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On 2-absorbing Primal Hyperideals Of Multiplicative Hyperrings

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Abstract

Let R be a commutative multiplicative hyperring. In this paper, we introduce the concept of 2-absorbing primal hyperideals.
A non zero hyperideal I of a multiplicative hyperring R is called a 2-absorbing primal hyperideal of R if the set of all elements in R, that are not 2-absorbing prime to I forms a hyperideal of R, denoted $\mu(I) = \{d \in\ R,\ d\ is\ not\ a\ 2-absorbing\ prime\ to\ I\}$. We study properties of 2-absorbing primal hyperideals and introduce a number of results concerning 2-absorbing primal hyperideals illustrated by several examples of 2-absorbing primal hyperideals.

keywords: Multiplicative hyperring, Prime hyperideal, Primary hyperideal, irreducible hyperideal, 2-absorbing hyperideals, 2-absorbing prime hyperideals, 2-absorbing primal hyperideals.

1 Introduction

Marty Krasner was the first researcher who gave the idea of hyperstructure theory in 1983, [9]. Hyperstructures have various application in applied and pure sciences such as Latices, Geometry, Cryptography.
Automata and Artificial Intelligence.

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In the sence of Matry, a hypergroup is a nonempty set H endowed by hyperstructure $\star : H \times H \longrightarrow P^*(H)$, where $P^*(H)$ is the set of all nonempty subsets of H, which satisfy associative law and product axioms. The hyperrings were introduced by Marty Krasner. Krasner hyperrings are a generalization of classical rings in which the multiplicative is a binary operation while the addition is a hyperoperation. The theory of hyperrings has been developed by many researchers see [1], [2], [7], [16]. There are various types of hyperrings and one of the important classes of hyperrings, called multiplicative hyperring, was introduced in [7]. 2-absorbing ideals of commutative ring have been introduced and studied by Badawi in [3], and continued to 2-absorbing ideals in semirings [5]. Then 2-absorbing primary hyperideals of multiplicative hyperrings was introduced in 2018, [12]. Also in 2018, 2-absorbing primal ideals was introduced in a commutative rings, [13].

This paper continue this study on 2-absorbing ideals, we introduce the concept of 2-absorbing primal hyperideals on commutative multiplicative hyperrings. We also study the effect of good homomorphisms on these hyperideals and characterize all 2-absorbing primals of any qoutient hyperring. We illustrate the results by several examples.

1.1 Preliminaries

There are various types of hyperrings and one of the important classes of hyperrings, called multiplicative hyperring, was introduced in [7].

- $(R, +, \star)$ is called multiplicative hyperring if
  1. $(R, +)$ is abelian group.
  2. $(R, \star)$ is hypersemigroup.
  3. For any $x, y, z \in R$, we have $x \star (y + z) \subseteq x \star y + x \star z$.
  4. For any $x, y, z \in R$, we have $(y + z) \star x \subseteq y \star x + z \star x$.
  5. For any $x, y \in R$, we have $x \star (-y) = (-x) \star y = -(x \star y)$.

     Here, we mean by a multiplicative hyperring a hypersemigroup by a nonempty set R with an associative hyperoperation $\star$, i.e,

     $$x \star (y \star z) = \cup_{t \in (y \star z)} x \star t = \cup_{s \in (x \star y)} s \star z = (x \star y) \star z,$$

     for all $x, y, z \in R$.

- If R is a multiplicative hyperring with $x \star y = y \star x, \forall\ x, y \in R$, then R is called a commutative multiplicative hyperring.

- $(R, +, \star)$ is called hyperring with identity element $1_R \in R$ if $x \star 1_R = 1_R \star x = x, \forall x \in R$, [1].

Throughout this paper $(R, +, \star)$ denotes a multiplicative hyperring, and all hyperrings are assumed to be commutative with identity.

A nonempty subset A of a hyperring R is a left (right) hyperideal iff

1. $a, b \in A \Rightarrow a - b \subseteq A$
2. $a \in A$, $r \in R \Rightarrow r \star a \in A$, $(a \star r \in A)$, [2].

Remark 1.1 *In a commutative hyperring a hyperideal is left if and only if it is right. So we call hyperideal with out distinguish between right and left hyperideals.*

Remark 1.2 *Let $(Z, +, .)$ be the ring of integers. Corresponding to every subset $A \in P^*(Z)(|A| \geq 2)$, there exists a commutative multiplicative hyperring $(Z_A, +, \star)$, called multiplicative hyperring over ring of integers induced by A (or simply, multiplicative hyperring Z_A of integers), $Z_A = Z$ and for any $x,\ y \in Z_A,\ x \star y = \{x.a.y : a \in A\}$. Moreover, every hyperideal of Z_A is principal hyperideal. i.e. Z_A is the set of integers with hyperoperation $\star$ defined as before, [10].*

- A hyperring R is called Noetherian if it satisfies the ascending chain condition on hyperideals of R, a hyperring R is called Artenian if it satisfies the descending chain condition on hyperideals of R, [2].

- Let M be a proper hyperideal of a hyperring R. The hyperideal M is called a maximal hyperideal of R if the only hyperideals of R that contains M are M itself and R, [2].

- A proper hyperideal P of a hyperring R is called a prime hyperideal of R if for every pair of elements $a, b \in R$ whenever $a \star b \subseteq P$, then either $a \in P$ or $b \in P$. A prime hyperideal P of a hyperring R is called a minimal prime hyperideal over a hyperideal I of R if it is minimal (with respect to inclusion) among all prime hyperideals of R containing I, [2].

It is well known that, in a commutative unitary hyperring R, for any proper hyperideal I of R, there exists a maximal hyperideal containing I. Moreover, in such a hyperring, each maximal hyperideal is prime hyperideal, so there exists at least one prime hyperideal in R, [2].

- Let Q be a proper hyperideal of a hyperring R. The hyperideal Q is called a primary hyperideal of R if for each $a, b \in R$ whenever $a \star b \subseteq Q$, then either $a \in Q$ or $b^n \subseteq Q$ for some $n \in N$, [7].

Definition 1.1 *[12] Let C be the class of all finite hyperproducts of elements of a multiplicative hyperring R. i.e.*
$C = \{r_1 \star r_2 \star r_3 \star \ldots r_n, r_i \in R, i = 1, 2, 3, \ldots n,\ n\ is\ finite\}$*. Let I be a hyperideal of R. If for any $A_J \subseteq C$, where A_J is the class of all J hyperproducts of elements of R, $(\cup_{J=1}^{n} A_J) \cap I \neq \emptyset \Rightarrow (\cup_{J=1}^{n} A_J) \subseteq I$, then I is said to be C-union hyperideal of R and denoted by C_u-hyperideal.*

- Let I be a hyperideal of a multiplicative hyperring $(R, +, \star)$. The intersection of all prime hyperideals of R containing I, is called the prime radical of I, being denoted by $Rad(I)$, $\sqrt{I} \subseteq Rad(I)$ where

$$\sqrt{I} = \{x, x^n \subseteq I,\ for\ some\ n \in N\}.$$

The equality holds when I is a C_u-hyperideal of R.
If the multiplicative hyperring R does not have any prime hyperideal containing I, we define $Rad(I) = R$, [10].

- Let I be a proper hyperideal of a hyperring R. The hyperideal I is called a 2-absorbing hyperideal of R if $a \star b \star c \subseteq I$, then $a \star b \subseteq I$ or $b \star c \subseteq I\ or\ a \star c \subseteq I$ for any $a,\ b,\ c \in R$, [12].

- Let I be a proper hyperideal of a hyperring R. The hyperideal I is called a 2-absorbing primary hyperideal of R if $a \star b \star c \subseteq I$, and $a \star b \nsubseteq I$ then $b \star c \subseteq \sqrt{I}\ or\ a \star c \subseteq \sqrt{I}$ for any $a,\ b,\ c \in R$, [12].

Theorem 1.1 *[12] If $P_1,\ P_2$ are prime hyperideals of R, then $P_1 \cap P_2$ is a 2-absorbing hyperideal of R.*

It clear that every 2-absorbing hyperideal is a 2-absorbing primary hyperideal. The converse is not true, as is shown in the following example.

