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Fuzzy soft separation axioms in fuzzy soft topological spaces

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Abstract. Fuzzy soft separation axioms was introduced by Mahanta and Das ([5]) using the definitions of a `fuzzy soft point' and `the complement of a fuzzy soft point is a fuzzy soft point', and `distinct of fuzzy soft points' in there sense. In this paper we, introduce fuzzy soft separation axioms in terms of the modified definitions of a `fuzzy soft point', the complement of a fuzzy soft point is a fuzzy soft set' and `distinct of fuzzy soft points'([7]). Also, we study some of their properties. Finally, we discuss fuzzy soft topological property for such spaces.

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Keywords: Fuzzy set, soft set; fuzzy soft set, fuzzy soft topology, fuzzy soft separation axioms; fuzzy soft T_i −spaces $(i = 0,1,2)$, fuzzy soft regular; fuzzy soft normal, fuzzy soft topological property.

1. Introduction

Prof. L. A. Zadeh[14] in 1965, introduced the concept of fuzzy set and fuzzy set operations.Chang [2] introduced the concept of fuzzy topology on a set X by axiomatizing a collection T of fuzzy subsets of X.

The concept of soft sets was first introduced by Molodtsov [8] in 1999 as a general mathematical tool for dealing with uncertain objects. Cagman et al. [1] and Shabir et al. [12] introduced soft topological spaceindependently in 2011. Maji et al. [6] introduced the concept of fuzzy soft set and some of its properties. Tanay et al. [13] introduced the definition of fuzzy soft topology over a subset of the initial universe set. Later, Roy and Samanta [11] gave the definition of fuzzy soft topology over the initial universe set. In [4], Kharal and Ahmed defined the notion of a mapping on classes of fuzzy soft sets.Mahanta and Das ([5]) introduced the fuzzy soft separation axioms $T_i(i = 0; 1; 2; 3; 4)$ by using the definitions of a `fuzzy soft point' and `the complement of a fuzzy soft point is a fuzzy soft point ', and `distinct of fuzzy soft points' in his sense.

In the present paper, we introduce fuzzy soft separation axioms $T_i(i = 0; 1; 2; 3; 4)$ in terms of the modified definitions of a `fuzzy soft point', the complement of a fuzzy soft point is a fuzzy soft set' and `distinct of fuzzy soft points' ([7]) and we study some of their properties. Finally, we discuss some fuzzy soft topological property for such spaces.

2. Preliminaries

First we recall basic definitions and results.

Definition2.1.([14]) Let$X$ be a non-empty set. A fuzzy set A in X is defined by a membership function$\mu_A: X \to [0,1]$ whose value $\mu_A(x)$ represents the "grade of membership" of x in A for $x \in X$. The set of all fuzzy sets in a set X is denoted by I^X, where I is the closed unit interval [0,1].

Definition 2.2. ([14]) If $A, B \in I^X$, then, we have:
(1) $A \leq B \Leftrightarrow \mu_A(x) \leq \mu_B(x),\ \forall x \in X$;
(2) $A = B \Leftrightarrow \mu_A(x) = \mu_B(x),\ \forall x \in X$;
(3) $C = A \vee B \Leftrightarrow \mu_C(x) = \max\left(\mu_A(x), \mu_B(x)\right),\ \forall x \in X$;
(4) $D = A \wedge B \Leftrightarrow \mu_D(x) = \min\left(\mu_A(x), \mu_B(x)\right), \forall x \in X$;
(5) $E = A^C \Leftrightarrow \mu_E(x) = 1 - \mu_A(x), \forall x \in X$.

Definition 2.3. ([8]) Let$X$ be an initial universe set and E be a set of parameters. Let $P(X)$ denotes the power set of X and $A \subseteq E$. A pair $(F, A)$is called a soft set over X if F is a mapping given by
$F : A \longrightarrow P(X)$.
In other words, a soft set is a parameterized family of subsets of the set X. For $e \in A$, $F(e)$ may be considered as the set of e −approximate elements of the soft set (F, A).

Definition 2.4. ([11]) Let X be an initial universe set and E be a set of parameters. Let $A \subseteq E$. A fuzzy soft set f_A over X is a mapping from E to I^X, i.e., $f_A: E \longrightarrow I^X$, where $f_A(e) \neq 0_X$ if $e \in A \subseteq E$, and $f_A(e) = 0_X$ if $e \notin A$, where 0_X denotes empty fuzzy set in$X$. The family of all fuzzy soft sets over X is denotes by $FSS(X, E)$.

Definition2.5.([11]) The fuzzy soft set $f_A \in FSS(X, E)$. is called null fuzzy soft set, denoted by$\tilde{0}_E$, if for all $e \in A$, $f_A(e) = 0_X$.

Definition2.6.([11]) Let $f_E \in FSS(X, E)$. The fuzzy soft set $f_E$is called absolute fuzzy soft set, denoted by $\tilde{1}_E$, if for all $e \in E$, $f_E(e) = 1_X$where $1_X(x) = 1$ for all$x \in X$.

Definition2.7.([11]) Let $f_A, g_B \in FSS(X, E)$.$f_A$is called a fuzzy soft subset of$g_B$if$A \subseteq B$ and$f_E(e) \leq g_B(e)$ for every $e \in E$and we write $f_A \sqsubseteq g_B$.$f_A$and $g_B$are said to be equal,denoted by $f_A = g_B$if $f_A \sqsubseteq g_B$and $g_B \sqsubseteq f_A$.

Definition2.8.([11]) Let $f_A, g_B \in FSS(X, E)$.The union (resp. intersection) of $f_A$and $g_B$isalso a fuzzy soft set h_C, defined by $h_C(e) = f_A(e) \vee g_B(e)$(resp.$h_C(e) = f_A(e) \wedge g_B(e)$) for all $e \in E$, where$C = A \cup B$ (resp $C = A \cap B$). Here we write $h_C = f_A \sqcup g_B$ (resp $h_C = f_A \sqcap g_B$).

Definition2.9.([11]) Let $f_A \in FSS(X, E)$. The fuzzy soft complement of f_A, denoted byf_A^c, is a fuzzy soft set defined by $f_A^c(e) = 1_X - f_E(e)$ for every $e \in E$.
Clearly $(f_A^c)^c = f_A$,$(\tilde{1}_E)^c = \tilde{0}_E$and $(\tilde{0}_E)^c = \tilde{1}_E$.

Definition 2.10.[11] Let $\mathfrak{T}$ be a collection of fuzzy soft sets over a universe $X$with a fixed set of parameters E, then $\mathfrak{T}$is called a fuzzy soft topology on Xif

(1) $\tilde{1}_E, \tilde{0}_E \in \mathfrak{T}$,

(2) The union of any members of $\mathfrak{T}$belongs to $\mathfrak{T}$,

(3) The intersection of any two members of $\mathfrak{T}$ belongs to $\mathfrak{T}$.

The triple$(X, \mathfrak{T}, E)$is called a fuzzy soft topological space over X. Also, each member of $\mathfrak{T}$ is called a fuzzy soft open set in $(X, \mathfrak{T}, E)$and their fuzzy soft complements are called fuzzy soft closed sets in $(X, \mathfrak{T}, E)$. The family of all fuzzy soft closed sets in $(X, \mathfrak{T}, E)$ is denoted by $\mathfrak{T}^c$

Definition. **2.12.** [10] Let $(X, \mathfrak{T}, E)$be a fuzzy soft topological space and $f_A \in FSS(X, E)$. The fuzzy soft closure of f_A, denoted by $cl(f_A)$, is defined as$cl(f_A) = \sqcap \{h_C : h_C \in \mathfrak{T}^c, f_A \sqsubseteq h_C\}$.Clearly, $cl(f_A)$is the smallest fuzzy soft closed set over Xwhich contains f_A.

