at the most fundamental level, as Bohm and others had always maintained with quantum mechanics, there is immediate contact. This has a tremendous bearing on all issues related to consciousness research, morphic resonance and *psi phenomena*. It was William James who made it clear that any adequate theory of consciousness must be able to reconcile the puzzling *wild facts* and *novel observations* (i.e. the anomalies) of paranormal, *psi phenomena* [81,100].

22. Golden Proportional Symmetry and Aesthetics

> "It is in this way, when they preserve the standard of the mean that all their works are good…. The greater and the lesser are to be measured in relation to one another(Φ:ϕ), but also to the establishment of the mean(Φ:1 &1:Φ and ϕ:1& 1:ϕ)…. Evidently we should divide the Science of Measurement into two parts…. One part comprises all the arts which measure number, length, depth, breadth, and thickness in relation to their opposites; the other comprises those which measure them in relation to the moderate, the fitting, the opportune, the needful, and all the other standards that are situated in the mean between the extremes."
>
> -Plato, *Statesman* 284A1-E8

Recall that both Pythagoras and Plato were initiated into the mysteries in ancient Egypt. Golden proportional symmetry was evident not only in the Great Pyramid of Giza, but throughout the tombs, temples, and artifacts. Alexander Badawy studied over 50 temples in Egypt and discovered they were designed using Fibonacci Numbers [101,126,127].Below in figures 5 and 6 we have the First and Second Egyptian Canons of Proportion, the oldest based upon Lucas numbers 18:11:7 and the more recent canon based upon Fibonacci numbers 21:13:8. According to tradition and existing fragments, the Greek sculptor Polyclitus, wrote his canon of proportion based upon golden proportional symmetry. The legend is that the canon was encoded into his magnificent bronze statue of Doryphorus, the spear bearer. Unfortunately, it was melted down by religious zealots. However, beautiful marble Roman copies, as in figure 7, still exist. The name Phi for the golden ratio was derived from Phidias, the Greek sculptor, who used it in the sculptures and buildings on the acropolis in Athens. The Roman Vitruvius
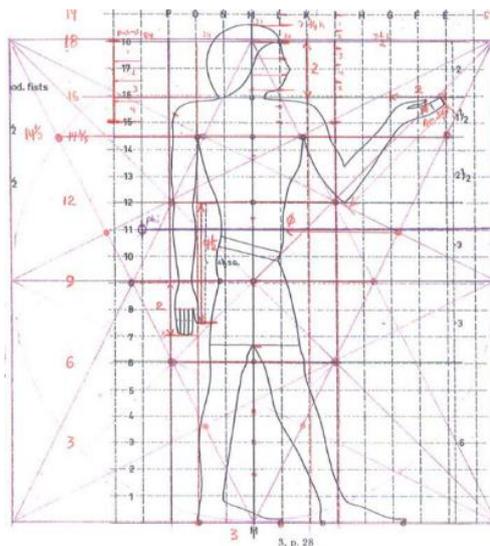

Figure 5. *1st Egyptian Canon by Lance Harding*

continued this golden proportional symmetry [102] which was later expressed by Leonardo Da Vinci in his rendition of the Vitruvian Man, see the figure 8 elaboration by Lance Harding [72].Da Vinci played a major role in the renaissance or rebirth of the classical Greek *golden mean number system* in illustrating the *Divina Proportione* written by Luca Pacioli, the monk drunk on beauty [103].Modern painters, sculptors and architects such as Georges Seurat, Piet Mondrian, Pablo Picasso, Paul Serusier, Hilma af Klint, Gino Severini, Constantin Brâncuși, Einar Jónsson, Father Desiderius Lenz,Le Corbusier, Frank Lloyd Wright, Buckminster Fuller, and Keith Critchlow have continued expressing the golden proportional symmetry. It is not uncommon for artists to frame their pictures in 3:2, or 5:3 rectangles, simple Fibonacci golden approximations. Salvador Dali's *Sacrament of the Last Supper* is a good example of 5:3 and includes a dodecahedron in the background.
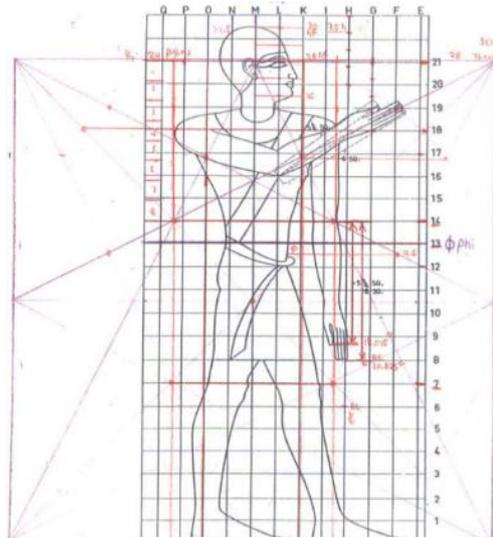