Example 1.1 *[12]*

(1) Let $R = (Z, +, \star)$ be the ring of integers for all $x,\ y \in Z$. We define the hyperoperation $x \star y = \{2xy, 5xy\}$ then $(Z, +, \star)$ is a multiplicative hyperring. The subsets $20Z = \{20n, n \in Z\}$ is a 2-absorbing primary hyperideal of Z that is not a 2-absorbing hyperideal of Z. Because $(2\star 2)\star 5 = \{80,\ 200,\ 500\} \subseteq 20Z$, but $2 \star 2 = \{20,\ 8\} \nsubseteq 20Z$ and also $2 \star 5 = \{20,\ 50\} \nsubseteq 20Z$.

Note that every primary hyperideal is a 2-absorbing primary hyperideal. In fact, let I be a primary hyperideal of R. Suppose that $a \star b \star c \subseteq I$ and $a \star b \nsubseteq I$ for any $a,\ b,\ c \in R$. Since I is a primary hyperideal, then $c \subseteq \sqrt{I}$. Hence there exist $n > 0$ such that $c^n \subseteq I$. Since I is a hyperideal, we have $a^n \star c^n \subseteq I$ *and* $b^n \star c^n \subseteq I$. Thus $a \star c \subseteq \sqrt{I}$ and $b \star c \subseteq \sqrt{I}$ and so I is a 2-absorbing primary hyperideal, [12]. The following example shows that a 2-absorbing primary hyperideal need not to be primary hyperideal.

Example 1.2 *[12]*

(1) Consider $R = (Z, +, \star)$ in Example 1.1(1). The hyperideal $20Z$ is a 2-absorbing primary hyperideal of Z. But $20Z$ is not a primary hyperideal of Z. Clearly $4 \star 5 = \{40,\ 100\} \subseteq 20Z$, but neither $4 \in 20Z$, nor $5^n \subseteq 20Z$, for any positive integer $n > 1$ and also neither $5 \in 20Z$, nor $4^n \subseteq 20Z$, for any positive integer $n > 1$.

2 On 2-absorbing Primal Hyperideal Of Multiplicative Hyperring

In this section, we introduce the concept of 2-absorbing primal hyperideal illustrated by several examples.

Definition 2.1 *An element k of R is said to be 2-absorbing prime to proper hyperideal I of R, if for any $a,\ b,\ c \in R$, $a \star b \star c \star k \subseteq I$, then $a \star b \subseteq I$ or $b \star c \subseteq I$ or $a \star c \subseteq I$.*

Definition 2.2 *An element d of R is said to be not 2-absorbing prime to proper hyperideal I of R, if there exist $a,\ b,\ c \in R$ with $a \star b \star c \star d \subseteq I$ such that $a \star b$, $b \star c$ and $a \star c \subseteq R \backslash I$. We denote by $\mu(I)$ the set of all elements in R that are not 2-absorbing prime to I.*

Definition 2.3 *Let I be a proper hypierideal of R, and $\mu(I)$ be the set of all $d \in R$ such that d is not a 2-absorbing prime to I. I is said to be 2-absorbing primal hyperideal of R if $\mu(I)$ forms a hyperideal in R.*

Definition 2.4 *An element $r \in R$ is prime to a proper hyperideal I of R, if $r \star s \subseteq I$, for any element $s \in R$, implies $s \in I$, that is, the residual*

$$(I : r) = \{s \in R, r \star s \subseteq I\} = I.$$

Note that $I \subseteq (I : r)$, for any hyperideal I. Thus r is prime to I if $(I : r) \subseteq I$.

Definition 2.5 *Let I be a hyperideal of R. The adjoint set of I, which is denoted as $adj(I) = \{a \in R : a \star b \subseteq I \ for \ some \ b \in R - I\}$. i.e. $adj(I)$ is the set of all elements that are not prime to I.*

Definition 2.6 *Let R be a multiplicative hyperring. A proper hyperideal I of R is said be primal hyperideal of R if $adj(I) = \gamma(I)$ forms a hyperideal of R.*

Lemma 2.1 *In the multiplicative hyperring of integers Z_A with scalar identity 1. Let I be a proper hyperideal of Z_A, let $\mu(I)$ be the set of elements of Z_A that are not 2-absorbing prime to I. Then $I \subseteq \mu(I)$.*

Proof. *Let $r \in I$. We can assume that $r \neq 0$ (since $0 \in \mu(I)$). As $0 \neq r = 1 \star 1 \star 1 \star r \subseteq I$ with $1 \notin I$, $1 \star 1 \nsubseteq I$, we must have r is not a 2-absorbing prime hyperideal to I, then $r \in \mu(I)$. Thus $I \subseteq \mu(I)$.*

Lemma 2.2 *Suppose that I is a proper hyperideal of R with scalar identity 1. Then $\gamma(I) \subseteq \mu(I)$.*

Proof. *Let $d \in \gamma(I)$. Then there exists $r \in R - I$ such that $r \star d \subseteq I$. Let $a = b = 1$ and $c = r$, then $a \star b \star c \star d \subseteq I$, with $a \star b$, $b \star c$ and $a \star c \subseteq R \backslash I$. Hence $d \in \mu(I)$.*

Theorem 2.1 *If I is a 2-absorbing primal hyperideal of R, with $\mu(I) \neq R$, then $\mu(I)$ is a prime hyperideal of R.*

Proof. Let $a,\ b \in R$ such that $a \star b \subseteq \mu(I)$. Then $\exists\ r,\ s,\ t \in R$, with $r \star s \star t \star (a \star b) \subseteq I$ such that $r \star s,\ r \star t$ *and* $s \star t \subseteq R \backslash I$.
Assume that $a \notin \mu(I)$. We must show that $b \in \mu(I)$.
Since $r \star (s \star b) \star t \star a \subseteq I$ and $a \notin \mu(I)$, we must have $r \star (s \star b)$ or $(s \star b) \star t$ or $r \star t \subseteq I$, but $r \star t \subseteq R \backslash I$. Thus $r \star (s \star b) \subseteq I$ *or* $(s \star b) \star t \subseteq I$. If $r \star (s \star b) \subseteq I$, since $r \star s \nsubseteq I$ then $b \in \mu(I)$. Similarly, if $(s \star b) \star t \subseteq I$, since $s \star t \nsubseteq I$ then $b \in \mu(I)$. Therefore, $\mu(I)$ is a prime hyperideal of R.

Example 2.1 *Let* $R = (Z, +, \star)$ *be the ring of integers for all* $x, y \in Z$*. We define the hyperoperation* $x \star y = \{2xy, 4xy\}$*, then* $(Z, +, \star)$ *is a multiplicative hyperring. The hyperideal* $I = 8Z$ *is a 2-absorbing primal hyperideal of* R *with* $\mu(I) = Z$*. Since* $1 \in Z$, $1 \star 1 \star 1 \star 1 = \{2, 4\} \star 1 \star 1 = \{4, 8, 16\} \star 1 = \{8, 16, 32, 64\} \subseteq 8Z = I$*, but* $1 \star 1 = \{2, 4\} \nsubseteq 8Z$*. So* $1 \in \mu(I)$*. Now, for any* $a \in Z$, $1 \star 1 \star 1 \star a = \{8a, 16a, 32a, 64a\} \subseteq 8Z = I$*, with* $1 \star 1 = \{2, 4\} \nsubseteq 8Z$*. Hence* $a \in \mu(I)$*. Therefore,* $\mu(I) = Z$*.*

Theorem 2.2 *Let* $R = (Z, +, \star)$ *be the ring of integers for all* $x, y \in Z$*. Define the hyperation:*

$x \star y = \{pxy,\ qxy,$ *where* p *and* q *are prime numbers with gcd(p, q)=1*$\}$*. Then*

(i) $I = pZ$, $J = qZ$ *are 2-absorbing primal hyperideals of* R *with* $\mu(pZ) = pZ$, $\mu(qZ) = qZ$*.*

(ii) $J = pqZ$*, is not a 2-absorbing primal hyperideal of* R *with* $\mu(pqZ) = pZ \cup qZ$*.*