Definition 2.13.[5,9] Let $(X, \mathfrak{T}, E)$be a fuzzy soft topological space and $Y \subseteq X$. Leth_E^Y be a fuzzy softset over (Y, E)i.e.,$h_E^Y : E \longrightarrow I^Y$such that

$$h_E^Y(e)(x) = \begin{cases} 1 & if x \in Y \\ 0 & if x \notin Y \end{cases}$$

Let$\mathfrak{T}_Y = \{h_E^Y \sqcap g_B : g_B \in \mathfrak{T}\}$, then $\mathfrak{T}_Y$ is a fuzzy soft topology on (Y, E)called a fuzzy soft subspace topology for $(Y, E)$and $(X, \mathfrak{T}_Y, E)$is called a fuzzy soft subspace of $(X, \mathfrak{T}, E)$. If $h_E^Y \in \mathfrak{T}$[resp. $h_E^Y \in \mathfrak{T}^c$] then $(X, \mathfrak{T}_Y, E)$is called fuzzy soft open [resp. closed]subspace of $(X, \mathfrak{T}, E)$.

Definition 2.13.[5] A fuzzy soft set $g_A$is said to be afuzzy soft point, denoted by e_{gA}, if for the element$e \in A$, $g_A(e) \neq 0_X$ and $g_A(e') = 0_X, \forall e' \in A - \{e\}$.

Definition 2.14.[5] The complement of a fuzzy soft point$e_{gA}$is a fuzzy soft point $(e_{gA})^c$such that $g_A^c(e) = 1_X - g_A(e)$ and $g_A^c(e') = 0_X$, $\forall e' \in A - \{e\}$.

Definition 2.15.[5] A fuzzy soft point e_{gA} is said to bein a fuzzy soft set h_C, denoted by $e_{gA} \in h_C$ if for theelement $e \in C \cup A$, $g(e) \leq h(e)$.

Theorem 2.16.[5] A fuzzy soft pointe_{gA}satisfy the following properties.

(1) If $e_{gA} \tilde{\in} h_A$ then e_{gA} may or may not belong to h_A^c,

(2) If$e_{gA} \tilde{\in} h_A \nRightarrow (e_{gA})^c \tilde{\in} h_A^c$,

(3) The union of all the fuzzy soft points of a fuzzy soft set is equal to the fuzzy soft set.

Definition 2.17.[7] A fuzzy soft point e_{x_α}over X is a fuzzy soft set over Xdefined as follows:

$$e_{x_\alpha}(e') = \begin{cases} x_\alpha & if\ e' = e \\ 0_X & if e' \in E - \{e\}, \end{cases}$$

where x_α is the fuzzy point([11]) in $X$with support $x$and value α, $\alpha \in (0,1)$.

A fuzzy soft point $e_{x_\alpha}$is said to belong to a fuzzy soft set f_A, denoted by $e_{x_\alpha} \tilde{\in} f_A$if $\alpha < f_A(e)(x)$.Two fuzzy soft points $e_{x_\alpha}$and $e'_{y_\beta}$are said to be distinct if $x \neq y$or$e \neq e'$.

Definition 2.18. ([4]) Let $FSS(X,E)$ and $FSS(Y,K)$ be the families of all fuzzy soft sets over X and Y , respectivly. Let $u : X \longrightarrow Y$ and $p : E \longrightarrow K$ be two mappings. Then a fuzzy soft mapping f_{up} : $FSS(X,E) \longrightarrow FSS(Y,K)$ is defined as follows: for a fuzzy soft set $f_A \in FSS(X,E)$, $\forall\, k \in p(E) \subseteq K$ and $y \in Y$, we have

$$f_{up}(f_A)(k)(y) = \begin{cases} \vee_{x\in u^{-1}(y)} \vee_{e\in p^{-1}(k)\cap A} f_A(e)(x) if & u^{-1}(y) \neq \varphi, p^{-1}(k) \cap A \neq \varphi, \\ 0, & \text{otherwise.} \end{cases}$$

$f_{up}(f_A)$is called the fuzzy soft image of a fuzzy soft set f_A.

Definition 2.19. ([4]) Let $u : X \longrightarrow Y$and $p : E \longrightarrow K$ be two mappings.
Let f_{up} : $FSS(X,E) \longrightarrow FSS(Y,K)$ be fuzzy soft mapping and $g_B \in FSS(Y,K)$. Then $f_{up}^{-1}(g_B)$, is a fuzzy soft set in $FSS(X,E)$, defined by

$$f_{up}^{-1}(g_B)(e)(x) = g_B\big(p(e)\big)\big(u(x)\big), \qquad \forall e \in E\,, x \in X.$$

$f_{up}^{-1}(g_B)$ is called the fuzzy soft inverse image of g_B.

If u and p are injective then the fuzzy soft mapping f_{up} is said to befuzzy soft injective. If u and p are surjective then the fuzzy soft mapping f_{up} is said to be fuzzy soft surjective. The fuzzy soft mapping f_{up} is calledfuzzy soft constant, if u and p are constant.f_{up} is said to be fuzzy soft bijective if f_{up} is fuzzy soft injective and fuzzy soft surjective mapping.

Definition 2.20.([10])Let $(X,\mathfrak{T}_1,E)$and$(Y,\mathfrak{T}_2,K)$be two fuzzy soft topological spacesand $f_{up} : (X,\mathfrak{T}_1,E) \longrightarrow (Y,\mathfrak{T}_2,K)$ be a fuzzy soft mapping. Then f_{up} is called

(1) fuzzy soft continuous if$f_{up}^{-1}(g_E) \in \mathfrak{T}_1$, for all $g_E \in \mathfrak{T}_2$,

(2) fuzzy soft open if$f_{up}(f_E) \in \mathfrak{T}_2$, for all $f_E \in \mathfrak{T}_1$.

(3) fuzzy soft homeomorphism if f_{up} is fuzzy soft bijective, fuzzy soft continuous and fuzzy soft open.

Theorem 2.21.([10])Let $(X,\mathfrak{T}_1,E)$and$(Y,\mathfrak{T}_2,K)$be two fuzzy soft topological spaces and $f_{up}: (X,\mathfrak{T}_1,E) \longrightarrow (Y,\mathfrak{T}_2,K)$ be a fuzzy soft mapping. Then the following are equivalent:
(1) f_{up} is fuzzy soft continuous,
(2)$f_{up}^{-1}(g_E) \in \mathfrak{T}_1^c$for each $g_E \in \mathfrak{T}_2^c$.

Proposition2.22.([4]) Let $FSS(X,E)$ and $FSS(Y,K)$ be two families of fuzzy soft sets. For the fuzzy soft mapping $f_{up}: FSS(X,E) \longrightarrow FSS(Y,K)$, the following statements hold,
(1) $f_{up}^{-1}(g_B)^c = (f_{up}^{-1}(g_B))^c \forall g_B \in FSS(Y,K)$.
(2) $f_{up}(f_{up}^{-1}(g_B)) \sqsubseteq g_B \forall g_B \in FSS(Y,K)$. If f_{up} is fuzzy soft surjective, the equality hold.
(3) $f_A \sqsubseteq f_{up}^{-1}(f_{up}(f_A)) \forall f_A \in FSS(X,E)$. If f_{up} is fuzzy soft injective, the equality hold.
(4) $f_{up}(\tilde{0}_E) = \tilde{0}_K, f_{up}(\tilde{1}_E) \sqsubseteq \tilde{1}_K$. If f_{up} isfuzzy soft injective, the equality hold.
(5) $f_{up}^{-1}(\tilde{1}_K) = \tilde{1}_E$ and $f_{up}^{-1}(\tilde{0}_K) = \tilde{0}_E$.