Figure 6.2nd Egyptian Canon by Lance Harding

"These three special ratios [Root 3, Root 2 & the Golden Mean] are directly related … to the triangle, square and pentagon. They recur consistently throughout the proportioning of the great sacred buildings of mankind."
-Critchlow, K., *Time Stands Still*, p. 86 [139,133]

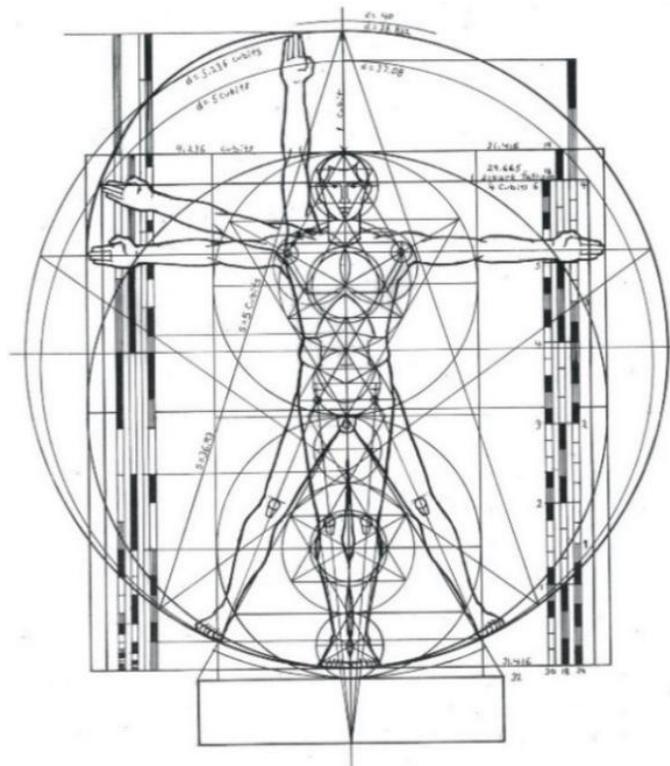*Figure 7*. *Doryphorus by Polyclitus*

Figure 8. *Da Vinci's Vitruvian Man by Lance Harding [72].*

23. The Pythagorean Quadrivium, Harmonics and the Music of the Spheres

Harmonics (number in time) was one of four disciplines studied in the Pythagorean Quadrivium, together with Arithmetic (pure number), Geometry (number in space), and Spherics (number in space and time). The golden ratio is a theme common to all. The structure of both rhythm and harmony is based upon intervals of simple integer ratios. The most simple and pleasing musical intervals, the octave (2:1) and the fifth (3:2), are the first Fibonacci approximations to the golden ratio. The Fibonacci series continues with the major and minor sixths (5:3 and 8:5, respectively). The scale itself holds the next step (13:8), for astonishingly, if we include the octave (2:1), musicians play eight notes in a scale, taken from thirteen chromatic notes. Finally, simple major and minor chords consist of the 1st, 3rd, 5th and 8th notes of the scale [72].

The golden ratio has been used by composers from Dufay to Bach, Bartok, and Sibelius, as a way of structuring a work of music. Russian musicologist Sabaneev discovered in 1925 that the golden ratio predominates in compositions by Beethoven (97% of works), Haydn (97%), Arensky (95%), Chopin (92%, including almost all of his *Etudes*), Schubert (91%), Mozart (91%), and Scriabin (90%) [71,72].

For Pythagoras, it was ultimately all contained in his vision of the Music of the Spheres. When one passes through the disciplines of the Quadrivium, one combines the sublimity of pure arithmetic, harmonics and geometry, in the harmonic rhythms of the three dimensional planetary objects in a cosmic synchronous dance through time. We know that the Pythagoreans carefully guarded their deepest secrets. And Aristotle made it clear that Plato was a follower of the Pythagoreans and had an unwritten doctrine, the agrapha dogmata (αγραφα δογματα), in which he apparently disclosed (reportedly in an enigmatic manner) some of the deeper mathematical mysteries of his One and Indefinite Dyad philosophy in his unwritten lectures On the Good [See the Tübingen school, 104,105].