Proof.
(i) Let $d \in \mu(I)$, $\exists\ a,\ b,\ c \in Z$ *such that* $a \star b \star c \star d \subseteq pZ$*. So* $\{p^2abc, pqabc, q^2abc\} \star d = \{p^3abcd, p^2qabcd, pq^2abcd, q^3abcd\} \subseteq pZ$*, implies that* p *divides any elements in* $a \star b \star c \star d$*. Thus* $p \backslash abcd$*. If* $d = 1$*, then* $p \backslash abc$*. So* $p \backslash a$ *or* $p \backslash b$ *or* $p \backslash c$*. Thus* $a \star b \subseteq pZ$ *or* $a \star c \subseteq pZ$ *or* $b \star c \subseteq pZ$*. Hence* $1 \notin \mu(I)$*, which implies that* $\mu(I) \neq Z$*. Let* $d \in pZ$*, then* $a = b = c = 1$ *satisfies* $a \star b = a \star c = b \star c = \{p,\ q\} \nsubseteq pZ$ *with* $a \star b \star c \star d = \{p^3d,\ p^2qd,\ pq^2d,\ q^3d\} \subseteq pZ$*. Hence* $d \in \mu(pZ)$*. So* $pZ \subseteq \mu(pZ)$*.*
Now, let $d \in \mu(pZ)$*. Then* $\exists\ a,\ b,\ c \in Z$ *such that* $a \star b \star c \star d \subseteq pZ$ *with* $a \star b$, $a \star c$ *and* $b \star c \nsubseteq pZ$*. Hence* $p \not\backslash a$, $p \not\backslash b$ *and* $p \not\backslash c$*. But* $p \backslash abcd$ *implies* $p \backslash d$ *and therefore,* $d \in pZ$*. So* $\mu(pZ) \subseteq pZ$ *and hence* $\mu(pZ) = pZ$ *is a hyperideal of* R*. Thus* $I = pZ$ *is a 2-absorbing primal hyperideal of* R*, with* $\mu(pZ) = pZ$*. Similarly,* $J = qZ$ *is 2-absorbing primal hyperideal of* R*, with* $\mu(qZ) = qZ$*.*

(ii) Let $d \in pZ$*, then* $a = b = 1$ *and* $c = q$ *satisfy that* $a \star b = \{p,\ q\} \nsubseteq pqZ$, $a \star c = b \star c = \{qp,\ q^2\} \nsubseteq pqZ$ *with* $a \star b \star c \star d = \{p^3qd, p^2q^2d, pq^3d, q^4d\} \subseteq pqZ$*, since* $d \in pZ$*. Thus* $pZ \subseteq \mu(pqZ)$*. Now, let* $d \in qZ$*, then* $a = b = 1$ *and* $c = p$ *satisfy that* $a \star b$, $a \star c$ *and* $b \star c \nsubseteq pqZ$ *with* $a \star b \star c \star d \subseteq pqZ$*. Then* $d \in \mu(pqZ)$*. Hence* $qZ \subseteq \mu(pqZ)$*. Therefore,* $pZ \cup qZ \subseteq \mu(pqZ)$*.*

Let $d \in \mu(pqZ)$. Then $\exists\ a,\ b,\ c \in R$ such that $a \star b \star c \star d \subseteq pqZ$ with $a \star b$, $a \star c$ and $b \star c \nsubseteq pqZ$. Thus pq divides any elements in $a \star b \star c \star d$. But $a \star b$, $a \star c$ and $b \star c \nsubseteq pqZ$ implies $a \star b \star c \nsubseteq pqZ$. Hence we have $p \backslash d$ or $q \backslash d$. Thus $d \in pZ \cup qZ$ and hence $\mu(pqZ) = pZ \cup qZ$ is not a hyperideal of R. So $J = pqZ$ is not a 2-absorbing primal hyperideal of R.

Example 2.2 *Let $R = (Z, +, \star)$ be the ring of integers for all $x, y \in Z$. We define the hyperoperation $x \star y = \{3xy, 2xy\}$, then $(Z, +, \star)$ is a multiplicative hyperring. The hyperideal $I = 2Z$ is a 2-absorbing primal hyperideal of R with $\mu(I) = 2Z$, Also The hyperideal $J = 3Z$ is a 2-absorbing primal hyperideal of R with $\mu(J) = 3Z$, by Theorem 2.2 (i).*

Example 2.3 *If R and the hyperoperation are defined as in Example 2.2, then $I = 20Z$ is a 2-absorbing primal hyperideal of R, with $\mu(I) = Z$.*
In fact, $2 \in Z$, $2 \star 2 \star 5 \star 1 = \{12, 8\} \star 5 \star 1 = \{120, 180, 80\} \star 1 = \{240, 360, 540, 160\} \subseteq 20Z$, while $2 \star 2 = \{12, 8\} \nsubseteq 20Z$, $2 \star 5 = \{20, 30\} \nsubseteq 20Z$, So $1 \in \mu(20Z)$. Also, $\forall a \in Z$, $2 \star 2 \star 5 \star a = \{240, 360, 540, 160\} \subseteq 20Z$ with $2 \star 2 \nsubseteq 20Z$, $2 \star 5 = \{20, 30\} \nsubseteq 20Z$, implies $a \in \mu(20Z)$. Thus $\mu(20Z) = Z$. Also, a hyperideal $J = 24Z$ is a 2-absorbing primal hyperideal of R, because $2,\ 3,\ 4 \in Z$, with $2 \star 3 \star 4 \star 1 = \{12, 18\} \star 4 \star 1 = \{96, 144, 216\} \star 1 = \{192, 288, 432, 648\} \subseteq J$, but $2 \star 3 = \{12, 18\} \nsubseteq J$, $2 \star 4 = \{16, 24\} \nsubseteq J$, $3 \star 4 = \{24, 36\} \nsubseteq J$. So $1 \in \mu(J)$. Thus $\mu(J) = Z$, since $\mu(J)$ is a hyperideal of R, then $J = 24Z$ is a 2-absorbing primal hyperideal of R.

Now, we start with the following result about 2-absorbing primal hyperideals of multiplicative hyperring.

Theorem 2.3 *Every prime hyperideal of R with scalar identity 1 is a 2-absorbing primal hyperideal of R, with $\mu(I) = I$.*

Proof. *Let I be a prime hyperideal of R. It is clear by Lemma 2.1 that $I \subseteq \mu(I)$. Now, let $d \in \mu(I)$. Then $\exists\ a,\ b,\ c \in R$, such that $a \star b \star c \star d \subseteq I$, with $a \star b$, $b \star c$ and $a \star c \nsubseteq I$. Therefore $a,\ b,\ c \notin I$, because I is a hyperideal of R. Now $a \star (b \star c \star d) \subseteq I$, where I is a prime hyperideal with $a \notin I$ implies that $b \star c \star d \subseteq I$. Similarly I is a prime hyperideal with $b \notin I$ implies $c \star d \subseteq I$. Now, since $c \notin I$ and I is a prime hyperideal, then $d \in I$ and therefore, $\mu(I) \subseteq I$. Thus $\mu(I) = I$ is a hyperideal of R. So I is a 2-absorbing primal hyperideal of R.*

Theorem 2.4 *Every primal hyperideal of R with scalar identity 1 is a 2-absorbing primal hyperideal.*

Proof. *Let I be a primal hyperideal of R. Then $\gamma(I)$ is a hyperideal of R, we need to show that I is a 2-absorbing primal hyperideal of R. We must show that $\mu(I)$ is a hyperideal of R. There are 2-cases:*
If $\mu(I) = R$, then I is a 2-absorbing primal hyperideal of R.
If $\mu(I) \neq R$, Then $1 \notin \mu(I)$. We show that $\gamma(I) = \mu(I)$. It is clear that $\gamma(I) \subseteq \mu(I)$, by Lemma 2.2. Let $d \in \mu(I)$, then there exist $a,\ b,\ c \in R$ with $a \star b \star c \star d \subseteq I$ such that $a \star b$, $b \star c$ and $a \star c \nsubseteq I$. If $a \star b \star c \subseteq I$, then $1 \in \mu(I)$. Thus we can show that $\mu(I) = R$ which is a contradiction. So $a \star b \star c \nsubseteq I$ and $d \in \gamma(I)$. Thus $\mu(I) \subseteq \gamma(I)$, which implies that $\gamma(I) = \mu(I)$. Therefore I is a 2-absorbing primal hyperideal.