(6) If $f_A \sqsubseteq g_B$, then $f_{up}(f_A) \sqsubseteq f_{up}(g_B) \forall f_A, g_B \in FSS(X, E)$.
(7) If$f_A \sqsubseteq g_B$. Then $f_{up}^{-1}(f_A) \sqsubseteq f_{up}^{-1}(g_B) \forall f_A, g_B \in FSS(Y, K)$.
(8)$f_{up}^{-1}\left(\sqcup_{i\in J}(g_B)_i\right) = \sqcup_{i\in J} f_{up}^{-1}(g_B)_i$ and $f_{up}^{-1}\left(\sqcap_{i\in J}(g_B)_i\right) = \sqcap_{i\in J} f_{up}^{-1}(g_B)_i \ \forall (g_B)_i \in FSS(Y, K)$.
(9) $f_{up}\left(\sqcup_{i\in J}(f_A)_i\right) = \sqcup_{i\in J} f_{up}(f_A)_i$ and $f_{up}\left(\sqcap_{i\in J}(f_A)_i\right) \sqsubseteq \sqcap_{i\in J} f_{up}(f_A)_i \ \forall (f_A)_i \in FSS(X, E)$.
If f_{up} is fuzzy soft injective, the equality hold.

3. Fuzzy soft separation axioms

Mahanta and Das ([5]) had introduced the concepts fuzzy soft T_0-spaces and fuzzy soft T_1-spaces using the definitions of a `fuzzy soft point' and `the complement of a fuzzy soft point is a fuzzy soft point', in his sense. Here we define fuzzy soft T_0-space and fuzzy soft T_1-space in terms of the modified definitions of a `fuzzy soft point',' the complement of a fuzzy soft point is a fuzzy soft set' and `distinct of fuzzy soft points' in Definition 2.17.

Remark 3.1. Instead of the notation e_{x_α}in Definition 2.17, we shall use the notation(e, x_α). Therefore, the fuzzy soft points(e, x_α) and $(\acute{e}, y_\beta)$ are said to be distinct in (X, E) if $e \neq \acute{e}$ or $x \neq y$.

The proof of the following theorem follows directly from definition of fuzzy softpoint and therefore omitted.

Theorem 3.2. A fuzzy soft point (e, x_α)satisfying the following properties:

(1) If $(e, x_\alpha) \tilde{\in} f_A$ then (e, x_α) may or may not belongs tof_A^c,

(2) If $(e, x_\alpha) \sqcap f_A = \tilde{0}_\theta$then $(e, x_\alpha) \tilde{\notin} f_A$ and $(e, x_\alpha) \tilde{\in} f_A^c$,

(3) If $(e, x_\alpha) \tilde{\in} f_A$ and $\alpha > 0.5$, then $(e, x_\alpha) \tilde{\notin} f_A^c$,

(4) If$(e, x_\alpha) \tilde{\in} f_A \nRightarrow (e, x_\alpha)^c \tilde{\in} f_A^c$,

(5) The union of all fuzzy soft points of a fuzzy softset is equal to the fuzzy soft set.

Definition 3.3. A fuzzy soft topological space $(X, \mathfrak{T}, E)$ is said to be a fuzzy soft T_0 −space if forevery pair of distinct fuzzy soft points (e, x_α),$(\acute{e}, y_\beta)$there exists a fuzzy softopen set containing one of the points but not the other.

Example 3.4. Let $X = \{x^1, x^2\}$,$E = \{e^1, e^2\}$and
$\mathfrak{T} = \{\tilde{1}_E, \tilde{0}_E, (f_E)_1, (f_E)_2, (f_E)_3, (f_E)_4, (f_E)_5, (f_E)_6\}$ where
$(f_E)_1 = \left\{\left(e^1 = \left\{\frac{x^1}{1}, \frac{x^2}{0}\right\}\right), \left(e^2 = \left\{\frac{x^1}{0}, \frac{x^2}{0}\right\}\right)\right\}$, $(f_E)_2 = \left\{\left(e^1 = \left\{\frac{x^1}{0}, \frac{x^2}{0}\right\}\right), \left(e^2 = \left\{\frac{x^1}{1}, \frac{x^2}{0}\right\}\right)\right\}$,
$(f_E)_3 = \left\{\left(e^1 = \left\{\frac{x^1}{0}, \frac{x^2}{0}\right\}\right), \left(e^2 = \left\{\frac{x^1}{0}, \frac{x^2}{1}\right\}\right)\right\}$, $(f_E)_4 = \left\{\left(e^1 = \left\{\frac{x^1}{1}, \frac{x^2}{0}\right\}\right), \left(e^2 = \left\{\frac{x^1}{0}, \frac{x^2}{1}\right\}\right)\right\}$,
$(f_E)_5 = \left\{\left(e^1 = \left\{\frac{x^1}{1}, \frac{x^2}{0}\right\}\right), \left(e^2 = \left\{\frac{x^1}{1}, \frac{x^2}{0}\right\}\right)\right\}$,$(f_E)_6 = \left\{\left(e^1 = \left\{\frac{x^1}{0}, \frac{x^2}{0}\right\}\right), \left(e^2 = \left\{\frac{x^1}{1}, \frac{x^2}{1}\right\}\right)\right\}$,

Then, clearly $\mathfrak{T}$ is a fuzzy soft topology over (X,E). Also for every pair of distinct fuzzy soft points there exists fuzzy soft open set containing one of the points but not the other. Hence $(X,\mathfrak{T},E)$ is a fuzzy soft T_0-space.

Example 3.5. Let $X=\{x^1,x^2\}$, $E=\{e^1,e^2\}$ and $\mathfrak{T}=\{\tilde{1}_E,\tilde{0}_E,f_E,g_E\}$ where
$f_E=\left\{\left(e^1=\left\{\frac{x^1}{1},\frac{x^2}{1}\right\}\right)\right\}$, $g_E=\left\{\left(e^2=\left\{\frac{x^1}{1},\frac{x^2}{1}\right\}\right)\right\}$.
Then $\mathfrak{T}$ is a fuzzy soft topology over (X,E) but $(X,\mathfrak{T},E)$ is not a fuzzy soft T_0 −space.

Example 3.6. The discrete fuzzy soft topological space is a fuzzy soft T_0 −space**,** but the indiscrete fuzzy soft topological space is not fuzzy soft T_0.

Theorem 3.7. A fuzzy soft subspace $(X,\mathfrak{T}_Y,E)$ of a fuzzy soft T_0-space $(X,\mathfrak{T},E)$ is fuzzy soft T_0.

Proof. Let (e,x_α), $(\acute{e},y_\beta)$ be two distinct fuzzy soft points in (Y,E). Then these fuzzy soft points are also in (X,E). Hence, there exists a fuzzy soft open set f_E in $\mathfrak{T}$ containing one of the points say, (e,x_α), but not $(\acute{e},y_\beta)$. Thus, $h_E^Y\sqcap f_E$ is a fuzzy soft open set in $(X,\mathfrak{T}_Y,E)$ containing (e,x_α) but not $(\acute{e},y_\beta)$. Therefore, $(X,\mathfrak{T}_Y,E)$ is fuzzy soft T_0**.**

Definition 3.8. A fuzzy soft topological space $(X,\mathfrak{T},E)$ is said to be a fuzzy soft T_1 −space if for every pair of distinct fuzzy soft points (e,x_α), $(\acute{e},y_\beta)$ there exist fuzzy soft open sets f_E and g_E such that $(e,x_\alpha)\,\tilde{\in}\,f_E$, $(\acute{e},y_\beta)\,\tilde{\notin}\,f_E$ and $(\acute{e},y_\beta)\,\tilde{\in}\,g_E$, $(e,x_\alpha)\,\tilde{\notin}\,g_E$.

