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From Vortex Mathematics to Smith Numbers: Demystifying Number Structures and Establishing Sieves Using Digital Root

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Abstract: Proficiency in number structures depends on a continuous development and blending of intricate combinations of different types of numbers and its related characteristics. The purpose of this paper is to unpack the mechanisms and underlying notions that elucidate the potential process of number construction and its inherent structures. By employing the concept of digital root, we show how juxtaposed assumptions can play in delineating generalized models of number structures bridging the abstract, the numerical, and the physical worlds. While there are numerous proposed ways of constructing *Smith* numbers, developing a generalized algorithm could help provide a unified approach to generating number structures with inherent commonalities.

In this paper, we devise a sieve for all *Smith* numbers as well as other related numbers. The sieve works on the principle of digital roots of both S_d(N), the sum of the digits of a number *N* and that of S_p(N), the sum of the digits of the extended prime divisors of *N*. Starting with $S_p(N) = S_p(p.q.r\ldots)$, where p,q,r,…, are the prime divisors whose product yields N and whose digital root (n) equals to that of S_d(N) thus $S_d(N) = n + 9x$; $x \in$ N. The sieve works on finding the proper value of *x* that renders a *Smith* number *N*. In addition to the sieve, new related numbers could emerge.

Keywords: Digital roots; *Smith* numbers; Vortex mathematics; Prime factorization; *Hoax numbers.*

1. Introduction

Nikola Tesla, the worldwide prominent electrical engineer and physicist famously remarked "If you only knew the magnificence of the 3,6, 9 then you have the key to the universe" (1919).

Ancient beliefs that a divine element is present in numbers have been timeless. The inherent perception that to have power of numbers is to have control over how the world works is deeply embedded in the human mind. To describe things using numbers is an essential step toward knowing and building awareness of the prime reality we live in. Hence, a mastery of numbers has always been seen as a necessary means of insight into the human centers of imagination.

Proficiency in number structures depends on a continuous development and blending of intricate combinations of different types of numbers and its related characteristics. Hence, a deep conceptual understanding of the definitions underlying the structures of different types of numbers is essential to facilitate systematic computation strategies and to establish possible relationships among the different rules. *Smith* numbers, *hoax* numbers, *beast* numbers (Wang, 1994) and numerous other related numbers illustrate the remarkable mechanisms that humans have created and appropriated to generate infinite abstract number structures. Such numbers although detached from counting concrete objects and uncommonly used, however, they are set and ready. As educators, we arguably perceive this insight as important as it is simple.

The purpose of this paper is to unpack the mechanisms and underlying notions that elucidate the potential process of number construction and its inherent structures. The ultimate goal is to shed some light on the role that juxtaposed assumptions can play in delineating generalized models of number structures bridging the abstract, the numerical, and the physical worlds. To illustrate our perspective, we employ the concept of digital root to explore numerical and functional underpinnings of what has been called "vortex-based mathematics" in relationship to electromagnetic fields. We further examine numbers generated through the employment of digital root mechanisms such as *Smith* numbers, and *Hoax* numbers and we propose "new" related numbers based on similar assumptions. The fundamental premise underlying our stance is to provide insight on the versatility, interdisciplinary and the wide scope of application of several mathematical concepts such as digital root.

2. Digital Root

By definition, the *digital root*(or repeated digital sum) of an integer N is a single-digit integer n, designated by $\rho(N) = n$, obtained by successive additions of the digits of N and of those of the outcomes (Hoffmann, 1998). In other words, if the sum of the digits of N, designated by $S_d(N)$, is more than 9 then these digits are added again and again until a one-digit sum is obtained. In fact, this is similar to the old process of casting out nines from N. In modern terms, we use modular arithmetic with a modulo operator (*mod*) to denote: $N \equiv \rho(N) \equiv n \ (mod\ 9)$; $n \neq 0$.

Example:

$N = 75{,}342{,}873$, has $S_d(N) = 39$, where $3 + 9 = 12$ and $1 + 2 = 3$; hence $\rho(75{,}342{,}873) = 3$ or $75{,}342{,}873 \equiv 3 \ (mod$ $9)$.

Thus, all integers in N fall into nine sets, called residual classes modulo 9, denoted as:

$\overline{1}, \overline{2}, \overline{3}, \overline{4}, \overline{5}, \overline{6}, \overline{7}, \overline{8}$, and $\overline{9}$

The set $E = \{\overline{1}, \overline{2}, \overline{3}, \overline{4}, \overline{5}, \overline{6}, \overline{7}, \overline{8}, \overline{9}\}$forms under the multiplication operation in Abelian group.

