Journal of Progressive Research in Mathematics(JPRM)
ISSN: 2395-0218

SCITECH
RESEARCH ORGANISATION

Volume 15, Issue 3
Published online: October 07, 2019|

Journal of Progressive Research in Mathematics
www.scitecresearch.com/journals

Fully Fuzzy Quadratic programming with unrestricted Fully Fuzzy variables and Parameters

Mervat *M. Elshafei*
Department of Mathematics, Faculty of Science, Helwan University, Cairo, Egypt.

Abstract

There exist several methods for solving fuzzy linear or nonlinear programming problems under positivity fuzzy variables and restricted fuzzy coefficients. Due to the limitation of these methods, they can't be applied for solving fully fuzzy linear or non- linear programming problems with unrestricted fuzzy coefficients and fuzzy variables. In this paper an efficient method to find the fuzzy optimal solution for fully fuzzy quadratic programming (FFQP) problem with unrestricted variables and parameters has been proposed. All the coefficients and decision variables of both objective functions and the constraints are triangular fuzzy numbers. The proposed method is based on converted FFQP problem into crisp quadratic programming (CQP) problem. Finally an illustrative numerical example has been given to clarify the proposed solution method.

Keywords: Fully Fuzzy linear programming; Quadratic Programming; Fuzzy arithmetic; Fuzzy Optimal solution; Triangular Fuzzy numbers.

1. Introduction

The quadratic programming problems are topic of great importance in nonlinear programming [11,14,15].They are useful in many fields such as production planning, financial, and cooperative planning, health care and hospital planning.

Tanaka [16] was first proposed the concept of fuzzy mathematical programming (FMP) problem. Zimmermann [18] was first proposed the formulation of fuzzy linear programming. Also Sakawa [12] first formulated the fuzzy bi-level programming problem and developed a fuzzy programming method to solve it.
.Many researchers adopted this concept for solving fuzzy linear, quadratic and nonlinear programming problem [2,9,10,17].

In recent years the fully fuzzy linear programming (FFLP) problems in which the coefficients and decision variables are described by fuzzy numbers has been an attractive topic for the researchers. Allahviranloo et al. [1] developed a method using ranking function for solving fully fuzzy linear programming problem. Kour et al.[5]solved the fully fuzzy linear programming problems with unrestricted fuzzy variables and obtain the exact fuzzy optimal solution. Jayalokshmi et al.[4] introduced a new method for finding an optimal fuzzy solution for fully fuzzy linear programming problems. Safaei [13] introduced a new method for solving Fully Fuzzy linear Fractional Programming with a triangular fuzzy numbers. Loganathan [6] proposed a method for solving Fully Fuzzy Nonlinear Programming with Inequality Constraints. There are a number of researchers who have developed and presented new method in this field such as [3,7,8].

The aim of this paper is to develop an efficient method for solving FFQP problem with

2. Preliminaries

In this section, we give some basic notations which are essential tools for describing our main results [4,5,6,7,13,17].

Definition1. Let X denotes a universal set. Then a fuzzy subset $\widetilde{A}$ of X is defined by its membership function $\mu_{\widetilde{A}} : X \to [0, 1]$; which assigns a real number $\mu_{\widetilde{A}}(x)$ in the interval [0,1]; to each element x $\in$X, where the value of $\mu_{\widetilde{A}}(x)$ at x shows the grade of membership of x in $\widetilde{A}$.

A fuzzy subset $\widetilde{A}$ can be characterized as a set of ordered pairs of element x and grade $\mu_{\widetilde{A}}(x)$, $\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x)); x \in X\}$.

Definition2. A fuzzy number $\widetilde{A} = (a,b,c)$ is said to be a triangular fuzzy number if its membership function is given

$$\mu_{\widetilde{A}}(x) = \begin{cases} \dfrac{x-a}{b-a} & , \quad a \le x \le b, \\ \dfrac{x-c}{b-c} & , \quad b \le x \le c, \\ 0 & , \quad otherwise. \end{cases} \qquad (1)$$

Definition3. A triangular fuzzy number $\widetilde{A} = (a,b,c)$ is said to be nonnegative fuzzy number if and only if $a \ge 0$. The set of non-negative fuzzy numbers may be represented by $F(R^+)$.

Definition4. A triangular fuzzy number $\widetilde{A} = (a,b,c)$ is said to be unrestricted fuzzy number if $a,b,c \in R$. The set of unrestricted fuzzy numbers can be represented by F (R).

Definition5. Let $\widetilde{A} = (a,b,c)$, $\widetilde{B} = (d,e,f)$ be two triangular fuzzy numbers then,

(i) $\quad \widetilde{A} \oplus \widetilde{B} = (a,b,c) \oplus (d,e,f) = (a+d, b+e, c+f)$

(ii) $\quad \widetilde{A} - \widetilde{B} = (a,b,c) - (d,e,f) = (a-f, b-e, c-d)$

(iii) $\quad \widetilde{A} \otimes \widetilde{B} = (\min\ (\gamma), be, \max(\gamma)) \quad where,\ \gamma = \{ad, af, cd, cf\}$.