Example 3.9. Let $X=\{x^1,x^2\}$, $E=\{e^1,e^2\}$ and
$\mathfrak{T}=\{\tilde{1}_E,\tilde{0}_E,(f_E)_1,(f_E)_2,(f_E)_3,(f_E)_4,(f_E)_5,(f_E)_6,(f_E)_7,(f_E)_8,(f_E)_9,(f_E)_{10},(f_E)_{11},(f_E)_{12}\}$ where
$(f_E)_1=\left\{\left(e^1=\left\{\frac{x^1}{1},\frac{x^2}{0}\right\}\right),\left(e^2=\left\{\frac{x^1}{0},\frac{x^2}{1}\right\}\right)\right\}$, $(f_E)_2=\left\{\left(e^1=\left\{\frac{x^1}{0},\frac{x^2}{1}\right\}\right),\left(e^2=\left\{\frac{x^1}{1},\frac{x^2}{0}\right\}\right)\right\}$,
$(f_E)_3=\left\{\left(e^1=\left\{\frac{x^1}{1},\frac{x^2}{1}\right\}\right),\left(e^2=\left\{\frac{x^1}{0},\frac{x^2}{0}\right\}\right)\right\}$, $(f_E)_4=\left\{\left(e^1=\left\{\frac{x^1}{0},\frac{x^2}{0}\right\}\right),\left(e^2=\left\{\frac{x^1}{1},\frac{x^2}{1}\right\}\right)\right\}$,
$(f_E)_5=\left\{\left(e^1=\left\{\frac{x^1}{1},\frac{x^2}{0}\right\}\right),\left(e^2=\left\{\frac{x^1}{0},\frac{x^2}{0}\right\}\right)\right\}$, $(f_E)_6=\left\{\left(e^1=\left\{\frac{x^1}{0},\frac{x^2}{0}\right\}\right),\left(e^2=\left\{\frac{x^1}{0},\frac{x^2}{1}\right\}\right)\right\}$,
$(f_E)_7=\left\{\left(e^1=\left\{\frac{x^1}{0},\frac{x^2}{1}\right\}\right),\left(e^2=\left\{\frac{x^1}{0},\frac{x^2}{0}\right\}\right)\right\}$, $(f_E)_8=\left\{\left(e^1=\left\{\frac{x^1}{0},\frac{x^2}{0}\right\}\right),\left(e^2=\left\{\frac{x^1}{1},\frac{x^2}{0}\right\}\right)\right\}$,
$(f_E)_9=\left\{\left(e^1=\left\{\frac{x^1}{1},\frac{x^2}{1}\right\}\right),\left(e^2=\left\{\frac{x^1}{0},\frac{x^2}{1}\right\}\right)\right\}$, $(f_E)_{10}=\left\{\left(e^1=\left\{\frac{x^1}{1},\frac{x^2}{0}\right\}\right),\left(e^2=\left\{\frac{x^1}{1},\frac{x^2}{1}\right\}\right)\right\}$,
$(f_E)_{11}=\left\{\left(e^1=\left\{\frac{x^1}{1},\frac{x^2}{1}\right\}\right),\left(e^2=\left\{\frac{x^1}{1},\frac{x^2}{0}\right\}\right)\right\}$, $(f_E)_{12}=\left\{\left(e^1=\left\{\frac{x^1}{0},\frac{x^2}{1}\right\}\right),\left(e^2=\left\{\frac{x^1}{1},\frac{x^2}{1}\right\}\right)\right\}$.
Then $(X,\mathfrak{T},E)$ is a fuzzy soft T_1 −space.

Example 3.10. The discrete fuzzy soft topological space is a fuzzy soft T_1 −space**,** but the indiscrete fuzzy soft topological space is not fuzzy soft T_1.

Theorem3.11. A fuzzy soft subspace $(X,\mathfrak{T}_Y,E)$ of a fuzzy soft T_1 −space $(X,\mathfrak{T},E)$ is fuzzy soft T_1.

Proof. It is similar to the proof of Theorem 3.7.

Theorem 3.12. If every fuzzy soft point(e,x_α) of a fuzzy soft topological space $(X,\mathfrak{T},E)$ is fuzzy soft closed such that $\alpha > 0.5$, then $(X,\mathfrak{T},E)$ is fuzzy soft T_1.

Proof. Suppose that (e^1,x_α) and (e^2,y_β)two distinct fuzzy soft points in (X,E). By hypothesis, (e^1,x_α) and (e^2,y_β) are fuzzy soft closed sets and $\alpha > 0.5, \beta > 0.5$. Hence, $(e^1,x_\alpha)^c$and $(e^2,y_\beta)^c$ are fuzzy soft open sets where $(e^1,x_\alpha) \tilde{\in} (e^2,y_\beta)^c$, $(e_2,y_\beta) \tilde{\notin} (e^2,y_\beta)^c$and $(e^1,x_\alpha) \tilde{\notin} (e^1,x_\alpha)^c$, $(e^2,y_\beta) \tilde{\in} (e^1,x_\alpha)^c$. Therefore, $(X,\mathfrak{T},E)$ is fuzzy soft T_1.

The condition $\alpha > 0.5$ in theorem 3.12, is necessary as shown by the following examples:

Example 3.13. Let $X = \{x^1,x^2\}$,$E = \{e^1\}$. Consider the collection $\mathfrak{T}$ of fuzzy soft sets over(X,E),$\mathfrak{T} = \{\tilde{1}_E, \tilde{0}_E, f_E, g_E, h_E\}$ where f_E, g_E and h_E are as follows:

$f_E = \left\{\left(e^1 = \left\{\frac{x^1}{1-\alpha}, \frac{x^2}{1}\right\}\right)\right\}$,$g_E = \left\{\left(e^1 = \left\{\frac{x^1}{1}, \frac{x^2}{1-\beta}\right\}\right)\right\}$,$h_E = \left\{\left(e^1 = \left\{\frac{x^1}{1-\alpha}, \frac{x^2}{1-\beta}\right\}\right)\right\}$.

Then$\mathfrak{T}$ is fuzzy soft topology over (X,E) and $(e^1,x^1_\alpha), (e^1,x^2_\beta)$ are two distinct fuzzy soft points in (X,E) such that for any fuzzy soft open set which containing (e^1,x^1_α)also containing (e^1,x^2_β). Hence, $(X,\mathfrak{T},E)$is not fuzzy soft T_1.

Remark 3.14. If $(X,\mathfrak{T},E)$is a fuzzy soft T_1 −space, then (e,x_α)maybe not a fuzzy soft closed setas the following example shows.

Example3.15. In Example 3.9$(X,\mathfrak{T},E)$is a fuzzy soft T_1 −space, but$(e^1,x_\alpha)$is not fuzzy soft closed set. To show this, let $(e^1,x^1_\alpha) = \left\{\left(e^1 = \left\{\frac{x^1}{\alpha}\right\}\right)\right\}$.

Then$(e^1,x^1_\alpha)^c = \left\{\left(e^1 = \left\{\frac{x^1}{1-\alpha}, \frac{x^2}{1}\right\}\right), \left(e^2 = \left\{\frac{x^1}{1}, \frac{x^2}{1}\right\}\right)\right\}$ is not fuzzy soft open set i.e., $(e^1,x^1_\alpha)$is not fuzzy soft closed set.

Definition 3.16.([7]) A fuzzy soft topological space $(X,\mathfrak{T},E)$is said to be a fuzzy soft T_2-space if for every pair of distinct fuzzy soft points (e,x_α), $(\acute{e},y_\beta)$there exists disjoint fuzzy softopen sets f_E and $g_E$such that $(e,x_\alpha) \tilde{\in} f_E$ and $(\acute{e},y_\beta) \in g_E$.