3. Applications of Digital Root: Vortex-Based Mathematics

An interesting significance of digital root lies in its inherent potential to uncover symmetrical and cyclical properties of specific number groups such as figurate numbers and Fibonacci sequence using a combination of geometric and numerical depiction (Ghannam, 2012). A consideration of digital roots has prompted the basics for the development of what has been known as vortex-based mathematics (Rodin, 2010). The main premise of vortex-based mathematics, is that unobserved or invisible energy can be mathematically modeled following oscillating paths between certain numbers. It is believed that such energy could be the driving force behind reality and the initial impulse form behind creation. The energy path is characterized by a coil motion following a logarithmic spiral of infinity that is non-decaying and eternal. To describe the path of this energy, proponents of vortex-based mathematics use what they call a "circle of life" or a "circle of enlightenment", a seemingly mathematical decryption and a model for sound and harmonics. The underlying hypothesis is that since simple, base 10 single-digit numbers follow specific patterns, these numbers depict a rhythmic and polarized motion creating the effects that make visible the phenomenon they represent. In the realm of vortex mathematics, unfolding all the patterns that underlie a combination of these numbers using the digital root functionality helps model a higher dimensional energy. In a clockwise direction, we simply denote the digits 1 through 9 on a circle as seen in Fig.1.

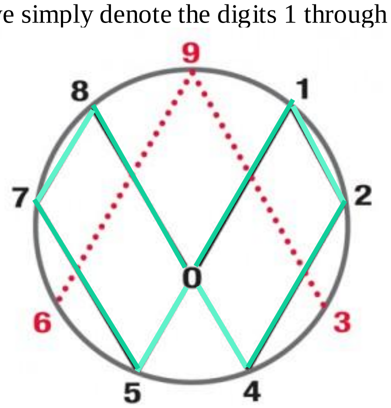


Fig. 1 A cyclical depiction of the single-digit numbers.

On the circle, the numbers are connected using straight line vectors signifying doubling and taking the digital root of the resultant. For example, 2 is the doubling of 1 hence it is connected to 1, 4 is the doubling of 2, 8 is the doubling of 4, 16 is the doubling of 8 taking its digital root it becomes 1+6=7, hence 8 is connected to 7, 32 is the doubling of 16, whose digital root is 5, hence 7 is connected to 5 and finally, 64 is the doubling of 32, and its digital root is 1, closing the circle. We can continue indefinitely doubling and taking the digital roots of numbers around the same bounded ∞-shaped trajectory that is constantly an indefinitely in motion as the numbers increase. For example, if we take the number 22 whose digital root is 2+2= 4, then doubling 22 gives 44 whose digital root is 4+4= 8. No matter what combination we take, there is no way to break this doubling sequence.

Furthermore, the sequencing of the digits depicted in the circle shown in Fig. 1, comprises three major elements: the infinity symbol composed of a path connecting the digits (1248751); the pyramid denoted by a trajectory (396) and the primal point of unity denoted by 0. Predictably, underlying the process of doubling and taking the digital root of the results are patterns of single-digit numbers sequenced on the circle as per the three elements. The sequence also holds true and is valid using halving, the inverse of doubling (See Fig. 2). For example, half of 1 is 0.5 hence 1 is connected to 5, and half of 0.5 is 0.25 whose digital root is 7, and half of 0.25 is 0.125 with a digital root equals 8, half of 0.125 gives 0.0625 with a digital root of 4, half of 0.0625 is 0.03125 with a digital root equals 1, half of 0.03125 is 0.015625 whose digital root is 1, and half of 0.015625 is 0.0078125 which closes the circuit with a digital root equals 5. Similarly, starting with any number, say 221 whose digital root is 2+2+1=5, then half of 221 has a digital root equals 7 and so on.

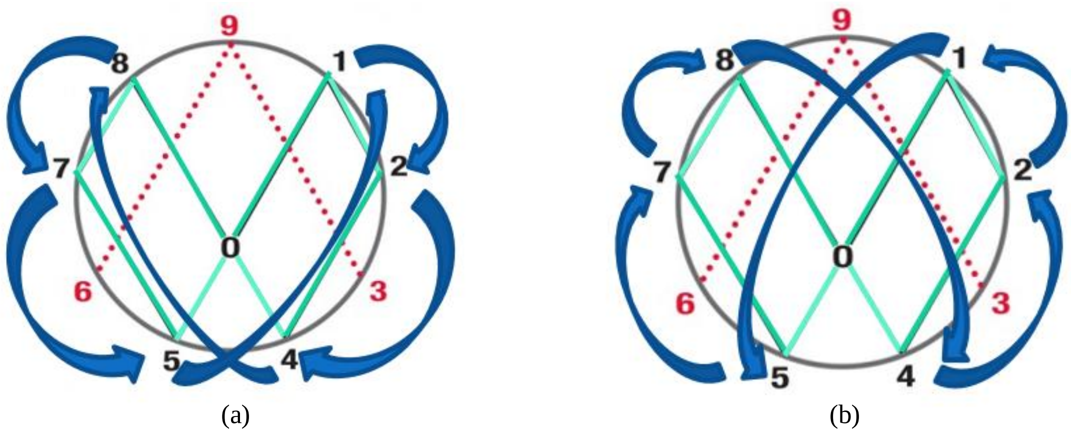


Fig. 2 The sequential pattern depicted using two infinite series:
a) number doubling and digital root; b) number halving and digital root.

Hence doubling and halving using digital root arithmetic signifies trailing motions or spin continuum with inverted directions. The significance of this algorithmic sequencing is believed to explain how vibrations and motion occur (Rodin, 2010). As per the circle, the energy that signifies the source of motion, vibrations, and what keeps time moving forward is represented by the number 9.