Definition6. Two triangular fuzzy numbers $\widetilde{A} = \{a,b,c\}$, $\widetilde{B} = \{d,e,f\}$ are said to be equal if and only if a = d, b = e, c = f.

Definition7. A ranking function is a function $R : F(R) \to R$, which maps each fuzzy number into the real line.

Let $\widetilde{A} = (a,b,c)$ be any triangular fuzzy number, then $R(\widetilde{A}) = \dfrac{1}{4}(a+2b+c)$.

Definition8. Let $\widetilde{A}=\{a,b,c\}$, $\widetilde{B}=\{d,e,f\}$ be two triangular fuzzy numbers, then,

(i) $\quad \widetilde{A}\le\widetilde{B}$ iff $R(\widetilde{A})\le R(\widetilde{B})$.

(ii) $\quad \widetilde{A}<\widetilde{B}$ iff $R(\widetilde{A})<R(\widetilde{B})$.

Definition9. Let $\widetilde{A}=(a,b,c)$, $\widetilde{B}=(x,y,z)$ be two triangular numbers. Then we have,

$$\widetilde{A}\otimes\widetilde{B}=\begin{cases}\min\ (ax,cx)\ ,\ by\ ,\ \max\ (az,cz)\ ,\ if\ \ a\ge 0\ , \\ \min\ (az,cx)\ ,\ by\ ,\ \max\ (ax,cz)\ ,\ if\ \ a<0,c>0, \\ \min\ (az,cz)\ ,\ by\ ,\ \max\ (ax,cx)\ ,\ if\ \ c\le 0.\end{cases} \tag{2}$$

Then,

$$\widetilde{A}\times\widetilde{B}=\begin{cases}\left(\{0.5(a+c)x-0.5(c-a)|x|\},by,\{0.5\ (a+c)z+0.5(c-a)|z|\}\right),if\ \ a\ge 0 \\ \left(\{0.5(az+cx)-0.5|cx-ax|\},by,\{0.5(ax+cz)+0.5|cz-ax|\}\right),if\ \ a<0,c>0, \\ \left(\{0.5(a+c)z-0.5(c-a)|z|\},by,\{0.5\ (a+c)x+0.5(c-a)|x|\}\right),if\ \ c\le 0\end{cases} \tag{3}$$

Now, we introduce splitting technique for fuzzy coefficients matrix.
Let $\widetilde{A}$ be m x n triangular fuzzy matrix, then we split $\widetilde{A}$ into the following:

$\widetilde{A}=\widetilde{A}^{+}\oplus\widetilde{A}^{\mp}\oplus\widetilde{A}^{-}$ where,

$$\left(\widetilde{A}^{+}\right)_{mxn}=\left(\left(a_{ij},b_{ij},c_{ij}\right)\right)_{mxn}=\begin{cases}(a_{ij},b_{ij},c_{ij})\ ,\ if\ \ a_{ij}\ge 0, \\ (0,0,0)\ ,\ otherwise.\end{cases} \tag{4}$$

$$\left(\widetilde{A}^{\mp}\right)_{mxn}=\left(a_{ij},b_{ij},c_{ij}\right)_{mxn}=\begin{cases}(a_{ij},b_{ij},c_{ij})\ ,\ if\ \ a_{ij}<0,c_{ij}>0, \\ (0,0,0)\ ,\ otherwise\ .\end{cases} \tag{5}$$

$$\left(\widetilde{A}^{-}\right)_{mxn}=\left(a_{ij},b_{ij},c_{ij}\right)_{mxn}=\begin{cases}(a_{ij},b_{ij},c_{ij})\ ,\ if\ \ c_{ij}<0, \\ (0,0,0)\ ,\ otherwise.\end{cases} \tag{6}$$

3. Problem Formulation

Consider the following fully fuzzy quadratic programming (FFQP) problem:
(FFQP):

$$Max\,(or\ Min)\ \ \widetilde{Z}=\widetilde{C}\,\widetilde{X}\oplus\widetilde{X}^{T}\ \widetilde{D}\,\widetilde{X}$$

subject to

$$\widetilde{A}\otimes\widetilde{X}\ (\le,=,\ge)\ \widetilde{B},$$

$\widetilde{X}$ is unrestricted triangular fuzzy numbers,

$$\widetilde{X}=(\widetilde{x}_1,\widetilde{x}_2,...,\widetilde{x}_n)^T,\widetilde{C}=(\widetilde{c}_1,\widetilde{c}_2,...,\widetilde{c}_n),\widetilde{B}=(\widetilde{b}_1,\widetilde{b}_2,...,\widetilde{b}_m)^T,$$

$$\widetilde{A}=\begin{pmatrix}\widetilde{a}_{11} \dots \widetilde{a}_{1n}\\ \vdots \\ a_{m1} \dots \widetilde{a}_{mn}\end{pmatrix},\ \widetilde{D}=\begin{pmatrix}\widetilde{d}_{11} \dots \widetilde{d}_{1n}\\ \vdots \\ \widetilde{d}_{n1} \dots \widetilde{d}_{nn}\end{pmatrix}.$$

Where, $\widetilde{C},\widetilde{D},\widetilde{A}, and\ \widetilde{B}$ are triangular fuzzy numbers and $\widetilde{X}$ is triangular fuzzy variables.