> "Aristotle, Heracides, Hestiaeus, and other associates of Plato attended these (lectures On the Good) and wrote them down in the enigmatic style in which they were delivered."
> -Simplicius on Aristotle's *Physics 202b36*.

It was the brilliant Ernest McClain, philosopher and professor emeritus of music at Brooklyn College who in *The Pythagorean Plato: Prelude to the Song Itself* began to uncover some of the Pythagorean musical content and puzzles in Plato's dialogues [77].McClain discovered in his study of the Music of the Spheres in Plato that there appeared to be a universal wisdom tradition of similar numbers and parallel harmonic mathematical insights secretly embedded in Plato which McClain also found in other cultures including Egypt, Babylon, Sumer and India. See for example the Pythagorean 12 tone just tuning system, with the pentatonic and heptatonic scales employed in the construction of the Parthenon. Apparently, the ancient Phrygian mode of the heptatonic scales was the preferred mode of Plato [78].

Kepler followed up on the Pythagorean Harmony of the Spheres, *Harmonice Mundi*, with his nested Platonic solids and their golden and root ratios mirrored in the orbits of the planetary solar system (see figure 9). This was later confirmed by John Martineau in his *Little Book of Coincidence* where he took the mean orbits and mean diameters of the planets and discovered that Kepler was right (see for example figure 10, *Kiss of Venus*). Thus, the *golden mean number system*, root ratios and Fibonacci numbers are reflected in the solar system [73,107-108].
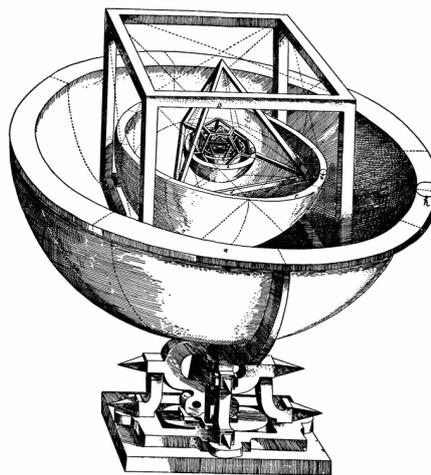
Figure 9. Kepler's nested Platonic solids model of the solar system.
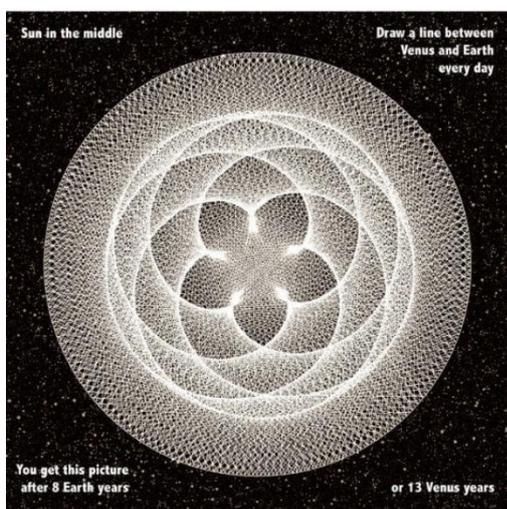
This Pythagorean Harmony of the Spheres was transferred by Danish physicist Niels Bohr into his model of the quantized microcosm of elementary particles and radiation.

Figure 10. John Martineau, "Kiss of Venus." Venus draws a beautiful fivefold rosette around Earth every 8 years (or 13 Venusian years) relating to Fibonacci numbers 13:8:5, reflecting $\Phi:1:\phi$.

Bohr found "a relationship between atomic structure and the discrete nature of electromagnetic radiation emitted by atoms…. the grandiose idea of the harmony of the Pythagoreans' celestial spheres seems to be equally present in the beauty of the dance of electrons and the way this manifests itself in the magnificent existence of spectral lines characteristic for each element. In accordance with the E infinity theory… the elementary particles of nature arise with the same elegance as a harmonious set of musical notes."
[see 109: Marcal de Oliveira Neto, 2006]

> "Geometry has two great treasures: one is the theorem of Pythagoreas; the other, the division of a line into extreme and mean ratio [the golden cut]. The first we may compare to a measure of gold; the second we may name a precious jewel."
>
> -Johannes Kepler, *Harmonice Mundi*[107]