The second element in the vortex depiction is the pyramid comprising of the three digits, 3, 6 and 9 with 9 being the apex of the pyramid and where 3 and 6 are never connected. Applying the mathematical functions of doubling and halving to 3 and 6, we see an oscillating pattern of motion illustrated by the infinite sequence 3,6,3,6,3,6, etc. In this context, 3 and 6 are referred to as “magnetic” numbers symbolizing a polarized pulse oscillating between two poles. However, and as depicted by the vortex circle, 3 and 6 are not connected to each other but rather are connected to digit 9. The significance of 9 lies in the fact that irrespective what mathematical function we apply, like halving and doubling, the digital root of the resultant is always 9. As such, 9 is seen as having a polarizing effect on the numbers and signifying an invisible axis of symmetry, a spindle separating the numbers 1 through 8 (See Fig.3). Therefore, 9 is seen as forming the beginning and conclusion of every sequence with the number sequences on both sides of the 9 axis, with the exception of the number 9 itself, becoming mirror images of each other.

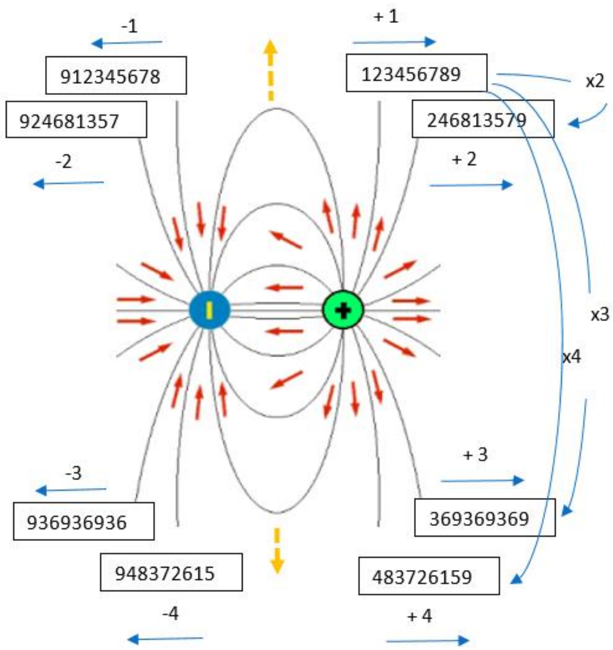


Fig. 3The polarizing effect of the digit 9 axis and the mirror images of number sequences

Consequently, the fundamental principles of Vortex mathematics center around the existence of six numbers namely, 1, 2, 4, 5, 7, and 8that embody the world of physical creation, and underlie the most prevalent geometrical form of creation in nature: the hexagon. Employing the above depiction, we can perform all arithmetic operations, i.e., addition, subtraction, multiplication, and division simultaneously and the potential outcomes are seemingly enclosed on the same circle. It is worth mentioning that performing the division function on the numbers instigates the emergence of three different family number groups triangulated across number triplets: Family group 1 encompasses numbers 1,4,7, family group 2 includes numbers 2, 5, 8 and family group 3 with numbers 3, 6, 9. Such groups are determined by the field represented by numbers 3 and 6. Thus, in a forward motion, 1 added to 3 gives 4, 4 plus 3 equals 7, and 7 plus 3 equals 10 whose digital root is 1. Similarly, 2 plus 3 equals 5, 5 plus 3 equals 8, and 8 plus 3 equals 11 whose digital root is 2. Excluding the 3, 6, 9 family group, the forward and backward motion represented by adding 3 and 6 results in the hexagonal trajectory (See Fig. 4).These number groups are repeated indefinitely by successive addition of number 3 and taking the digital root of the sum.

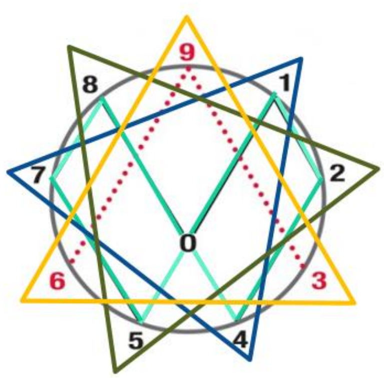


Fig. 4 Family number groups determined by adding 3 and 6 in a forward and backward motions.

The circle then represents a modeling of biological, physical, and chemical phenomena in the universe. Interestingly, in vortex mathematics the numbers are stationary as per the circle depiction however, the different mathematical functions are moving, designating different capabilities thus showing motions across space and time.

4.Background:*Smith* Numbers

The term "Smith" numbers was originally coined by Albert Wilansky (1982) who defined properties of the numbers and provided an explanation of the name, "The largest *Smith* number known is due to my brother-in-law H. Smith who is not a mathematician. It is his telephone number: 4937775!" (p. 21). In 1987, Wayne McDaniel showed that there were indeed infinitely many *Smith* numbers and proposed the first generalization of *Smith* numbers, the *k-Smith* number.

But what exactly are "Smith" numbers? To answer this question, we present some important definitions.