Example3.17.([7]**)** Let$X = \{x^1,x^2\}$,$E = \{e^1,e^2\}$ and

$\mathfrak{T} = \{\tilde{1}_E, \tilde{0}_E, (f_E)_1, (f_E)_2, (f_E)_3, (f_E)_4, (f_E)_5, (f_E)_6, (f_E)_7, (f_E)_8, (f_E)_9, (f_E)_{10}, (f_E)_{11}, (f_E)_{12}, (f_E)_{13}, (f_E)_{14}\}$ where

$(f_E)_1 = \left\{\left(e^1 = \left\{\frac{x^1}{1}, \frac{x^2}{0}\right\}\right), \left(e^2 = \left\{\frac{x^1}{0}, \frac{x^2}{0}\right\}\right)\right\}$, $(f_E)_2 = \left\{\left(e^1 = \left\{\frac{x^1}{0}, \frac{x^2}{1}\right\}\right), \left(e^2 = \left\{\frac{x^1}{0}, \frac{x^2}{0}\right\}\right)\right\}$,

$(f_E)_3 = \left\{\left(e^1 = \left\{\frac{x^1}{0}, \frac{x^2}{0}\right\}\right), \left(e^2 = \left\{\frac{x^1}{1}, \frac{x^2}{0}\right\}\right)\right\}$, $(f_E)_4 = \left\{\left(e^1 = \left\{\frac{x^1}{0}, \frac{x^2}{0}\right\}\right), \left(e^2 = \left\{\frac{x^1}{0}, \frac{x^2}{1}\right\}\right)\right\}$,

$(f_E)_5 = \left\{\left(e^1 = \left\{\frac{x^1}{1}, \frac{x^2}{1}\right\}\right), \left(e^2 = \left\{\frac{x^1}{0}, \frac{x^2}{0}\right\}\right)\right\}$, $(f_E)_6 = \left\{\left(e^1 = \left\{\frac{x^1}{0}, \frac{x^2}{1}\right\}\right), \left(e^2 = \left\{\frac{x^1}{1}, \frac{x^2}{0}\right\}\right)\right\}$,

$(f_E)_7 = \left\{\left(e^1 = \left\{\frac{x^1}{0}, \frac{x^2}{0}\right\}\right), \left(e^2 = \left\{\frac{x^1}{1}, \frac{x^2}{1}\right\}\right)\right\}$, $(f_E)_8 = \left\{\left(e^1 = \left\{\frac{x^1}{1}, \frac{x^2}{0}\right\}\right), \left(e^2 = \left\{\frac{x^1}{0}, \frac{x^2}{1}\right\}\right)\right\}$,

$(f_E)_9 = \left\{\left(e^1 = \left\{\frac{x^1}{1}, \frac{x^2}{0}\right\}\right), \left(e^2 = \left\{\frac{x^1}{1}, \frac{x^2}{0}\right\}\right)\right\}$, $(f_E)_{10} = \left\{\left(e^1 = \left\{\frac{x^1}{0}, \frac{x^2}{1}\right\}\right), \left(e^2 = \left\{\frac{x^1}{1}, \frac{x^2}{0}\right\}\right)\right\}$,
$(f_E)_{11} = \left\{\left(e^1 = \left\{\frac{x^1}{1}, \frac{x^2}{1}\right\}\right), \left(e^2 = \left\{\frac{x^1}{1}, \frac{x^2}{0}\right\}\right)\right\}$, $(f_E)_{12} = \left\{\left(e^1 = \left\{\frac{x^1}{0}, \frac{x^2}{1}\right\}\right), \left(e^2 = \left\{\frac{x^1}{1}, \frac{x^2}{1}\right\}\right)\right\}$,
$(f_E)_{13} = \left\{\left(e^1 = \left\{\frac{x^1}{1}, \frac{x^2}{0}\right\}\right), \left(e^2 = \left\{\frac{x^1}{1}, \frac{x^2}{1}\right\}\right)\right\}$, $(f_E)_{14} = \left\{\left(e^1 = \left\{\frac{x^1}{1}, \frac{x^2}{1}\right\}\right), \left(e^2 = \left\{\frac{x^1}{0}, \frac{x^2}{1}\right\}\right)\right\}$.
Then, clearly $\mathfrak{T}$ is fuzzy soft topology over (X, E). Also, for every pair of distinct fuzzysoft points, there exist disjoint fuzzy soft open sets in(X, E) containing them. Hence$(X, \mathfrak{T}, E)$ is afuzzy soft T_2-space.

Example3.18. Thediscrete fuzzy soft topological space is a fuzzy soft T_2-space, but the indiscrete fuzzy soft topological space is not a fuzzy soft T_2.

Theorem 3.19. A fuzzy soft subspace $(X, \mathfrak{T}_Y, E)$of fuzzy soft T_2-space$(X, \mathfrak{T}, E)$is fuzzy soft T_2.

Proof. Let (e, x_α), $(\acute{e}, y_\beta)$be two distinct fuzzy soft points in (Y, E). Then, these fuzzy soft points are alsoin (X, E). Hence, there exists disjoint fuzzy soft open sets f_E and g_E in $\mathfrak{T}$ such that $(e, x_\alpha) \tilde{\in} f_E$ and $(\acute{e}, y_\beta) \tilde{\in} g_E$. Thus, $h_E^Y \sqcap f_E$ and $h_E^Y \sqcap g_E$ are disjoint fuzzy soft open sets in $\mathfrak{T}_Y$ such that $(e, x_\alpha) \tilde{\in} h_E^Y \sqcap f_E$and $(\acute{e}, y_\beta) \tilde{\in} h_E^Y \sqcap g_E$. So, $(X, \mathfrak{T}_Y, E)$is a fuzzy soft T_2-space.

Remark 3.20. From definitions one deduce the following implication hold:

fuzzy soft $T_2 \Longrightarrow$ fuzzysoft T_1 $\Longrightarrow$fuzzysoft T_0

The inverse implications may not be true as shows is by the following examples.

Example3.21. In Example 3.4, $(X, \mathfrak{T}, E)$is a fuzzysoft T_0 −space but not fuzzysoft T_1 −space. Since (e^1, x_β^2),$(e^2, x_\alpha^1)$are distinct fuzzy soft points and the only fuzzysoft open set which containing (e^1, x_β^2) is $\tilde{1}_E$ also containing(e^2, x_α^1). Hence $(X, \mathfrak{T}, E)$is not fuzzysoft T_1.
In Example 3.9, $(X, \mathfrak{T}, E)$is a fuzzysoft T_1-space but not fuzzysoft T_2-space. Since (e^1, x_α^1),(e^2, x_β^2) are distinct fuzzy soft points and the only fuzzysoft open sets which containing (e^1, x_α^1), (e^2, x_β^2) are $(f_E)_1$, $(f_E)_2$ but they are not disjoint. Hence $(X, \mathfrak{T}, E)$is not fuzzysoft T_2.

Definition 3.22. Let $(X, \mathfrak{T}, E)$be a fuzzy soft topological space. If for every fuzzy soft closed seth_E and every and fuzzy soft point(e, x_α)such that $(e, x_\alpha) \sqcap k_E = \tilde{0}_E$ there exists disjoint fuzzysoft open sets f_E and g_E suchthat $(e, x_\alpha) \tilde{\in} f_E$ and $k_E \sqsubseteq g_E$. Then $(X, \mathfrak{T}, E)$is called fuzzy soft regular space.

Definition 3.23. A fuzzy soft topological space$(X, \mathfrak{T}, E)$ is called a fuzzy soft T_3 −space if it is fuzzy soft T_1 and fuzzy soft regular.