24. The Silent Pulse, Synchrony and Quantum Coherent States of Consciousness

It was the Dutchman, Christian Huygens, who in 1665 gave one of the first explanations of what scientists later referred to as entrainment or the "mutual phase-locking of two oscillators." He had observed that two pendulum clocks left hanging on a wall beside each other, tended to fall into a synchronous rhythmic swing together. He maintained that there was a kind of "sympathy" established between them in which they sought to keep the same time – attuned with one another in a synchronous pulse. This universal phenomenon is now known to occur "whenever two or more oscillators in the same field are pulsing at nearly the same time, they tend to 'lock in' so that they are pulsing at exactly the same time." [82]

Madame H.P. Blavatsky, author of the *Secret Doctrine*, once wrote: "all is vibration." [91] We now know that the brain waves of musicians fall into synchronization [74]. Dance performers have observed that there have been occasions during rehearsals when all the female dancers would suddenly go absent, attending to the onset of their menstrual cycles occurring at "essentially" the same time. We observe these synchronous moments in the flight of flocks of birds, the sudden simultaneous movements of a school of fish, swimming bacteria, swarms of bees or marching locusts. Threshold levels are now being observed as smaller groups move from chaos to order as their numbers grow. Modifications in the environment, such as introduction of predators, can show what appears to be change of the operating rules by the synchronous animals as recently observed in jackdaw flocks [93,110,111].Two detailed studies of the neural activity of Egyptian fruit bats and mice, respectively, demonstrate that brain oscillations fall into synchrony during social interaction [137,138].

We should mention here of course the important work of Rupert Sheldrake on morphogenetic fields where learned behavior is carried forward through morphic resonance to other members of that species [112,113]. And most importantly for our views here, the work of Brian Goodwin, who viewed morphic resonance as structured by the ratios of the Platonic solids, including of course, the golden ratio, $\sqrt{2}$ and $\sqrt{3}$[114,115].

The interdisciplinary study of *synchrony* centers on coupled oscillators. Recall that in section 16 we noted that the Indefinite Dyad of the Greater Φ (1.6180339…) and the Lesser ϕ(0.6180339…) also emerge from E-Infinity theory in the eigenvalue solutions to the coupled oscillator equation modeling the brain, quark masses and the double-slit experiment.[51,52] In his book, *Sync: How Order Emerges from Chaos in the Universe,* Steven Stogratz writes:

> "Groups of fireflies, planets, or pacemaker cells are all *collections of oscillators* – entities that cycle automatically, that repeat themselves over and over again at more or less regular time intervals. Fireflies flash; planets orbit; pacemaker cells fire. *Two or more oscillators are said to be coupled* if some physical or chemical process allows them to influence one another. Fireflies communicate with light. Planets tug on one another with gravity. Heart cells pass electrical currents back and forth. As these examples suggest, nature uses every available channel to allow its oscillators to talk to one another. And the result of those conversations is often *synchrony*, in which *all the oscillators begin to move as one*."[116]

Recently the red dwarf star Trappist-1 was found to have 7 earth-sized planets, 3 in the Goldilocks zone, exhibiting a harmonic Fibonacci dance similar to Pythagoras', Plato's, Kepler's, Martineau's and Hartmut Warm's music of the spheres. The orbital periods include 8, 5, 3, 2, 4/3 (musical fourth) and 1. "The original discoverers noted that those orbits were almost exactly in what scientists call 'resonance.' That is, the second planet completes five orbits in almost the same time the first planet makes eight. The third planet completes three orbits for every five orbits of the second planet, and the fourth planet makes two orbits for every three orbits of the third planet." [117]

In physics, we find the most amazing phenomena of superconductivity and superfluidity at or near absolute zero. Everything moves in lock-step coherence, unrestricted by the normal constraints of gravity and other typical barriers. It may well be that quantum coherent states of conscious, where we go into lock-step resonance, into *synchrony* with broader and broader fields of awareness, may be inherent within the very fabric of existence (of so-called spacetime – the quantum foam that makes up everything)! Perhaps the universal-total-cosmic awareness is potentially present within us. The state of cosmic consciousness may in fact be our birthright: the whole universe is holographically co-present within each of us. Historical, cultural, philosophical and religious evidence abounds to suggest that through proper discipline, focus, and selflessness one can enter a quantum coherent state of resonance, of profound synchrony in which the state of realization, full awareness, at-one-ment in total bliss, or cosmic union is possible.