Prime partition of an integer

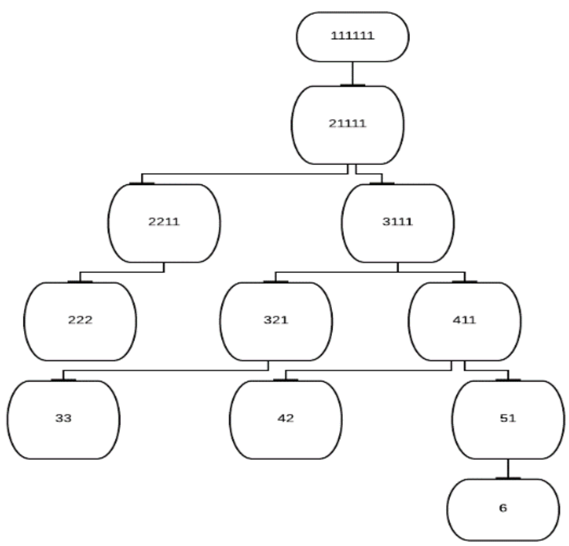
Broadly, any integer N can be expressed as a sum of smaller integers (with or without repetition), such as:

$$6 = 1 + 2 + 3;\ 10 = 4 + 3 + 2 + 1;\ \ldots$$

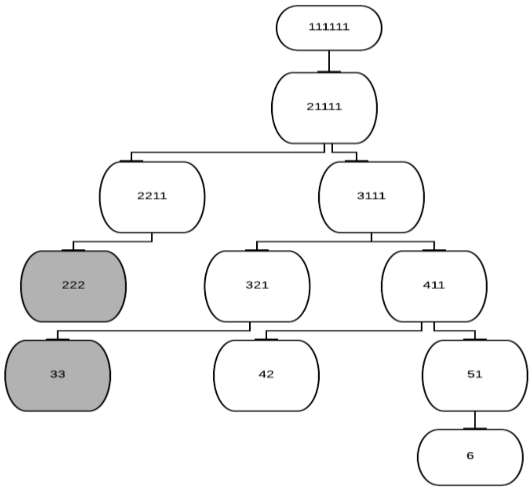
It should be noted that N admits a finite set of partitions such as

$$\begin{aligned}6 &= 6\\ &= 5 + 1\\ &= 4 + 2 = 4 + 1 + 1\\ &= 3 + 3 = 3 + 2 + 1 = 3 + 1 + 1 + 1\\ &= 2 + 2 + 2 = 2 + 2 + 1 + 1 = 2 + 1 + 1 + 1 + 1\\ &= 1 + 1 + 1 + 1 + 1 + 1\end{aligned}$$

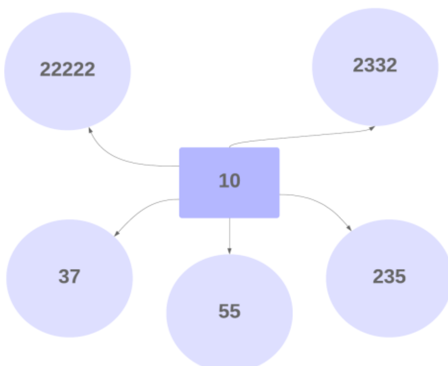
We can also represent the partitions using integer partition trees as follows:



In case the chosen partition of N consists of primes only, then it is called a *prime partition* of N. For example, 6 has two prime partitions: 6=2+2+2 and 6= 3+3(See highlighted classes below).



Similarly, N = 10 admits 5 prime partitions as represented in the integer partition tree below:
10= 2+2+2+2+2=2+2+3+3=2+3+5=3+7=5+5



Hence N=10 admits the following prime partitions: (2,2,2,2,2); (2,2,3,3) ;(2,3,5); (3,7); and (5,5)
Considering an integer $N \geq 2$ such that

$$N = p_1 * p_2 * \dots *p_r$$

where p_1 , p_2 , ... ,p_r are the prime divisors of N (not necessarily distinct), and the sum of the digits of the prime divisor p_i is designated by $S_d(p_i)=\sum_{i=1}^{r} n_i$ .Therefore, the prime partition of N denotes the set of primes p_i whose sum equals N(Gupta & Luthera, 1955).$S_p(N)$, the sum of the digits of the extended prime divisors is given by

$$(n_1, n_2, \dots ,n_r)$$

As an example, consider N such that $S_p(N)$ = 22. A prime partition of 22 is (2^3, 3, 4, 7), where the prime 2 has three prime representatives, which are 2,11 or 101. In fact, S_d (2) = 2, has only three prime representatives as mentioned earlier. Although, it might be conjectured that a sequence of zeros between two ones such as: 101, 1001, 10001, 100001, ... could qualify to be included in the partition class, however, 101 is the only prime number, the rest are all composite.