Example3.24. Let $X = \{x^1, x^2\}$,$E = \{e^1\}$and $\mathfrak{T} = \{\tilde{1}_E, \tilde{0}_E, f_E, g_E\}$ where
$f_E = \left\{\left(e^1 = \left\{\frac{x^1}{1}\right\}\right)\right\}, g_E = \left\{\left(e^1 = \left\{\frac{x^2}{1}\right\}\right)\right\}$.
Then $\mathfrak{T}$ isfuzzy soft topology over (X, E).Now, $\mathfrak{T}^c = \{\tilde{1}_E, \tilde{0}_E, f_E^c, g_B^c\}$ where
$f_E^c = \left\{\left(e^1 = \left\{\frac{x^2}{1}\right\}\right)\right\}$, $g_E^c = \left\{\left(e^1 = \left\{\frac{x^1}{1}\right\}\right)\right\}$.
For the fuzzy soft point $(e^1, x_\alpha^1) \sqcap f_E^c = \tilde{0}_E$there exist fuzzysoftopen sets f_E and $g_E$such that$(e^1, x_\alpha^1) \tilde{\in} f_E, f_E^c \sqsubseteq g_E$ and $f_E \sqcap g_E = \tilde{0}_E$.
For the fuzzy soft point $(e^1, x_\beta^2) \sqcap g_E^c = \tilde{0}_E$there exist fuzzysoftopen sets$g_E$and $f_E$such that$(e^1, x_\beta^2) \tilde{\in} g_E, g_E^c \sqsubseteq f_E$ and $g_E \sqcap f_E = \tilde{0}_E$.
Then $(X, \mathfrak{T}, E)$is a fuzzy soft T_3 −space.

Example3.25. Let$X = \{x^1, x^2\}$,$E = \{e^1, e^2\}$and $\mathfrak{T} = \{\tilde{1}_E, \tilde{0}_E, f_E, g_E\}$where
$f_E = \left\{\left(e^1 = \left\{\frac{x^1}{1}, \frac{x^2}{1}\right\}, 1\right)\right\}, g_E = \left\{\left(e^2 = \left\{\frac{x^1}{1}, \frac{x^2}{1}\right\}, 1\right)\right\}$.
Then $\mathfrak{T}$ isa fuzzy soft topology over (X, E).Now, $\mathfrak{T}^c = \{\tilde{0}_\theta, \tilde{1}_\Delta, f_E^c, g_E^c\}$ where
$f_E^c = \left\{\left(e^2 = \left\{\frac{x^1}{1}, \frac{x^2}{1}\right\}\right)\right\}$, $g_E^c = \left\{\left(e^1 = \left\{\frac{x^1}{1}, \frac{x^2}{1}\right\}\right)\right\}$.
By used fuzzy soft regularity on fuzzy soft closed sets as follows:
$(e^1, x_\alpha^1) \sqcap f_E^c = \tilde{0}_E \Longrightarrow \exists\, g_E^c, g_E \in \mathfrak{T}$ such that $(e^1, x_\alpha^1) \tilde{\in} g_E^c, f_E^c \sqsubseteq g_E$ and $g_E^c \sqcap g_E = \tilde{0}_E$,
$(e^1, x_\beta^2) \sqcap f_E^c = \tilde{0}_E \Longrightarrow \exists\, g_E^c, g_E \in \mathfrak{T}$ such that $(e^1, x_\beta^2) \tilde{\in} g_E^c, f_E^c \sqsubseteq g_E$ and $g_E^c \sqcap g_E = \tilde{0}_E$,
$(e^2, x_{\alpha'}^1) \sqcap g_E^c = \tilde{0}_E \Longrightarrow \exists\, f_E^c, f_E \in \mathfrak{T}$ such that $(e^2, x_{\alpha'}^1) \tilde{\in} f_E^c, g_E^c \sqsubseteq f_E$ and $f_E^c \sqcap f_E = \tilde{0}_E$,
$(e^2, x_{\beta'}^2) \sqcap g_E^c = \tilde{0}_E \Longrightarrow \exists\, f_E^c, f_E \in \mathfrak{T}$ such that $(e^2, x_{\beta'}^2) \tilde{\in} f_E^c, g_E^c \sqsubseteq f_E$ and $f_E^c \sqcap f_E = \tilde{0}_E$.
Then $(X, \mathfrak{T}, E)$is a fuzzy soft regular space, but not a fuzzy softT_1 −space and hence not fuzzy soft T_3.

Proposition 3.26. If $(X, \mathfrak{T}, E)$is a fuzzy soft regular space, then for any fuzzy soft open set g_E and a fuzzy soft point(e, x_α) in (X, E) such that $(e, x_\alpha) \sqcap g_E^c = \tilde{0}_E$, then there exists a fuzzysoft open set s_E such that $(e, x_\alpha) \tilde{\in} s_E \sqsubseteq cl(s_E) \sqsubseteq g_E$.

Proof. Suppose that $(X, \mathfrak{T}, E)$is a fuzzy soft regular space. Let $g_E$be a fuzzysoft open setin(X, E) such that $(e, x_\alpha) \sqcap g_E^c = \tilde{0}_E$.Now$g_E^c$ is fuzzysoft closed set in(X, E) such that $(e, x_\alpha) \sqcap g_E^c = \tilde{0}_E$ and $(X, \mathfrak{T}, E)$isa fuzzy soft regular, therefore there exist two disjoint fuzzysoft open sets $s_E$and $w_E$such that$(e, x_\alpha) \in s_E$and $g_E^c \sqsubseteq w_E$. Now, w_E^c is a fuzzysoft closed set in(X, E)such that$s_E \sqsubseteq w_E^c \sqsubseteq g_E$. Thus, $(e, x_\alpha) \in s_E \sqsubseteq cl(s_E)$ and $s_E \sqsubseteq w_E^c \sqsubseteq g_E$ and hence,$cl(s_E) \sqsubseteq g_E$. This proves that$(e, x_\alpha) \in s_E \sqsubseteq cl(s_E) \sqsubseteq g_E$.

Theorem 3.27. Every fuzzy soft regular space, in which every fuzzy soft point(e, x_α) is fuzzysoft closed,is a fuzzy soft T_2 −space.

Proof. Let (e, x_α), $(\acute{e}, y_\beta)$be two distinct fuzzy soft points of a fuzzy soft regular space$(X, \mathfrak{T}, E)$.By hypothesis, $(e', y_\beta)$is fuzzysoft closed set and $(e, x_\alpha) \sqcap (\acute{e}, y_\beta) = \tilde{0}_E$. From the fuzzy soft regularity, there exist disjoint fuzzysoft open sets f_E and g_E such that

$(e,x_\alpha)\,\tilde{\in}\, f_E$ and $(\acute{e},y_\beta)\sqsubseteq g_E$. Thus, $(e,x_\alpha)\,\tilde{\in}\, f_E$ and $(\acute{e},y_\beta)\sqsubseteq g_E$. Thus $(e,x_\alpha)\,\tilde{\in}\, f_E$ and $(\acute{e},y_\beta)\,\tilde{\in}\, g_E$. Therefore, $(X,\mathfrak{T},E)$ is a fuzzy soft T_2 −space.

Corollary 3.28. Every fuzzy soft T_3 −space, in which every fuzzy soft point(e,x_α) is fuzzysoft closed is a fuzzy soft T_2.

Theorem 3.29. A fuzzy soft subspace $(Y,\mathfrak{T}_Y,E)$ of a fuzzy soft T_3 −space$(X,\mathfrak{T},E)$is fuzzy soft T_3.

Proof. By Theorem 3.11,$(Y,\mathfrak{T}_Y,E)$is a fuzzy soft T_1 −space. Now, we want to prove that$(X,\mathfrak{T}_Y,E)$is a fuzzy soft regular space. Let $k_E$be a fuzzysoft closed set in$(Y,E)$and (e,y_β)be afuzzy soft point in(Y,E)such that $(e,y_\beta)\sqcap k_E=\tilde{0}_E$. Then, $k_E=h_E^Y\sqcap f_E$(Theorem 4.9, [5])for some fuzzysoft closed set $f_E$in(X,E).Hence, $(e,y_\beta)\sqcap(h_E^Y\sqcap f_E)=\tilde{0}_E$. But $(e,y_\beta)\,\tilde{\in}\, h_E^Y$, so $(e,y_\beta)\sqcap f_E=\tilde{0}_E$. Since $(X,\mathfrak{T},E)$is fuzzy soft regular. Then, there exist disjoint fuzzysoft open sets $s_E$and $w_E$in $\mathfrak{T}$such that $(e,y_\beta)\,\tilde{\in}\, s_E$and $f_E\sqsubseteq w_E$. It follows that,$h_E^Y\sqcap s_E$and $h_E^Y\sqcap w_E$are disjoint fuzzysoft open sets in $\mathfrak{T}_Y$such that $(e,y_\beta)\,\tilde{\in}\, h_E^Y\sqcap s_E$and $k_E\sqsubseteq h_E^Y\sqcap w_E$.Therefore, $(Y,\mathfrak{T}_Y,E)$is fuzzy soft regular and thus fuzzy soft T_3.