25. Music and Illumination in the Platonic Tradition; Path Back to the One

Aristotle, who had been with Plato in the Academy for nearly twenty years, once wrote: "For Plato, too, was right in raising this question and asking, as he used to do, 'are we on the way from or to the First Principles?'" [*NicomacheanEthics* 1095A].In the Platonic tradition, the intention was to lift the soul [consciousness] out of the realm of mere opinion (*doxa*) through attunement (resonance) with the ratios and proportions contained in the harmonies and rhythms of music [unfolded in the explicate order]. This allowed the soul [consciousness] to pass into the Intelligible realm of knowledge (*episteme*) [implicate order], passing through the realm of mathematical reasoning (*dianoia*)] up into direct intuition (*noesis*) of the pure Forms, i.e. the pure numbers and root ratios themselves. And at the highest level, it was possible at the summit of the Cave to experience the sublimity of the Indefinite Dyad of the Greater (Φ) and Lesser (ϕ).Then in an act of nonlocal illumination, experience the Source itself, the One, while simultaneously holding in full comprehension how the One enfolds $\Phi:1:\phi$ into the Many via the Indefinite Dyad and its *golden mean number system*.

26. A Glimpse into the Brain and Consciousness via the Picture of E-Infinity

Almost from its inception, E-infinity Cantorian-fractal spacetime theory was presented in two interrelated forms, a topological geometrical form and a quasi-mechanical form. The first, the topological geometrical form, is a union and intersection of infinitely many Cantor sets of the random Mauldin-Williams Cantor sets [1-4,19,24,28,45]. Then by putting together a hint from Professor M. Conrad regarding the transfiniteness of the brain and its relation to the

transfiniteness of E-Infinity theory, along with the Greater (Φ) and Lesser (ϕ) eigenvalue solutions of coupled oscillators, and Nobel laureate Gerald M. Eldelman's vibrating spring model of the brain, a second, the quasi-mechanical form, emerged as an infinite number of nested golden mean oscillating "springs" which had appeal in certain applications and was especially appropriate as a model for the brain [51-53].

However, in both forms, the basic hierarchical structure was identical and the underlying mathematics was preserved [16-52]. Nevertheless, when it came to what Penrose aptly labeled the missing science of consciousness [53,79,80] there is no doubt that considerable advantage is gained in understanding this subject, whose importance cannot be overstated, by using both forms simultaneously or in a complementary fashion. That way the topological interpretation and the nested oscillators' interpretation of E-Infinity theory lead to an incredibly beautiful and complete picture. This is particularly true when applied to the eminent and ingenious work on consciousness by Penrose and Hameroff and their followers [53,79,80,99] called Orchestrated Objective Reduction (Orch-OR) in which microtubules, as essentially quantum computers, structure or "orchestrate" quantum coherent conscious events. Realizing the almost self-evident tenet that the center of consciousness must be the brain and applying the nested oscillators above to it, a tantalizing similarity between the cosmos as a whole and the brain is found including golden mean holography and Lie symmetry group topology [49-52].

> "The main lure for E-Infinity as a model for the brain comes mainly from the ability of having a structure of almost infinite capacity to store and process information which can be confined in a finite portion of space. Transfinite fractals bridge with elegant simplicity the gap and the contradictions between the discrete and the continuum as well as the finite and the infinite. Taking the classical Cantor set as an example, it is a completely disjointed set, nevertheless it possesses the cardinality of the continuum. In ordinary language, a Cantor set has as many points as a continuous line." El Naschie, The Brain and E-Infinity, 2006. [52]

Edelman took his "tangle of randomly coupled and stressed elastic springs' model" as a "metaphor" for the dynamic core of his N-dimensional neural space. However, in view of the nested vibration model of E-Infinity theory [51] and its connection to biology [118,119] we view Edelman's vibrating string model as much more than a mere metaphor. As a model its dimension $\sim < n > = 4 + \phi^3$ may be seen as the result of an infinite number of elementary nested vibrations [120]. Thus, E-Infinity extends Edelman's model to n=∞ because

$$\sim < n > = (0)(\phi)^0 + (1)(\phi) + (2)(\phi)^2 + 3(\phi)^3 + \dots = 4 + \phi^3 \sum_{0}^{\infty} \ n\phi^n = 4 + \phi^3(50)$$

where ϕ may be regarded as the frequency of a unit two degrees of freedom oscillator joined in infinitely many ways and combinations following the Southwell and Denkerly eigenvalue theorems. [51,52]

27. Returning to the Mathematics of Harmony

"Let no one unversed in geometry enter here."
-Above entryway to Plato's Academy.