Similarly, the number 4 represents all the primes p_i whose S_d (p_i) = 4, thus

$$p_i \in \{13, 31, 103, 211, 1021,\dots\},$$

and the number 7 represents all primes qi whose S_d (qi) = 7 thus

$$qi \in \{7, 43, 61, 151, 223,\dots\}.$$

In this very sense, we use the term "the prime partitions of an integer $n = S_d(N)$". According to this convention, another prime partition of 22 is given by (2, 5, 7, 8), which is equivalent to one of the following:

(2, 5, 7, 8), (11, 5, 7, 53), (2, 23, 43, 71), (101, 41, 61, 107),…

In fact, there are many such prime partitions whose sum of digits equals 22, with

$$S_d(p_1) = 2, \quad S_d(p_2) = 5, \quad S_d(p_3) = 7, \quad S_d(p_4) = 8.$$

5. *Smith* Numbers

We consider a positive integer N a *Smith* number, if the sum of its digits $S_d(N)$ equals the sum of the digits of its *extended* prime divisors $S_p(N)$; that is

N is a *Smith* number $\Longleftrightarrow S_d(N) = S_p(N)$.

Examples:

The numbers below are *Smith* numbers since they satisfy the following:

1) $N = 6036 = 2.2.3.503$; $S_d(N) = 6 + 0 + 3 + 6 = \mathbf{15}$; $S_p(N) = 2+2+3+5+0+3=\mathbf{15}$.

2) $N = 9985 = 5.1997$; $S_d(N) = 9+9+8+5=\mathbf{31}$; $S_p(N) = 5+1+3+3+3+3+3+3+7=\mathbf{31}$.

3) $N = 4, 937, 775 = 3.5.5.65837$; $S_d(N) = 4+9+3+7+7+7+5=\mathbf{42}$; $S_p(N) = 3+5+5+6+5+8+3+7=\mathbf{42}$.

Note that all prime composite numbers are trivial cases of *Smith* numbers, for example:

$$p = 13; \ S_d(13) = S_p(13) = 4.$$

6. The Proposed Sieve

We create a sieve to help us establish a generalized mathematical model to generate *Smith* numbers. The main target of the sieve is to find all possible prime partitions of the number (n) whose products P yield a digital root equals $\rho(n)$, consequently obtaining a generalized form of N. To illustrate, we consider the *Smith* number N whose $S_d(N) = S_p(N) = n$. Next, we build tables of all possible products of the various values of the prime factors of N; these products equal the sum of the digits of its extended prime divisors $S_p(N) = \rho(n) + 9x$, x Є N and $S_d(N) = n$. The major operation of the sieve is to pick out the numbers N among these products whose $S_p(N) = n$. Consequently, we prepare for lists of primes according to their sum of digits such as

$S_p(p) = 2$ Є {2; 11; 101}

$S_p(p) = 4$Є {13; 31; 103; 211;…}

$S_p(p) = 5$ Є {5; 23; 41; 113; 131,…}

Note that there are no primes of digital root = 3, 6, or 9 (Grant & Ghannam, 2019).

To demonstrate this procedure, we consider N whose $S_d(N) = S_p(N) = 13$, and to find such *Smith* numbers N we start with $\rho(13) = 4$, then find all possible prime partitions of (13) whose products P yield a digital root 4 as shown in Table 1:

Table 1

Partition	Sum	Product (P)	ρ(P)	Result
(2,11)	2 + 11 = 13	2x11 = 22	4	Correct
(2,4,7)	2+4+7= 13	2x4x7=56	2	Incorrect
(2,3,3,5)	2+3+3+5=13	2x3x3x5= 90	9	Incorrect
(3,10)	3+10=13	3x10= 30	3	Incorrect
(3,5,5)	3+5+5=13	3x5x5=75	3	Incorrect
(5,8)	5+3+5=13	5x8=40	4	Correct

Hence, the form $N = p.q$; where $S_p(p) = 2$ and $S_p(q) = 11$ or $S_p(p) = 5$ and $S_p(q)$ is 5 or 3; in both cases the sum is 13 and $S_d(N) = 4 + 9x$; with $x=1$.

To show how the sieve works, we first build Tables for both of products as shown in Tables 2&3.

Table 2

q	2.q	11.q	101.*q*
11	22	121	1111
29	58	319	2929
47	94	517	4747
83	166	913	8383
137	247	1507	13837
173	346	1903	17473
191	382	2101	19291
227	454	2497	22927
263	526	2893	26563
281	562	3091	28381
317	634	3487	32017
353	706	3883	35653

Table 3

q	5.*q*	23.*q*	41.*q*	113.*q*
8	40	184	328	904
17	85	391	697	1921
53	265	1219	2173	5989
71	355	1633	2911	8023
107	535	2461	4387	12091
33	1165	5359	9553	26329
251	1255	5773	10291	28363

Thus the sieve picks out the numbers (underlined in the Tables) whose $S_d(N) = 13$ or $x = 1$ and discards the others.

7.*k-Smith* Numbers

We can further explore *k-Smith* numbers. By definition (Miller, Hereen, & Hornsby, 2002), a positive integer N is called a *k-Smith* number if

$$S_p(N) = k \times S_d(N)$$

Examples:

1) $N = 316 = 2^2 . 79$; $S_d(N) = 10$; $S_p(N) = 20 = 2. S_d(N)$.

2) $N = 26011 = 19.37^2$; $S_d(N) = 10$; $S_p(N) = 30 = 3. S_d(N)$.

3) $N = 4{,}000{,}000{,}002 = 2.3.66{,}666{,}6667$; $S_d(N) = 6$; $S_p(N) = 60 = 10. S_d(N)$.