The converse of Theorem 2.3 need not be true.

Example 2.4 *Consider the ring $(Z_4, \bigoplus, \star)$, that $\overline{a} \bigoplus \overline{b}$ and $\overline{a} \star \overline{b}$ are remainder of $\frac{a+b}{4}$ and $\frac{a.b}{4}$ which $+$ and . are ordinary addition and multiplication for all $\overline{a}, \overline{b} \in Z_4$. For all $\overline{a},\ \overline{b} \in Z_4$, we define the hyperoperation $\overline{a} \star \overline{b} = \{\overline{0},\ \overline{ab},\ \overline{2ab},\ \overline{3ab}\}$. $(Z_4, \bigoplus, \star)$ is a commutative multiplicative hyperring.*

The hyperoperation and multiplication as in the following table:

$\bigoplus$	$\overline{0}$	$\overline{1}$	$\overline{2}$	$\overline{3}$
$\overline{0}$	$\{\overline{0}\}$	$\{\overline{1}\}$	$\{\overline{2}\}$	$\{\overline{3}\}$
$\overline{1}$	$\{\overline{1}\}$	$\{\overline{2}\}$	$\{\overline{3}\}$	$\{\overline{0}\}$
$\overline{2}$	$\{\overline{2}\}$	$\{\overline{3}\}$	$\{\overline{0}\}$	$\{\overline{1}\}$
$\overline{3}$	$\{\overline{3}\}$	$\{\overline{0}\}$	$\{\overline{1}\}$	$\{\overline{2}\}$

$\star$	$\overline{0}$	$\overline{1}$	$\overline{2}$	$\overline{3}$
$\overline{0}$	$\{\overline{0}\}$	$\{\overline{0}\}$	$\{\overline{0}\}$	$\{\overline{0}\}$
$\overline{1}$	$\{\overline{0}\}$	Z_4	$\{\overline{0}, \overline{2}\}$	Z_4
$\overline{2}$	$\{\overline{0}\}$	$\{\overline{0}, \overline{2}\}$	$\{\overline{0}\}$	$\{\overline{0}, \overline{2}\}$
$\overline{3}$	$\{\overline{0}\}$	Z_4	$\{\overline{0}, \overline{2}\}$	Z_4

The hyperideals: $I_0 = \{\overline{0}\}$, $I_1 = \{\overline{0}, \overline{2}\}$, where I_1 are maximal, prime hyperideal, which implies that I_1 is a 2-absorbing primal hyperideal, with $\mu(I_1) = I_1$ which is a hyperideal of R. Note that, I_0 is not a prime hyperideal, because $\overline{2} \star \overline{2} = \{\overline{0}\} \subseteq I_0$, but $\overline{2} \notin I_0$.

I_0 is a 2-absorbing primal hyperideal. In fact, $\overline{1},\ \overline{2},\ \overline{3} \in Z_4$, $\overline{1} \star \overline{2} \star \overline{3} \star d \subseteq I_0$, with $\overline{1}\star\overline{2},\ \overline{2}\star\overline{3}$ and $\overline{1}\star\overline{3} \nsubseteq I_0$, which implies that $d = \overline{0},\ \overline{2}$. The sets of all elements in Z_4 that are not 2-absorbing primal to I_0 denoted, $\mu(I_0) = \{\overline{0}, \overline{2}\}$ which is a hyperideal of R.

The converse of Theorem 2.4 need not be true.

Example 2.5 *In Example 2.2, $I = 12Z$ is a 2-absorbing primal hyperideal of R. In fact, $2,\ 3 \in Z$, $2 \star 3 \star 2 \star 1 = \{12, 18\} \star 2 \star 1 = \{48, 72, 108\} \star 1 = \{96, 144, 216, 324\} \subseteq I$, but $2 \star 3 = \{12, 18\} \nsubseteq I$, $2 \star 2 = \{8, 12\} \nsubseteq I$. So $1 \in \mu(I)$. Thus $\mu(I) = Z$, and $\mu(I)$ is a hyperideal of R. But I is not a primal hyperideal of R, because $\gamma(I) = 3Z \cup 2Z$ is not a hyperideal of R.*

Theorem 2.5 *If I is a primary hyperideal of R with scalar identity 1, then I is a 2-absorbing primal hyperideal of R.*

Proof. *Let I be a primary hyperideal of R, we need to show that I is a 2-absorbing primal hyperideal of R, we must show that $\mu(I)$ is a hyperideal of R.*
There are 2- cases: If $\mu(I) = R$, then I is a 2-absorbing primal hyperideal of R. If $\mu(I) \neq R$. To show that I is 2- absorbing primal hyperideal of R, it is enough to show that $\mu(I) = \sqrt{I}$. Let $a \in \sqrt{I}$, then there exists smallest positive integer n, such that $a^n \subseteq I$. By induction, if $n = 1$, then $a \in I \subseteq \mu(I)$. If $n > 1$. Suppose $x \star y \star a^{n-1} \star a \subseteq I$. Let $x = y = 1$ and $z = a^{n-1}$, then $a^{n-1} \star a \subseteq I$ and $a^{n-1} \nsubseteq I$, so $x \star y,\ y \star z$ and $x \star z \nsubseteq I$, so for we get that $a \in \mu(I)$. Thus, $\sqrt{I} \subseteq \mu(I)$. Conversely, let $a \in \mu(I)$, then there exist $x,\ y,\ z \in R$ such that $x \star y \star z \star a \subseteq I$ with $x \star y,\ y \star z$ and $x \star z \subseteq R \backslash I$. Since $x \star y \nsubseteq I$, we have that $z \star a \subseteq \sqrt{I}$, because I is a primary hyperideal and since $z \nsubseteq \sqrt{I}$, because if $z \subseteq \sqrt{I}$, then let m be the smallest positve integer such that, $z^m \subseteq I$ which implies that $1 \in \mu(I)$ which is a contradiction, since we assumed that $\mu(I) \neq R$, therefore, $z^m \star a^m \subseteq I$, for some $m > 0$ and $z^m \nsubseteq I$ so, $a^m \subseteq \sqrt{I}$ implies $a \in \sqrt{I}$ and hence $\mu(I) \subseteq \sqrt{I}$. Thus, $\mu(I) = \sqrt{I}$ and so $\mu(I)$ is a hyperideal of R.

Remark 2.1 *In Example 2.2, $12Z$ is not C_u hyperideal of R.*
Because $(1 \star 1) \cup (6 \star 2) \cap 12Z = \{2, 3, 24, 36\} \cap 12Z = \{24, 36\} \neq \phi$. But $(1 \star 1) \cup (6 \star 2) = \{2, 3, 24, 36\} \nsubseteq 12Z$.

Theorem 2.6 *[10] If Q is a primary C_u hyperideal of a multiplicative hyperring $(R, +, \star)$, then $\sqrt{Q}$ is a prime hyperideal of R.*

Corollary 2.1 *Suppose that Q is a primary C_u hyperideal of R. Then Q, $\sqrt{Q}$ are 2-absorbing primal C_u hyperideals of R.*

Proof. *Follows From Theorems are 2.6, 2.3, 2.5.*

Theorem 2.7 *Let $R = (Z, \ ,+, \ \star)$ be the ring of integers for all $x, \ y \in Z$. We define the hyperoperation*

$$x \star y = \{pxy, \ qxy, \ where \ p, \ q \ are \ prime \ numbers \ with, \ gcd(p, q) \ = 1\},$$

then $(Z, +, \star)$ is a multiplicative hyperring. If $I = p^n Z$, with $n \geq 1$, then I is a 2-absorbing primal hyperideal of R with