Definition 3.30. Let $(X,\mathfrak{T},E)$be a fuzzy soft topological space. If for every disjoint fuzzy soft closedh_E, $k_E$there exist disjoint fuzzy soft open sets $s_E$and $w_E$such that $h_E\sqsubseteq s_E$, $k_E\sqsubseteq w_E$. Then$(X,\mathfrak{T},E)$is called fuzzy soft normal space. $(X,\mathfrak{T},E)$ is called a fuzzy soft T_4 −space if it is fuzzy soft normal and fuzzy soft T_1 −space.

Example3.31. Let$X=\{x^1,x^2\}$,$E=\{e^1,e^2\}$and $\mathfrak{T}=\{\tilde{1}_E,\tilde{0}_E,f_E,g_E,h_E,k_E\}$ where

$f_E=\left\{\left(e^1=\left\{\frac{x^1}{1},\frac{x^2}{0}\right\}\right),\left(e^2=\left\{\frac{x^1}{0},\frac{x^2}{0}\right\}\right)\right\}$, $g_E=\left\{\left(e^1=\left\{\frac{x^1}{1},\frac{x^2}{0}\right\}\right),\left(e^2=\left\{\frac{x^1}{0},\frac{x^2}{1}\right\}\right)\right\}$,

$h_E=\left\{\left(e^1=\left\{\frac{x^1}{0},\frac{x^2}{1}\right\}\right),\left(e^2=\left\{\frac{x^1}{1},\frac{x^2}{0}\right\}\right)\right\}$,$k_E=\left\{\left(e^1=\left\{\frac{x^1}{1},\frac{x^2}{1}\right\}\right),\left(e^2=\left\{\frac{x^1}{1},\frac{x^2}{0}\right\}\right)\right\}$.

Then $\mathfrak{T}$ isfuzzy soft topology over (X,E). Now, let$\mathfrak{T}^c=\{\tilde{1}_E,\tilde{0}_E,f_E^c,g_E^c,h_E^c,k_E^c\}$ where

$f_E^c=\left\{\left(e^1=\left\{\frac{x^1}{0},\frac{x^2}{1}\right\}\right),\left(e^2=\left\{\frac{x^1}{1},\frac{x^2}{1}\right\}\right)\right\}$,$g_E^c=\left\{\left(e^1=\left\{\frac{x^1}{0},\frac{x^2}{1}\right\}\right),\left(e^2=\left\{\frac{x^1}{1},\frac{x^2}{0}\right\}\right)\right\}$,

$h_E^c=\left\{\left(e^1=\left\{\frac{x^1}{1},\frac{x^2}{0}\right\}\right),\left(e^2=\left\{\frac{x^1}{0},\frac{x^2}{1}\right\}\right)\right\}$,$k_E^c=\left\{\left(e^1=\left\{\frac{x^1}{0},\frac{x^2}{0}\right\}\right),\left(e^2=\left\{\frac{x^1}{0},\frac{x^2}{1}\right\}\right)\right\}$.

For the two disjoint fuzzy soft closed sets g_E^c and h_E^c there exists $g_E,h_E\in\mathfrak{T}$ such that $h_E^c\sqsubseteq g_E, g_E^c\sqsubseteq h_E$ and $g_E\sqcap h_E=\tilde{0}_E$.

For the two disjoint fuzzy soft closed sets g_E^c and k_E^c there exists $h_E,g_E\in\mathfrak{T}$ such that $g_E^c\sqsubseteq h_E, k_E^c\sqsubseteq g_E$and $h_E\sqcap g_E=\tilde{0}_E$.

Then $(X,\mathfrak{T},E)$is a fuzzy soft normal space, but not aT_1 −space and hence not T_4.

Proposition 3.32. If $(X,\mathfrak{T},E)$is a fuzzy soft normal space, then for each fuzzy soft closed set $k_E$in(X,E) and any fuzzysoftopen set$g_E$in(X,E) such that $k_E\sqcap g_E^c=\tilde{0}_E$then there exists a fuzzysoftopen set $s_E$such that $k_E\sqsubseteq s_E\sqsubseteq cl(s_E)\sqsubseteq g_E$.

Proof. Let$(X,\mathfrak{T},E)$be a fuzzy soft normal space. Let $k_E$be a fuzzysoft closed setin(X,E) and $g_E$be a fuzzy soft open setin(X,E) such that $k_E \sqcap g_E^c = \tilde{0}_E$,then $k_E \sqsubseteq g_E$. Now, k_E and $g_E^c$are two disjoint fuzzysoft closed sets in (X,E). Since $(X,\mathfrak{T},E)$is a fuzzy soft normal, so there exist two disjoint fuzzysoft open sets $s_E$and $w_E$such that$k_E \sqsubseteq s_E$, $g_E^c \sqsubseteq w_E$ and $s_E \sqcap w_E = \tilde{0}_E$, we have$s_E \sqsubseteq w_E^c$, but w_E^c is a fuzzysoft closed set and hence$cl(s_E) \sqsubseteq w_E^c$. Thus, we have $k_E \sqsubseteq s_E \sqsubseteq cl(s_E) \sqsubseteq g_E$.

Theorem 3.33. A fuzzy soft closed subspace $(Y,\mathfrak{T}_Y,E)$of a fuzzy soft normal space$(X,\mathfrak{T},E)$is fuzzy soft normal.

Proof. Let $(k_E)_1$and $(k_E)_2$be disjoint fuzzysoft closed sets in (Y,E). Then, $(k_E)_1 = h_E^Y \sqcap (f_E)_1and(k_E)_2 = h_E^Y \sqcap (f_E)_2$(Theorem 4.9 in [5]) for some fuzzysoft closed sets $(f_E)_1$, $(f_E)_2$in (X,E). Since h_E^Y is fuzzy soft closed in (X,E), then $(k_E)_1$, $(k_E)_2$ are disjoint fuzzy soft closed sets in (X,E). Since $(X,\mathfrak{T},E)$is fuzzy soft normal, then there exist disjoint fuzzy soft open sets $(g_E)_1$and $(g_E)_2$in (X,E)such that $(k_E)_1 \sqsubseteq (g_E)_1$, $(k_E)_2 \sqsubseteq (g_E)_2$, and then,$(k_E)_1 \sqsubseteq (s_E)_1 = h_E^Y \sqcap (g_E)_1$, $(k_E)_2 \sqsubseteq (s_E)_2 = h_E^Y \sqcap (g_E)_2$. From definition of $\mathfrak{T}_Y$, we have $(s_E)_1, (s_E)_2 \in \mathfrak{T}_Y$ are fuzzy soft open sets in$(Y,E)$and $(s_E)_1 \sqcap (s_E)_2 = [h_E^Y \sqcap (g_E)_1] \sqcap [(h_E^Y \sqcap (g_E)_2] = h_E^Y \sqcap [(g_E)_1 \sqcap (g_E)_2] = h_E^Y \sqcap \tilde{0}_E = \tilde{0}_E$. Therefore, $(Y,\mathfrak{T}_Y,E)$is fuzzy soft normal.

Theorem 3.34. Every fuzzy soft normal space, in which every fuzzy soft point$(e,x_\alpha)$is fuzzysoft closed, is a fuzzy soft regular space.