This paper is presenting a picture in which there appears to be a foundational mathematics of harmony lying behind nature and the cosmos. There is evidence that the ancients, including Pythagoras and Plato, were well aware of this. Euclid's *Elements* were in fact intended to make this demonstrably evident. It was the Platonic philosopher Proclus who asserted (in what has come to be called Proclus' Hypothesis) that Euclid wrote his *Elements* primarily in order to give a systematic theory of the proportional relationships of the 5 Platonic Solids culminating in their construction in the final Book XIII.The purpose behind it was to demonstrate the golden proportional harmonies and the other root ratios ($\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, etc.) reflected in the mathematics actually inherent in nature and the cosmos. Unfortunately, this underlying Pythagorean harmony of the Elements was missed by modern scholars and teachers of mathematics. Instead the primary emphasis was thought to be on the axiomatic approach to geometry.

Alexey Stakhov has spent a lifetime trying to rectify this error. Much of his writing and teaching has been centered on reestablishing the actual mathematics of harmony throughout nature and the cosmos. His focus has been centered on the role of the *golden mean number system,* including eneralized Fibonacci and golden proportions, extending its ramifications throughout a variety of fields. His work has entered into the fields of computer science, a new theory of coding and cryptography, number theory, and hyperbolic models of nature [128-130,71].

28. Concluding Remarks

Leading experimental scientists know that in any accurate experiment such as COBE and WMAP, testing a cosmological theory entails in general finding certain numbers to decide if a theory is right or wrong. Hence on this account alone when some call number theory mere numerology, then at a minimum this would be immensely misguided and short-sighted. It was the invention of the zero by the ancient Indian civilization and its sweeping implementation by the Arabs in the decimal number system that gave us a solid basis for the subsequent modern sciences. This simple step must thus be considered one of the most important intellectual steps taken by humanity undergirding modern civilization. The same could be said and much more about modern set theory and the deep meaning of countable infinity and incommensurate infinity. Even far more advanced than all of that and far more efficient than the binary number system employed in modern digital computers, is the almost miraculous *golden mean number system* which surpasses anything we could think of and deserves to be labeled a transfinite Alan Turing machine or golden supercomputer for short [41-42,119].

Thepresent paper starts from those ancient times and takes seriously the ancient theory of Pythagoras and Plato of a *golden mean number system*and uses its basic premises to solve the

mystery of dark energy of the cosmos and the fundamental question about the unity of art and scienceas well as the bigger than life question of consciousness as seen via the pioneering work of Penrose and Hameroff [6,52-54,79-80,99].

The Principles of the One and Indefinite Dyad make up the superimplicate order giving birth to the implicate order or *golden mean number system*, which in turn is enfolded or *infused into* the explicate order of Lucas and Fibonacci numbers (and root ratios) throughout nature and the cosmos. Thus, the One and Indefinite Dyad are an ontological pre-number superstructure that enters at the outset into the fabric of reality as life (hence, hylozoism – universal life) and consciousness (panpsychism – universal sentient awareness). This is the basis of nature's ability to combine and complexify in its evolutionary unfoldment of consciousness, and ultimately for the individual to enter into the climactic cosmic nonlocal state of awareness, or samadhi. David Bohm in "A new theory of the relationship of mind and matter" writes: "For the human being, all of this implies a thoroughgoing wholeness in which mental and physical sides participate very closely in each other…. There is no real division between mind and matter, psyche and soma…. Each human being participates in an inseparable way in society and in the planet as a whole…. Such participation goes on to a collective mind, and perhaps ultimately to some yet more comprehensive mind in principle capable of going indefinitely beyond even the human species as a whole." [76] This, of course,raises the more speculative question of whether or not there is a profound cosmopsychism or universal consciousness at work here. But here we must rest content with our more humble effort of proposing a possible unification of the Sciences, Arts and Consciousness through the *golden mean number system*. And we simply add the observation that it appears to have been known long ago by the recognized king of philosophers, the Pythagorean initiate of the mysteries, Aristocles, who was renamed Plato for the profound "breadth of his vision" (called *epopteia* in the mystery schools). And today it is most beautifully encoded in the magnificent E-Infinity theory for which so many have labored and contributed [1,3,86].

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