(i) $\mu(I) = pZ$, *for $n = 1$ and $n = 2$.*

(ii) $\mu(I) = Z$, *if $n \geq 3$, with n is a positive integer.*

Proof.
(i) Follows From Theorem 2.2 (i), $I = p^n Z$ is a 2-absorbing primal hyperideal of R with $\mu(I) = pZ$, for $n = 1$.
If $d \in \mu(I)$, $n = 2$, then $\exists \ a, \ b, \ c \in Z$ such that $a \star b \star c \star d \subseteq p^2 Z$, so $\{p^3 abcd, \ p^2 qabcd, \ pq^2 abcd, \ q^3 abcd\} \subseteq p^2 Z$. Now $p^2 \backslash abcd$. If $d = 1$, which implies that $p^2 \backslash abc$. So $p^2 \backslash ab$ or $p^2 \backslash ac$ or $p^2 \backslash bc$. Thus $a \star b$ or $a \star c$ or $b \star c \ \subseteq \ p^2 Z$. Hence $1 \notin \mu(I)$, thus $\mu(I) \neq Z$. Let $a = b = 1$, $c = p$, then $a \star b = \{p, \ q\} \not\subseteq p^2 Z$, $a \star c = b \star c = \{p^2, \ pq\} \not\subseteq p^2 Z$. Therefore, $pZ \subseteq \mu(I)$. Now, let $d \in \mu(I)$, then $\exists \ a, \ b, \ c \in Z$ such that $a \star b \star c \star d \subseteq p^2 Z$, with $a \star b$, $a \star c$, and $b \star c \not\subseteq p^2 Z$. Hence $p^2 \not\backslash a$, $p^2 \not\backslash b$ and $p^2 \not\backslash c$. But $p^2 \backslash abcd$, hence $p \backslash d$, and $d \in pZ$. Thus $\mu(I) \subseteq pZ$. So $\mu(I) = pZ$, which is a hyperideal of R. Therefore, $I = p^n Z$ is a 2-absorbing primal hyperideal of R, with $\mu(I) = pZ$, for $n = 2$.

(ii) If $d \in \mu(I)$, $n = 3$, then $\exists \ a, \ b, \ c \in Z$ such that $a \star b \star c \star d \subseteq p^3 Z$, so $\{p^3 abcd, \ p^2 qabcd, \ q^2 pabcd, \ q^3 abcd\} \subseteq p^3 Z$. Now $p^3 \backslash abcd$.
Let $a = b = c = p$, then $a \star b$, $a \star c$, $b \star c = \{p^3, \ p^2 q\} \not\subseteq p^3 Z$. Therefore, $d = 1 \in \mu(I)$. Hence $\mu(I) = Z$. Similarly, for $n > 3$, the hyperideal $p^n Z$ is a 2-absorbing primal hyperideal of R with $\mu(I) = Z$.
In general, if we take the hyperideal $I = p^n Z$, with $n \geq 3$, then I is a 2-absorbing primal hyperideal of R with $\mu(I) = Z$.

Example 2.6 *If we take a hyperideal $I = 8Z$ with R and the hyperoperation as in Example 2.2, then I is a 2-absorbing primal hyperideal of R with $\mu(I) = Z$, by Theorem 2.7(ii). It is easy to see that $8Z$ is not a 2-absorbing hyperideal of R. Since $2 \star 2 \star 2 = \{32, 48, 72\} \subseteq 8Z$, while $2 \star 2 = \{8, 12\} \nsubseteq 8Z$.*

Theorem 2.8 *Let $R = (Z, +, \star)$ be the ring of integers for all $x,\ y \in Z$. Define the hyperation*

$x \star y = \{pxy,\ qxy,$ *where p and q are prime numbers with gcd(p, q)=1*$\}$. *Then*

(i) *$I = kZ$, with k is a prime number which is a relatively prime with p and q (i.e. $\gcd(p,\ q) = \gcd(p,\ k) = \gcd(q,\ k) = 1$), is a 2-absorbing primal hyperideal with $\mu(kZ) = kZ$.*

(ii) *$J = ktZ$, where k and t are prime numbers with $\gcd(k,\ p) = \gcd(k,\ t) = \gcd(k,\ q) = \gcd(t,\ q) = \gcd(p,\ q) = 1$, is not a 2-absorbing primal hyperideal of R with $\mu(ktZ) = kZ \cup tZ$.*

Proof.
*(i) Let $d \in kZ$, then $a = b = c = 1$ satisfies $a \star b$, $a \star c$ and $b \star c \nsubseteq kZ$ with $a \star b \star c \star d = \{p^3d,\ p^2qd,\ pq^2d,\ q^3d\} \subseteq kZ$. Hence $kZ \subseteq \mu(kZ)$.
Now, let $d \in \mu(kZ)$. Then $\exists\ a,\ b,\ c \in Z$ such that $a \star b \star c \star d \subseteq kZ$ with $a \star b$, $a \star c$ and $b \star c \nsubseteq kZ$. Hence $k \not| a$, $k \not| b$ and $k \not| c$. But $k \backslash abcd$ implies $k \backslash d$ and therefore, $d \in kZ$. So $\mu(kZ) \subseteq kZ$ and hence $\mu(kZ) = kZ$ is a hyperideal of R. Thus $I = kZ$ is a 2-absorbing primal hyperideal of R.*

*(ii) Let $d \in kZ$, then $a = b = 1$ and $c = t$ satisfy that $a \star b$, $a \star c$ and $b \star c \nsubseteq ktZ$ with $a \star b \star c \star d \subseteq ktZ$. Thus $kZ \subseteq \mu(ktZ)$.
Now, let $d \in tZ$, then $a = b = 1$ and $c = k$ satisfy that $a \star b$, $a \star c$ and $b \star c \nsubseteq ktZ$ with $a \star b \star c \star d \subseteq ktZ$. Hence $tZ \subseteq \mu(ktZ)$.
Therefore, $kZ \cup tZ \subseteq \mu(ktZ)$. Let $d \in \mu(ktZ)$. Then $\exists\ a,\ b,\ c \in R$ such that $a \star b \star c \star d \subseteq ktZ$ with $a \star b$, $a \star c$ and $b \star c \nsubseteq ktZ$. Thus kt divides every elements in $a \star b \star c \star d$, which implies $kt \backslash abcd$, but $kt \not| ab$, $kt \not| ac$ and $kt \not| bc$. Therefore, $kt \not| abc$. Hence we have $k \backslash d$ or $t \backslash d$. Thus $d \in kZ \cup tZ$ and hence $\mu(ktZ) = kZ \cup tZ$ is not a hyperideal of R.
So $J = ktZ$ is not a 2-absorbing primal hyperideal of R.*

In the next result, we investigate the conditions that makes $\mu(\sqrt{I}) \subseteq \mu(I)$.

Theorem 2.9 *Let I be a proper hyperideal of R and I be a 2-absorbing primal hyperideal of R. If $\sqrt{I}$ is also a 2-absorbing primal hyperideal of R, then $\mu(\sqrt{I}) \subseteq \mu(I)$.*

Proof. *Let $a \in \mu(\sqrt{I})$, then there exist $r,\ s,\ t \in R$ with $r \star s \star t \star a \subseteq \sqrt{I}$ such that $r \star s,\ r \star t$ and $s \star t \subseteq R \backslash \sqrt{I}$, so there exists n ¿ 0 such that $r^n \star s^n \star t^n \star a^n \subseteq I$ and since $r^n \star s^n \nsubseteq I,\ r^n \star t^n \nsubseteq I$ and $s^n \star t^n \nsubseteq I$, then $a^n \subseteq \mu(I)$. Thus $a \in \sqrt{\mu(I)} = \mu(I)$, since $\mu(I)$ is a prime hyperideal in R or $\mu(I) = R$, by Theorem 2.1. Therefore, $\mu(\sqrt{I}) \subseteq \mu(I)$.*

Theorem 2.10 *Let I be a proper hyperideal of R with scalar identity 1. If $\sqrt{I}$ is a prime hyperideal of R, then I be a 2-absorbing primal hyperideal of R.*

Proof. *We shall prove that I is a primary hyperideal of R. Let $c \star d \subseteq I$, with $c \notin \sqrt{I}$. Then $c \star d \subseteq \sqrt{I}$, and $c \notin I$, since $I \subseteq \sqrt{I}$. So $d \in \sqrt{I}$, because $\sqrt{I}$ is a prime hyperideal. Hence there exists positive integer $n > 0$, such that $d^n \subseteq I$. Thus I is a primary hyperideal, with scalar identity 1. Therefore, by Theorem 2.5, I be a 2-absorbing primal hyperideal of R.*

The converse of Theorem 2.10 need not be true.