We note that the function of the *k*-sieve is similar to that of *1-Smith* numbers.

Let $S_d(N) = n$ then $S_p(N) = k.n$, hence the procedure targets all possible prime partitions of $(k.n)$ whose products have digital root of $\rho\ (n)$. The next step involves building the tables of the products of these prime partitions for all the primes in the partitions. Ultimately, the sieve picks out the required *k-Smith* numbers.

To illustrate, consider N whose $S_d(N) = 10$ and we look for the *2-Smith* number; $S_p(N) = 2 \times 10 = 20$. We find the possible prime partitions of 20, for instance $(2^2, 16)$ whose sum is 20 and product equals 64 of digital root ρ (64) = 1 which is the digital root of $S_d(N)$, thus we build the following table for:

$$N = p^2.q; S_p(p) = 2; \qquad S_p(q) = 16$$

Table 4

q	2^2.q	11^2.q	101^2. q
79	316	9,559	805,879
97	388	11,737	989,497
277	1,108	33,517	2,825,677
1,663	6,652	201,223	16,964,263
1,753	7,012	212,113	17,882,353
3,931	15,724	475,651	40,100,131

The sieve picks out the numbers whose $S_d(N) = 10$ (underlined, and discards the numbers) N whose $S_d(N) \neq 10$

5. *Hoax* Numbers

We consider another interesting number, the *Hoax* number. By definition (Tattersall, 2001), a positive integer N is called a *Hoax* number if

$$S_p(N) = S_q(\mathrm{N})$$

Where S_q (N) is the sum of the digits of the distinct prime divisors of N.

For example:

$N = 47{,}700 = 2^2 .3^2 .5^2 .53$; $S_d(N) = 18$; $S_q(\mathrm{N}) = 2 + 3 + 5 + 8 = 18$, while $S_p(N) = 4 + 6 + 10 + 8 = 28$; hence 47,700 is a *Hoax* number.

Also the integer $N = 2401 = 7^4$ admits $S_d(N) = 7$ and $S_q(\mathrm{N}) = 7$, while $S_p(N) = 4 \times 7 = 28$, is a *Hoax* as well as a *4-Smith* number. Another example is $N = 43{,}501 = 41.1061$; $S_d(N) = 13$ and $S_p(N) = S_q(\mathrm{N}) = 13$; thus 43,501 is a *Hoax* number and a *Smith* number as well.

It is important to note that all *Smith* numbers that have distinct (non-repeating) prime divisors are also *Hoax* numbers. The sieve devised for such *Smith* numbers can be applied for such *Hoax* numbers too. But in this case, since the prime divisors are repeated many times, we have to adjust the device or the procedure according to the digital root of $S_d(N)$. To illustrate, consider an integer N whose $S_d(N) = 10$, one of its prime partitions has a product P of digital root 2 : (2,8), the sum is 2 + 8 = 10 and the product 2 x 8 = 16 of digital root $\rho(16) = 7$, then we have to multiply 7 either by 2^n or 8^n to make the digital root $\rho(7 \times 2^n) = 1$ or $\rho(7 \times 8^n) = 1$; actually $n = 2$, for $\rho(7 \times 2^2) = \rho(28) = 11$. Thus, we adjust the prime partition to be $(2^3,8)$ and $N = p^3.q$where $S_p(p) = 2$ and $S_p(q) = 8$ is the required form for the sieve:

q	17	53	71	107	233	251	…
2	<u>136</u>	<u>424</u>	568	856	1,864	<u>2,008</u>	…
11	22,627	70,543	94,501	142,417	<u>310,123</u>	334,081	…

6. Newly Invented Numbers

Morowah **Numbers**

We create a number called *Morowah* number defined as follows: A positive integer N is a *Morowah* number if

$$S_d(N) = n^a \text{ and } S_p(N) = a^n;\ a, n \in \mathbb{N};\ a \neq n,$$

Examples:

1) N = 18 = 2.3.3; $S_d(N) = 3^2$; $S_p(N) = 2^3$.
2) N = 11,977 = 7.29.59; $S_d(N) = 5^2$; $S_p(N) = 2^5$
3) N = 26,978 = 2.7.41.47; $S_d(N) = 2^5$; $S_p(N) = 5^2$
4) N = 406,138,734 = $2.3^3.17.499.887$; $S_d(N) = 6^2$; $S_p(N) = 2^6$
5) N = 998,299,990 = 2.5.3823.26113; $S_d(N) = 2^6$; $S_p(N) = 6^2$
6) N = 919,999,999,800 = $2^3.3^2.5^2.7^2$.10,430,839; $S_d(N) = 3^4$; $S_p(N) = 4^3$
7) N = 99,299,998,000 = $2^4.5^3.7.79.89783$; $S_d(N) = 4^3$; $S_p(N) = 3^4$
8) N = 9,491,899 x 10^{12} = $2^{12}.5^{12}.17.281.1987$ $S_d(N) = 7^2$; $S_p(N) = 2^7$

The case ($a = n$) is a trivial one such as : $a = n = 2 \to 2^2$, $a = n = 3 \to 3^3$,… *and the case* a = 2, $n = 4 \to 2^4 = 4^2 = 16$. The function of the sieve in such numbers is exactly the same as before but we have to find at first, all prime partitions of $S_p(N) = a^n$, which yields products of digits equal to the digital root of n^a.