Proof. Let $(X,\mathfrak{T},E)$fuzzy soft normal space and (e,x_α)be a fuzzy soft point and $k_E$be a fuzzy soft closed set such that$(e,x_\alpha) \sqcap k_E = \tilde{0}_E$. Since (e,x_α) is a fuzzy soft closed set in (X,E), thenthere exist disjoint fuzzy soft open sets $s_E$and $w_E$such that $(e,x_\alpha) \sqsubseteq s_E$, $k_E \sqsubseteq w_E$and thus,$(e,x_\alpha) \tilde{\in} s_E$, $k_E \sqsubseteq w_E$.Therefore, $(X,\mathfrak{T},E)$is fuzzy soft regular.

Corollary 3.35. Every fuzzy soft T_4 −space, in which every fuzzy soft point(e,x_α) is fuzzysoft closed, is a fuzzy soft T_3.

Theorem3.36. Let $f_{up}: (X,\mathfrak{T}_1,E) \longrightarrow (Y,\mathfrak{T}_2,K)$ be a fuzzy softbijective and fuzzy soft open mapping. If $(X,\mathfrak{T}_1,,E)$is a fuzzy soft T_i −space, then $(Y,\mathfrak{T}_2,K)$ is a fuzzy soft T_i-space, $i = 0; 1; 2$

Proof. We prove the theorem for ($i = 2$, for example), the other cases are similar.

Let$(X,\mathfrak{T}_1,E)$be a fuzzy soft T_2 −space and $f_{up}: (X,\mathfrak{T}_1,E) \longrightarrow (Y,\mathfrak{T}_2,K)$bea fuzzy softbijective fuzzy soft open mapping, we want to show that $(Y,\mathfrak{T}_2,K)$is a fuzzy soft T_2 −space. So, let $(\acute{e},x_\alpha)$, $(\acute{s},y_\beta)$ be two distinct fuzzy soft points in (Y,K). Since f_{up} is bijection mapping, then there exist two distinct fuzzy soft points (e, a_α),(s, b_β)in(X, E)such that $f_{up}(e, a_\alpha) = (\acute{e}, x_\alpha)$, $f_{up}(s, b_\beta) = (\acute{s}, y_\beta)$. But $(X, \mathfrak{T}_1, E)$is a fuzzy soft T_2 −space, so, there exists disjoint fuzzy soft open sets f_E and $g_E$in(X, E) such that$(e, a_\alpha) \tilde{\in} f_E$, $(s, b_\beta) \tilde{\in} g_E$.

It follows that, $f_{up}(e, a_\alpha) = (\acute{e}, x_\alpha) \tilde{\in} f_{up}(f_E)$,$f_{up}(s, b_\beta) = (\acute{s}, y_\beta) \tilde{\in} f_{up}(g_E)$and $f_{up}(f_E) \sqcap f_{up}(g_E) = f_{up}(f_E \sqcap g_E) = f_{up}(\tilde{0}_E) = \tilde{0}_K$(from Proposition 2.22).

Since $f_E, g_E \in \mathfrak{T}_1$, and f_{up} is a fuzzy soft open mapping, $f_{up}(f_E)$,$f_{up}(g_E) \in \mathfrak{T}_2$. Now, there exists disjoint fuzzy soft open sets $f_{up}(f_E)$ and $f_{up}(g_E)$ in (Y, K) such that$(\acute{e}, x_\alpha) \tilde{\in} f_{up}(f_E)$ and $(\acute{s}, y_\beta) \tilde{\in} f_{up}(g_E)$. Hence,$(Y, \mathfrak{T}_2, K)$ is a fuzzy soft T_2 −space.

Definition 3.37. The property P is called a fuzzy soft topological property if it is preserved under a fuzzy soft homeomorphism mapping.

Corollary 3.38. The property of being fuzzy soft T_i −space $(i = 0; 1; 2)$ is a fuzzy soft topological property.

Theorem 3.39. The property of being fuzzy soft T_i −space $(i = 3; 4)$ is a fuzzy soft topological property.

Proof. We prove the theorem for ($i = 3$, for example), the other cases are similar.
Since, the property of being fuzzy soft T_1 −space is a fuzzy soft topological property, we only show that the property of fuzzy soft regularity is a fuzzy soft topological property.

Let$f_{up}: (X, \mathfrak{T}_1,, E) \longrightarrow (Y, \mathfrak{T}_2, K)$ be a fuzzy soft homeomorphism and$(X, \mathfrak{T}_1, E)$is a fuzzy soft regular space. Let $k_E$be a fuzzy soft closed set in(Y, K) and let $(\acute{e}, y_\alpha)$ be a fuzzy soft point in (Y, K) such that $(\acute{e}, y_\alpha) \sqcap k_E = \tilde{0}_K$Since f_{up}isfuzzy soft surjective, there exists a fuzzy soft point(e, x_α)in(X, E)such that $f_{up}(e, x_\alpha) = (\acute{e}, y_\alpha)$. Since $f_{up}$is fuzzy soft continuous and $k_E$fuzzy soft closed in(Y, K), we have $f_{up}^{-1}(k_E)$ a fuzzy soft closed set in$(X, E)$from Theorem 2.21. Now, $f_{up}(e, x_\alpha) = (\acute{e}, y_\alpha)$, implies that $(e, x_\alpha) = f_{up}^{-1}(\acute{e}, y_\alpha)$ [as $f_{up}$is fuzzy soft injective] and since $(\acute{e}, y_\alpha) \sqcap k_E = \tilde{0}_K$, which implies that $f_{up}^{-1}((\acute{e}, y_\alpha) \sqcap k_E) = f_{up}^{-1}(\tilde{0}_K) = \tilde{0}_E$. Then $(e, x_\alpha) \sqcap f_{up}^{-1}(k_E) = \tilde{0}_E$. Now, $f_{up}^{-1}(k_E)$ is fuzzy soft closed set in(X, E) and(e, x_α) is a fuzzy soft point in(X, E) such that $(e, x_\alpha) \sqcap f_{up}^{-1}(k_E) = \tilde{0}_E$. But $(X, \mathfrak{T}_1, E)$ is a fuzzy soft regular space, so, there exist disjoint fuzzy soft open sets f_E and $g_E$in (X, E) such that$(e_1, x_\alpha) \tilde{\in} f_E$, $f_{up}^{-1}(k_E) \sqsubseteq g_E$ and therefore,

$f_{up}(e, x_\alpha) = (\acute{e}, y_\alpha) \tilde{\in} f_{up}(f_E)$, $f_{up}(f_{up}^{-1}(k_E)) = k_E \sqsubseteq f_{up}(g_E)$ [as f_{up} is fuzzy soft surjective] and $f_{up}(f_E) \sqcap f_{up}(g_E) = f_{up}(f_E \sqcap g_E) = f_{up}(\tilde{0}_E) = \tilde{0}_K$[from Proposition 2.22]. Since f_{up} is a fuzzy soft open mapping, then $f_{up}(f_E), f_{up}(g_E) \in \mathfrak{T}_2$. Now, there exist disjoint fuzzy soft open sets $f_{up}(f_E)$ and $f_{up}(g_E)$in(Y, K)such that $(\acute{e}, y_\alpha) \tilde{\in} f_{up}(f_E)$ and $k_E \sqsubseteq f_{up}(g_E)$. Thus, $(Y, \mathfrak{T}_2, K)$ is a fuzzy soft regular space.

4.Conclusion

In the present work, we introduce fuzzy soft separation axioms $T_i(i = 0; 1; 2; 3; 4)$ in terms of the modified definitions of a `fuzzy soft point', the complement of a fuzzy soft point is a fuzzy soft set' and presented fundamentals properties such as fuzzy soft hereditary, fuzzy soft topological property. For future works, we consider to study on fuzzy soft separation axioms T_i $(i = 0; 1; 2; 3; 4)$ ingeneralized fuzzy soft topological spaces.

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