Example 2.7 *In Example 2.5, $I = 12Z$ is a 2-absorbing primal hyperideal of R. But $\sqrt{12Z} = 6Z$ is not a prime hyperideal of R. Because $2 \star 3 = \{12, 18\} \subseteq \sqrt{12Z}$ with neither 2 nor 3 in $\sqrt{12Z}$. Thus $\sqrt{12Z}$ is not a prime hyperideal of R.*

We will shown in the next example that if I is a 2-absorbing primal hyperideal of R, then $\sqrt{I}$ need not to be 2-absorbing primal hyperideal of R.

Example 2.8 *Continue Example 2.7, $I = 12Z$ is a 2-absorbing primal hyperideal of R. But $\sqrt{12Z} = 6Z$ is not a 2-absorbing primal hyperideal of R, with $\mu(\sqrt{12Z}) = 2Z \cup 3Z$ is not a hyperideal of R, since by Theorem 2.2 (ii).*

Example 2.9 *The hyperideal $\sqrt{8Z} = Z$ in Example 2.1. Note that $1^2 = \{2, 4\}, 1^3 = \{4, 8, 16\}, 1^4 = \{8, 16, 32, 64\} \subseteq 8Z$. So $1 \in \sqrt{8Z}$. Hence $\sqrt{8Z} = Z$. But in Example 2.2, $\sqrt{8Z} = 2Z$ is a prime hyperideal of R.*

Corollary 2.2 *[4] If $I_1,\ I_2,\ I_3, \ldots,\ I_n$ are hyperideals of a hyperring R, then $\bigcap_{i=1}^{n} I_i$ is a hyperideal of R.*

Theorem 2.11 *Let P be a prime hyperideal of R with scalar identity 1 and let $I_1,\ I_2,\ I_3, \ldots,\ I_n$ be 2-absorbing primal hyperideals of R, such that $\sqrt{I_i} = P$, for any $i = 1, 2, 3,\ \ldots,\ n$, then $\bigcap_{i=1}^{n} I_i$ is a 2-absorbing primal hyperideal of R.*

Proof. *Clearly $\sqrt{\bigcap_{i=1}^{n} I_i} = \bigcap_{i=1}^{n} \sqrt{I_i} = P$. Suppose $I_1,\ I_2,\ I_3,\ \ldots,\ I_n$ are 2-absorbing primal hyperideals of R, then $\mu(I_1),\ \mu(I_2),\ \ldots,\ \mu(I_n)$ forms hyperideals of R, so by Corollary 2.2, $\bigcap_{i=1}^{n} \mu(I_i)$ is a hyperideal of R. Since $\sqrt{I_i} = P$ is a prime hyperideal of R, then $\sqrt{I_i}$ is a 2-absorbing primal hyperideal of R, which implies that by Theorem 2.9, $\mu(\sqrt{I_i}) \subseteq \mu(I_i)$. Thus, $\bigcap_{i=1}^{n} \mu(\sqrt{I_i}) = \mu(P) \subseteq \bigcap_{i=1}^{n} \mu(I_i)$. Therefore, $\mu(\bigcap_{i=1}^{n} I_i) = \bigcap_{i=1}^{n} \mu(I_i)$ forms a hyperideal of $\bigcap_{i=1}^{n} I_i$. Thus $\bigcap_{i=1}^{n} I_i$ is a 2-absorbing primal hyperideal of R.*

Remark 2.2 *Let P be a proper hyperideal of R and let $I_1,\ I_2,\ I_3,\ \ldots,\ I_n$ be 2-absorbing primal hyperideals of R, such that $\sqrt{I_i} = P$, for any $i = 1,\ 2,\ 3,\ \ldots,\ n$, then $\bigcap_{i=1}^{n} I_i$ need not to be 2-absorbing primal hyperideal.*

Example 2.10 *Let $I = 12Z, J = 24Z$. Then $\sqrt{I} = \sqrt{J} = 6Z$ with respect the hyperoperation defined in Example 2.2. I, J are 2-absorbing primal hyperideals of R, see Examples 2.3 and 2.5 in which $I \bigcap J = 6Z$. But $I \bigcap J$ is not a 2-absorbing primal hyperideal of R, see Example 2.8.*

Note that, if I and J are 2-absorbing primal hyperideals of R with $\sqrt{I} \neq \sqrt{J}$, then $I \bigcap J$ may not be 2-absorbing primal hyperideal of R.

Example 2.11 *In Example 2.2, if we take $I = 5Z,\ J = 7Z$, then $I,\ J$ are 2-absorbing primal hyperideals of R, with $\mu(I) = I,\ \mu(J) = J$, by Theorem 2.8 (i). Now $\sqrt{I} \neq \sqrt{J}$, since $\sqrt{I} = 5Z$ and $\sqrt{J} = 7Z$. Note that $I \bigcap J = 35Z$ is not a 2-absorbing primal hyperideal of R, with $\mu(35Z) = 5Z \bigcup 7Z$ is not a hyperideal of R, by Theorem 2.8 (ii).*

Example 2.12 *If $I = 12Z, J = 20Z$ are two 2-absorbing primal hyperideals of R with $\mu(I) = \mu(J) = Z$, see Examples 2.3 and 2.5, and $\sqrt{I} \neq \sqrt{J}$, where $\sqrt{I} = 6Z,\ \sqrt{J} = 10Z$. It easy to see that $I \bigcap J = 30Z$ is a 2-absorbing primal hyperideal of R. To explain this, $5,\ 3,\ 2 \in Z$, $5 \star 3 \star 2 \star 1 = \{30, 45\} \star 2 \star 1 = \{120, 180, 270\} \star 1 = \{240, 360, 540, 810\} \subseteq 30Z$, while $5 \star 3,\ 3 \star 2$ and $5 \star 2 \nsubseteq 30Z$. So $1 \in \mu(30Z)$. Thus $\mu(30Z) = Z$, which is a hyperideal of R.*

Remark 2.3

(i) *A 2-absorbing primal hyperideal of R need not to be 2-absorbing hyperideal. From Example 2.12,* $30Z$ *is a 2-absorbing primal hyperideal of R. But* $30Z$ *is not a 2-absorbing hyperideal of R. Note that,* $2\star 3\star 5=\{120,180,270\}\subseteq 30Z$*, while* $2\star 3=\{12,18\}$*,* $2\star 5=\{20,30\}$*,* $3\star 5=\{30,45\}\nsubseteq 30Z$*. Also,* $I=8Z$*, see Example 2.6.*

(ii) *A 2-absorbing hyperideal of R need not to be 2-absorbing primal hyperideal. From Example 2.2,* $6Z$ *is a 2-absorbing hyperideal of R. Let* $P_1=2Z$*,* $P_2=3Z$*, then* $P_1\cap P_2=6Z$ *is a 2-absorbing hyperideal of Z, since,* P_1*,* P_2 *are prime hyperideals, by Theorem 1.1. But* $6Z$ *is not a 2-absorbing primal hyperideal of R, see Example 2.8.*

Definition 2.7 *Let I be a proper hyperideal of a hyperring R. The hyperideal I is called an irreducible hyperideal of R if* $I=J\cap K$*, where J, K are hyperideals of R, implies* $I=J$ *or* $I=K$*, [13].*

Theorem 2.12 *Let I be an irreducible hyperideal of R, then I is a 2-absorbing primal hyperideal of R.*

Proof. *To prove that I is a 2-absorbing primal hyperideal of R. We must show that* $\mu(I)$ *is a hyperideal of R. If* $\mu(I)=R$*, then I is a 2-absorbing primal hyperideal of R.*
Therefore, we may assume that $\mu(I)\neq R$*. Let* $a,\ b\in\mu(I)$*, then* $\exists\ x,\ y,\ z\in R$*, with* $x\star y,\ x\star z,\ y\star z\nsubseteq I$*, such that* $x\star y\star z\star a\subseteq I$*.*
If $I=(I:a)$*, then* $x\star y\star z\subseteq I$ *which implies that* $1\in\mu(I)$*. So* $\mu(I)=R$*, a contradiction. Therefore,* $I\subset(I:a)$*, similarly* $I\subset(I:b)$*.*
Thus, $I\subset(I:a)\bigcap(I:b)\subseteq(I:a+b)$*, since if* $I=(I:a)\bigcap(I:b)$*, then* $I=(I:a)$ *or* $I=(I:b)$*, hence* $a+b\in I$*, then* $a+b\in\mu(I)$*, by Lemma 2.1. Moreover, if* $r\in R,\ a\in\mu(I)$*, then* $I\subset(I:a)\subseteq(I:r\star a)$ *which implies that* $r\star a\subseteq\mu(I)$*. Hence* $\mu(I)$ *is a hyperideal of R and the proof is complete.*