For instance, $S_d(N) = 32$, $\rho(32) = 5$, then $S_p(N) = 25$ and next find all possible prime partitions of 25, such as $(4,7,14) \to S = 4 + 7 + 14 = 25$, $P = 392$ and $\rho(392) = 5 = \rho(32)$. Thus for $N = p.q.r$, where $S_p(p) = 4$; $S_p(q) = 7$; $S_p(r) = 14$; $S_d(N) = 5;14;23;32;41;\ldots;5 + 9m;\ldots$, where $m \in \mathbb{N}$. Hence, our sieve will pick out $S_d(N) = 32$, corresponding to $m = 3$ (underlined) in the following Table:

Table 5

r	13.7r	13.34r	13.61r	…	31.7r	31.43r	31.61r	…
59	5,369	23,981	<u>46,787</u>	…	12,803	<u>78,</u>647	111,569	…
149	13,559	83,291	118,157	…	32,333	<u>198,617</u>	<u>281,759</u>	…
167	15,197	93,353	132,431	…	36,239	222,611	<u>315,797</u>	…
239	21,749	133,601	<u>189,587</u>	…	51,863	<u>318,587</u>	<u>451,949</u>	…

257	23,387	143,663	203,801	…	55,769	342,581	485,949	…
293	26,663	163,787	232,349	…	63,581	390,569	554,063	…
347	31,577	193,973	275,171	…	75,299	462,551	656,177	…
383	34,853	214,097	303,719	…	83,111	510,539	724,253	…
419	38,129	234,221	332,267	…	90,923	558,527	792,329	…
. . .	. . .	. . .	. . .	…	. . .	. . .	. . .	. . .
1,049	49,459	586,391	831,857	…	227,633	1,983,317	1,983,481	…
…	…	…	…	…	…	…	…	…

7. *Yara* Numbers

For any integer N, we define the number $S_m(N)$ as the mean of $S_p(N)$ and $S_q(N)$, i.e.

$$S_m(N) = \frac{1}{2}[S_p(N) + S_q(N)]$$

We create a *Yara number,* which we define as *a positive integer N, whose sum of digits* $S_d(N)$ *equals its mean sum* $S_m(N)$, *i.e.*

$$N \text{ is a Yara number} \Longleftrightarrow S_d(N) = S_m(N)$$

It should be noted that $S_p(N)$ and $S_q(N)$ must be of the same parity.

Examples:

1) $N = 37395 = 3^3.5.277$

$$\begin{cases} S_p(N) = 30 \\ S_d(N) = 27 \\ S_q(N) = 24 \end{cases} \Longrightarrow S_m(N) = \frac{1}{2}[30 + 24] = 27 = S_d(N).$$

2) $N = 25{,}008{,}401 = 11^2.41.71^2$

$$\begin{cases} S_p(N) = 25 \\ S_d(N) = 20 \\ S_q(N) = 15 \end{cases} \Longrightarrow S_m(N) = 20 = S_d(N).$$

3) $N = 27{,}552 = 2^5.3.7.41$

$$\begin{cases} S_p(N) = 25 \\ S_d(N) = 21 \\ S_q(N) = 17 \end{cases} \Longrightarrow S_m(N) = 21 = S_d(N).$$

To generate a *Yara* number, we follow certain patterns that distinguish these numbers from the preceding numbers. These patterns depend on the following characteristics: first, the difference Δ between $S_p(N)$ and $S_q(N)$ has to be even, i.e.

$$\Delta = S_p(N) - S_q(N) = 2,4,6,8,10,\ldots.$$

The second characteristic is that *Yara* numbers must have repeated prime divisors; for instance, the simplest form is $N = p^a.q$, where $S_p(p) = 2,3,4,5,\ldots$, and where p is strictly prime. The prime q follows some pattern also. The exponent $[a]$ depends on Δ; if $\Delta = 2$, then $a = 2$ and $S_p(p) = 2$.

For example, if we take $\Delta = 2$, then $N = p^2.q$, where $p = 2,11$ or 101.Furthermore,$S_p(N) = 2 \text{ x } 2 + S_p(q)$ and $S_p(N) = 2 + S_p(q)$ which means that $S_m(N) = 3 + S_p(q)$. The pairs of numbers whose difference $\Delta = 2$ are $(4 + S_p(q), 2 + S_p(q))$; thus $S_p(q) = 4$ and
$N = 2^2.13; 2^2.31; 2^2.103; \ldots; 11^2.13; 11^2.31; 11^2.103;\ldots; 101^2.13; 101^2.31; 101^2.103;\ldots;$
Therefore, the function of the sieve is to extract the underlined numbers of these products as in Table 6:

Table 6

q	$2^2.q$	$11^2.q$	$101^2.q$	…
13	<u>52</u>	1,573	132,613	…
31	<u>124</u>	3,751	316,231	…
103	<u>412</u>	12,463	1,050,703	…
211	844	25,531	2,152,411	…
1,201	4,804	145,321	12,251,401	…
2,011	8,044	243,331	20,514,211	…
3,001	<u>12,004</u>	363,121	30,613,201	…
⋮	⋮	⋮	⋮	⋮

Actually $S_p(q) = 4 + 3.m$; $m = 0,1,2,3,\ldots$ in the preceding demonstration.
For $m = 1$ we have:

$$S_p(q) = 7 \text{ and } S_p(N) = 9, \text{ then } S_m(N) = S_d(N) = 10.$$

Table 7

q	7	43	61	151	223	241	313	331	421	601	…
$2^2.q$	<u>28</u>	<u>172</u>	<u>244</u>	<u>604</u>	892	964	<u>1,252</u>	<u>1,324</u>	1,684	<u>2,404</u>	…
$11^2.q$	847	<u>5,203</u>	7,381	18,271	26,983	29,161	37,873	<u>40,051</u>	50,941	72,721	…

If we consider $\Delta = 10$, again the simplest form is $N = p^a.q$;

$$\Delta = [a.\, S_p(p) + S_p(q)] - [S_p(p) + S_p(q)] = [a-1].\, S_p(p) = 10$$

This implies that $a = [10 \div S_p(p)] + 1$ which means that $S_p(p) = 2$ or 5.

For $S_p(p) = 5$, then $a = [10 \div 5] + 1 = 3$; hence $N = p^3.q$, where

$$S_m(N) = \frac{1}{2}\{[3.5 + S_p(q) + 5 + S_p(q)]\} = 10 + S_p(q).$$

The least value of $S_p(q)$ is 4, which implies that

$S_p(N) = 15 + 4 = 19$, $S_q(N) = 5 + 4 = 9$ and $S_m(N) = S_d(N) = 14$.

The digital root of $S_d(N) = 5$, hence the prime partition of $S_p(N)$ that yields a product of digital root of 5 is $(5^3,4)$ $[5^3 \text{ x } 4 = 500]$. The next value of $S_p(q)$ is 13 and the prime partition is

$(5^3,13)$ $[5^3 \text{ x } 13 = 1{,}625;\ \rho(1{,}625) = 5]$.

$S_p(N) = 15 + 13 = 28$, $S_q(N) = 5 + 13 = 18$ and $S_m(N) = S_d(N) = 10 + 13 = 23$.

The sieve sweeps through the table of different products of $N = p^3.q$ as shown in Table 8.

Table 8

q	$5^3.q$	$23^3.q$	$41^3.q$	…
67	<u>8,375</u>	815,189	4,617,707	…
139	<u>17,375</u>	<u>1,691,213</u>	9,580,019	…
157	<u>19,625</u>	<u>1,910,219</u>	10,820,597	…
193	24,125	<u>2,348,231</u>	<u>13,301,753</u>	…
229	<u>28,625</u>	2,786,243	15,782,909	…
283	<u>35,375</u>	<u>3,443,261</u>	19,504,643	…
337	42,125	<u>4,100,279</u>	23,226,377	…
373	<u>46,625</u>	4,538,291	25,707,533	…
409	51,125	4,976,303	28,188,689	…
463	57,875	<u>5,633,321</u>	<u>31,910,423</u>	…
⋮	⋮	⋮	⋮	⋮

Following the same procedure, we can establish infinite ways of developing other related numbers such as *k-Hoax* Numbers, *k-Yara* Numbers, etc.

8. Closing Thoughts

Going back deep into history, we find that originally number structures and sequences did not exist fully formed, but rather that it evolved stepwise from one numerical boundary to the next (Burton, 2007). Such "rudimentary" first stages explain a series of peculiarities inherent in what we call "mature" number structures. These early difficulties have been overcome by a deep analysis of the number structures and sequences. In the process of creating new and intelligible number structures, we establish basic laws governing both the number sequence and the written number symbols. The question of how these rules of succession are observed opens up a wide range of possibilities that bear witness to the inventiveness of the human mind and the potential conceptual difficulties with number structures that could be encountered (Weibul, 2000).

In this paper, we argue that as in the case of *Smith numbers*, *Hoax numbers* and other related numbers, the key to all such investigations lies in the meaning we attach to numbers from which we concoct other relationships and, consequently create new numbers. If we closely examine how our use of numbers has progressed, it is easy to discern that our sense of number structures transcends the symbolic representations that we create to manipulate and operate on numbers. We have employed numbers as attributes, as adjectives and we even document cultural histories using number systems. Looking back once more into history, we may recognize that highly perfected number structures may have been invented by more indigenous, non-Western cultures (Chahine & Naresh, 2013), although perhaps the West has made the greatest use of them and developed them to their highest state. As sources far from each other in space and time have come together for the development of number structures, it seems natural to claim that numbers are conceivably the manifestations of our thoughts and our system of concepts that help us understand and make sense of the sublime world we live in. If you accept the claim that human beings are born with an innate capacity to carry out simple arithmetic operations, then "mathematizing" ordinary ideas could bepart and parcel of what makes us human. As such, mathematical meaning is embedded in our daily experience and embodied in our mutual interaction with the world around us.

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