Theorem 2.13 *[4] Let* $(H,\star)$ *be group and let* $G=H\cup\{0,u,\nu\}$ *where* $u,\ \nu$ *are orthogonal idempotent elements and* $u\neq\nu$ *i.e.* $u\nu=\nu u=0$ *and* $u^2=u,\ \nu^2=\nu$*. Define the hyperaddition on G by*

$$g+0=0+g=\{g\}\ \text{for all}\ g\in G.$$
$$g+g=\{g,0\}\ \text{for all}\ g\in G.$$
$$\text{if}\ g_1\neq g_2, g_1+g_2=G\backslash\{g_1,g_2,0\}\ \text{for all}\ g_1,g_2\in G\backslash\{0\}.$$

The multiplication can be defined as,

$$g \star 0 = 0 \star g = \{0\} \text{ for all } g \in G.$$

$h \star u = u \star h = u$ $h \star \nu = \nu \star h = \nu$ *for all* $h \in H$, *and* $u \star \nu = \nu \star u = 0$. *Then* $(H, +, \star)$ *is a hyperring.*

Example 2.13 *Consider the set* Z_6, *let* $H = Z_6^* = \{1, 5\}$ *and the orthogonal idempotent elements of* Z_6 *are* 3, 4 *because* $3.4 = 0,\ 3^2 = 3,\ 4^2 = 4$.
Let $G = H \cup \{0,\ 3,\ 4\}$, *implies* $(G,\ +,\ \star)$ *is a hyperring. The hyperaddition and multiplication as in the following table:*

$+$	0	3	4	5	1
0	{0}	{3}	{4}	{5}	{1}
3	{3}	{0, 3}	{1, 5}	{ 1,4}	{5,4}
4	{4}	{1, 5}	{0, 4}	{3,1}	{3,5}
5	{5}	{1, 4}	{1, 3}	{0,5}	{3,4}
1	{1}	{1, 4}	{3, 5}	{3, 4}	{0, 1}

$\star$	0	3	4	5	1
0	{0}	{0}	{0}	{0}	{0}
3	{0}	{3}	{0}	{3}	{3}
4	{0}	{0}	{4}	{4}	{4}
5	{0}	{3}	{4}	{1}	{5}
1	{0}	{3}	{4}	{5}	{1}

The hyperideals: $I_1 = \{0, 3\}$, $I_2 = \{0, 4\}$, *where* I_1, I_2 *are maximal, irreducible, prime hyperideals, which implies that* I_1, I_2 *are* 2*-absorbing primal hyperideals with* $\mu(I_1) = \{0, 4\}$, $\mu(I_2) = \{0, 3\}$. *But* $I_1 \cap I_2 = \{0\}$ *is not a* 2*-absorbing primal hyperideal. Note that, Let* $3 \in G$, $3 \star 3 \star 3 \star d \subseteq I_1 \cap I_2$, *while* $3 \star 3 = \{3\} \nsubseteq I_1 \cap I_2$, *then d= 0, 4. Let* $4,\ 1,\ 5 \in G$, $4 \star 1 \star 5 \star d \subseteq I_1 \cap I_2$, *while* $4 \star 1 = \{4\}$, $1 \star 5 = \{5\}$, $4 \star 5 = \{4\} \nsubseteq I_1 \cap I_2$, *then* $d = 0,\ 3$. *Therefore,* $\mu(I_1 \cap I_2) = \{0, 3, 4\}$ *is not a hyperideal. Because* $3 + 4 = \{1, 5\} \nsubseteq I_1 \cap I_2$.

Definition 2.8 *[8] Let* R_1 *and* R_2 *be two hyperrings. A mapping* ϕ *from* R_1 *into* R_2 *is called a homomorphism if (i)* $\phi(a + b) \subseteq \phi(a) + \phi(b)$ *(ii)* $\phi(ab) \subseteq \phi(a)\phi(b)$ *and (iii)* $\phi(0) = 0$ *hold for all* $a, b \in R_1$.
The mapping ϕ *is called a good homomorphism or a strong homomorphism if (i)* $\phi(a + b) = \phi(a) + \phi(b)$ *(ii)* $\phi(ab) = \phi(a)\phi(b)$ *and (iii)* $\phi(0) = 0$ *hold for all* $a, b \in R_1$.

Definition 2.9 *[8] A homomorphism (resp., strong homomorphism). A mapping ϕ from hyperring R_1 into hyperring R_2 is said to be an isomorphism (res., strong isomorphism) if ϕ is one to one and onto. If R_1 is strongly isomorphic to R_2, then it is denoted by $R_1 \cong R_2$.*

Theorem 2.14 *[7] Let $f : R \longrightarrow S$ be a good homorphism and I, J be hyperideals of R and S, respectively. Then the followings are satisfied:*

(i) *If I is a C_u hyperideal of R containing $Ker(f)$ and f is an epimorphism, then $f(I)$ is a C_u hyperideal of S.*

(ii) *If J is a C_u hyperideal of S, then $f^{-1}(J)$ is a C_u hyperideal of R.*

Theorem 2.15 *Let $f : R \longrightarrow S$ be a good homorphism and I, J be proper hyperideals of R and S, respectively. Then the followings are satisfied:*

(i) *If I is a 2-absorbing primal hyperideal of R containing $Ker(f)$ and f is an epimorphism, then $f(I)$ is a 2-absorbing primal hyperideal of S.*

(ii) *If J is a 2-absorbing primal hyperideal of S, then $f^{-1}(J)$ is a 2-absorbing primal hyperideal of R.*

Proof. *(i) It is clear that by Theorem 2.14, $f(I)$ is a hyperideal of S. It is enough to show that $f(\mu(I)) = \mu(f(I))$ is a hyperideal of S.*
Let $y_1, \ y_2 \in f(\mu(I))$ and $s \in S$. Since f is onto, then there exist $x_1, \ x_2 \in \mu(I), \ r \in R$ such that $f(x_1) = y_1, \ f(x_2) = y_2$ and $f(r) = s$. Since I is a 2-absorbing primal hyperideal of R. Then $\mu(I)$ is a hyperideal of R, then $x_1 - x_2 \subseteq \mu(I), \ r \star x_1 \in \mu(I)$. So that

$$y_1 - y_2 = f(x_1) - f(x_2) = f(x_1 - x_2) \subseteq f(\mu(I)), \text{ and}$$
$$s \star y_1 = f(r) \star f(x_1) = f(r \star x_1) \subseteq f(\mu(I)).$$

So $f(\mu(I))$ is a hyperideal of S.
Finally, let $a \in \mu(f(I))$, then $a \in S$, and $\exists \ r_1, \ s_1, \ t_1 \in S$ such that $r_1 \star s_1 \star t_1 \star a \subseteq f(I)$, with $r_1 \star s_1, \ s_1 \star t_1$ and $r_1 \star t_1 \nsubseteq f(I)$. Since f is onto, then $\exists \ r, \ s, \ t, \ b \in R$ such that $f(r) = r_1, \ f(s) = s_1, \ f(t) = t_1$ and $f(b) = a$. Now, $f(r \star s \star t \star b) = f(r) \star f(s) \star f(t) \star f(b) = r_1 \star s_1 \star t_1 \star a \subseteq f(I)$. Thus $r \star s \star t \star b \subseteq I$, with $f(r \star s) = f(r) \star f(s) \nsubseteq f(I), \ f(r \star t) = f(r) \star f(t) \nsubseteq f(I)$ and $f(s \star t) = f(s) \star f(t) \nsubseteq f(I)$. Hence $r \star s, \ r \star t$ and $s \star t \nsubseteq I$. Thus $b \in \mu(I)$. Therefore, $a = f(b) \in f(\mu(I))$.

Conversly, let $y \in f(\mu(I))$, *implies* $y = f(d) \in S$ *for some* $d \in \mu(I)$. *Then there exist* $b,\ c,\ l \in R$ *such that* $b \star c \star l \star d \subseteq I$, *with* $b \star c,\ c \star l$ *and* $b \star l \nsubseteq I$. *Hence* $f(b \star c \star l \star d) = f(b) \star f(c) \star f(l) \star y \subseteq f(I)$, *with* $f(b) \star f(c)$, $f(c) \star f(l)$ *and* $f(b) \star f(l) \nsubseteq f(I)$. *Hence* $y = f(d) \in \mu(f(I))$. *So,* $\mu(f(I)) = f(\mu(I))$. *Therefore,* $f(I)$ *is a 2-absorbing primal hyperideal of* S.

(ii) It easy to see that $f^{-1}(J)$ *is a hyperideal of* R. *It is enough to show that* $f^{-1}(\mu(J)) = \mu(f^{-1}(J))$ *is a hyperideal of* R. *Let* $a_1,\ a_2 \in f^{-1}(\mu(J))$, $r \in R$, *then* $f(a_1),\ f(a_2) \in \mu(J),\ f(r) \in S$. *Since* J *is a 2-absorbing primal hyperideal of* S. *Then* $\mu(J)$ *is a hyperideal of* S.
So that $f(a_1) - f(a_2) = f(a_1 - a_2) \subseteq \mu(J)$, *and also* $f(r) \star f(a_1) = f(r \star a_1) \subseteq \mu(J)$. *Therefore* $a_1 - a_2 \subseteq f^{-1}(\mu(J))$, *and also* $r \star a_1 \subseteq f^{-1}(\mu(J))$.
Hence, $f^{-1}(\mu(J))$ *is a hyperideal of* R. *Finally, let* $b \in \mu(f^{-1}(J))$ *and* $b \in R$, *then* $\exists\ r,\ s,\ t \in R$ *such that* $r \star s \star t \star b \subseteq f^{-1}(J)$, *with* $r \star s,\ s \star t$ *and* $r \star t \nsubseteq f^{-1}(J)$. *Then* $\exists\ r_1,\ s_1,\ t_1,\ a \in S$ *such that* $f^{-1}(r_1) = r,\ f^{-1}(s_1) = s$, $f^{-1}(t_1) = t$ *and* $f^{-1}(a) = b$. *Now,*
$f^{-1}(r_1 \star s_1 \star t_1 \star a) = f^{-1}(r_1) \star f^{-1}(s_1) \star f^{-1}(t_1) \star f^{-1}(a) = r \star s \star t \star b \subseteq f^{-1}(J)$. *Thus* $r_1 \star s_1 \star t_1 \star a \subseteq J$, *with* $f^{-1}(r_1 \star s_1) = f^{-1}(r_1) \star f^{-1}(s_1) \nsubseteq f^{-1}(J)$, $f^{-1}(r_1 \star t_1) = f^{-1}(r_1) \star f^{-1}(t_1) \nsubseteq f^{-1}(J)$ *and* $f^{-1}(s_1 \star t_1) = f^{-1}(s_1) \star f^{-1}(t_1) \nsubseteq f^{-1}(J)$. *Hence* $r_1 \star s_1,\ r_1 \star t_1$ *and* $s_1 \star t_1 \nsubseteq J$.
Hence $a \in \mu(J)$. *Thus* $b = f^{-1}(a) \in f^{-1}(\mu(J))$.
Conversly, let $x \in f^{-1}(\mu(J))$, *then* $\exists\ y \in \mu(J)$ *such that* $f^{-1}(y) = x$, *which implies that there exists* $y_1,\ y_2,\ y_3 \in S$ *such that* $y_1 \star y_2 \star y_3 \star y \subseteq J$, *with* $y_1 \star y_2,\ y_2 \star y_3$ *and* $y_1 \star y_3 \nsubseteq J$.
Hence $f^{-1}(y_1 \star y_2 \star y_3 \star y) = f^{-1}(y_1) \star f^{-1}(y_2) \star f^{-1}(y_3) \star f^{-1}(y) \subseteq f^{-1}(J)$, *with* $f^{-1}(y_1) \star f^{-1}(y_2),\ f^{-1}(y_2) \star f^{-1}(y_3)$ *and* $f^{-1}(y_1) \star f^{-1}(y_3) \nsubseteq f^{-1}(J)$.
Hence $x = f^{-1}(y) \in \mu(f^{-1}(J))$. *Thus,* $f^{-1}(\mu(J)) = \mu(f^{-1}(J))$.
Therefore, $f^{-1}(J)$ *is a 2-absorbing primal hyperideal of* R.

Suppose that I is a hyprideal of R. Then qoutient abelian group $R/I = \{c + I :\ c \in R\}$, becomes a hyperring with the multiplication

$$(c + I) \star (d + I) = \{r + I : r \in c \star d\}.$$

In this case R/I is called quotient hyperring. One can show that all hyperideal of R/I is of the form J/I where J is a hyperideal of R containing I, since the natural homomorphism $\phi : R \longrightarrow R/I,\ \phi(r) = r + I$ is a good epimorphism, [7].
The next theorem investigate the relation between the 2-absorbing primal hyperideals of R and R/I, for some hyperideals I of R containing J.

Theorem 2.16 *Let I, J be proper hyperideals of R, with $J \subseteq I$. Then I is a 2-absorbing primal hyperideal of R iff I/J is a 2-absorbing primal hyperideal of R/J.*

Proof. To prove this result, we must show that $\mu(I/J) = \mu(I)/J$.
Let $a + J \in \mu(I/J)$, then there exist $r + J,\ s + J,\ t + J \in R/J$ with $r \star s \star t \star a + J \subseteq I/J$ such that $r \star s + J,\ r \star t + J,\ s \star t + J \nsubseteq I/J$. So $r \star s \star t \star a \subseteq I$ with $r \star s,\ r \star t,\ s \star t \nsubseteq I$. $Hence\ a \in \mu(I)$, therefore, $a + J \in \mu(I)/J$.
Conversely, let $a + J \in \mu(I)/J$, which implies that $a \in \mu(I)$, so there exist $r,\ s,\ t \in R$ with $r \star s \star t \star a \subseteq I$ such that $r \star s,\ r \star t,\ s \star t \nsubseteq I$. Therefore, $r + J,\ t + J, s + J \in R/J$ with $r \star s \star t \star a + J = (r \star s \star t + J)(a + J) \subseteq I/J$ such that $r \star s + J,\ r \star t + J,\ s \star t + J \nsubseteq I/J$ and so $a + J \in \mu(I/J)$. Hence $\mu(I/J) = \mu(I)/J$. The proof is complete.

From Theorem 2.16, we have the following main result.

Lemma 2.3 *Let J be a proper hyperideal of R, then there is one to one correspondence between 2-absorbing primal hyperideal I of R containing J and 2-absorbing primal hyperideal I/J of R/J.*

Corollary 2.3 *Let $f : R \longrightarrow S$ be a good homorphism and I, J be hyperideals of R and S, respectively. Then the followings are satisfied:*

(i) *If I is a C_u primary hyperideal containing $Ker(f)$ and f is an epimorphism, then $f(I)$ is a 2-absorbing primal hyperideal of S.*

(ii) *If J is a C_u primary hyperideal of S, then $f^{-1}(J)$ is a 2-absorbing primal hyperideal of R.*

Proof. (i) and (ii) Follows from Theorem 2.15 and Corollary 2.1.

Corollary 2.4 *[7] Suppose that $I \subseteq J$ are hyerideals of R. Then $\sqrt{J/I} = \sqrt{J}/I$.*

Corollary 2.5 *Let $I \subseteq Q$ be hyperideals of R with scalar identity 1 then*

(i) *If Q is primary hyperideal of R. Then Q/I is a 2-absorbing primal hyperideal of R/I.*

(ii) *If Q is a C_u primary hyperideal of R containing I. Then $\sqrt{Q}/I$ is a 2-absorbing C_u primal hyperideal of R/I.*

Proof.
(i) Follows from Theorem 2.5, 2.16.
(ii) Follows from Corollary 2.1, Theorem 2.16.

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