The function $\widetilde{X}^T D\widetilde{X}$ defines a quadratic form. The rank $(\widetilde{A},\widetilde{B})=$ rank $(\widetilde{A})=m.$
A method to solve FFQP problem and obtain the fuzzy optimal solution is proposed in the following section.

4. An Algorithm

Step1. By using the splitting technique write the FFQP problem as follows:

(FFQP)₁:

$$Max\,(or\ Min)\widetilde{Z}=\left(\sum_{j=1}^{n}\widetilde{c}_j\otimes\widetilde{x}_j\right)\oplus\left(\sum_{k=1}^{n}\sum_{j=1}^{n}\widetilde{x}_k\widetilde{d}_{kj}\widetilde{x}_j\right)$$

subject to

$$\left(\widetilde{A}^{+}\oplus\widetilde{A}^{\mp}\oplus\widetilde{A}^{-}\right)\otimes\widetilde{x}_j\le\widetilde{b}_i\ ,\ i=1,...,m,\quad j=1,...,n$$

$$\widetilde{x}_j\in F(R).$$

Step2. If all the parameters $\widetilde{c}_j,\widetilde{x}_j,\widetilde{d}_{kj},\widetilde{a}_{ij}^{+},\widetilde{a}_{ij}^{\mp},\widetilde{a}_{ij}^{-}$, and $\widetilde{b}_i$ are represented by triangular fuzzy numbers (p_j,q_j,r_j), (x_j,y_j,z_j), (s_{kj},t_{kj},v_{kj}), $\left(a_{ij}^{+},b_{ij}^{+},c_{ij}^{+}\right)$, $\left(a_{ij}^{\mp},b_{ij}^{\mp},c_{ij}^{\mp}\right),\left(a_{ij}^{-},b_{ij}^{-},c_{ij}^{-}\right)$, $and\ (b_i,g_i,h_i)\ respectively$, then (FFQP)₁ problem can be written as follows:

(FFQP)₂:

$$Max\,(or\ Min)\widetilde{Z}=\left(\sum_{\substack{j=1\\p_j\ge0}}^{n}(p_j,q_j,r_j)\otimes(x_j,y_j,z_j)\oplus\right.$$

$$\left.\sum_{\substack{j=1\\p_j<0\\r_j\ge0}}^{n}(p_j,q_j,r_j)\otimes(x_j,y_j,z_j)\oplus\sum_{\substack{j=1\\r_j<0}}^{n}(p_j,q_j,r_j)\otimes(x_j,y_j,z_j)\right)\oplus$$

$$\left(\sum_{k=1}^{n}\sum_{\substack{j=1\\s_j\ge0}}^{n}(x_k,y_k,z_k)\otimes(s_{kj},t_{kj},v_{kj})\otimes(x_j,y_j,z_j)\oplus\right.$$

$$\sum_{k=1}^{n} \sum_{\substack{j=1 \\ s_j<0 \\ v_j \geq 0}}^{n} (x_k, y_k, z_k) \otimes (s_{kj}, t_{kj}, v_{kj}) \otimes (x_j, y_j, z_j) \oplus$$

$$\left. \sum_{k=1}^{n} \sum_{\substack{j=1 \\ v_j<0}}^{n} (x_k, y_k, z_k) \otimes (s_{kj}, t_{kj}, v_{kj}) \otimes (x_j, y_j, z_j) \right)$$

subject to

$$\sum_{j=1}^{n} \left(a_{ij}^{+}, b_{ij}^{+}, c_{ij}^{+}\right) \otimes (x_j, y_j, z_j) \oplus \sum_{j=1}^{n} \left(a_{ij}^{\mp}, b_{ij}^{\mp}, c_{ij}^{\mp}\right) \otimes (x_j, y_j, z_j) \oplus$$

$$\sum_{j=1}^{n} \left(a_{ij}^{-}, b_{ij}^{-}, c_{ij}^{-}\right) \otimes (x_j, y_j, z_j) \leq (b_i, g_i, h_i),$$

$$(x_j, y_j, z_j) \in F(R) \ , \ i = 1, \ldots, m \ , \ j = 1, \ldots, n$$

Step3. Using Definition 9, the (FFQP)$_2$ problem can be written as follows:

(FFQP)$_3$:

$$Max\,(or\;Min)\widetilde{Z} = \sum_{\substack{j=1 \\ p_j \geq 0}}^{n} \left(\min\ (p_j x_j, r_j x_j), q_j y_j, \max\ (p_j z_j, r_j z_j)\right) \oplus$$

$$\sum_{\substack{j=1 \\ p_j<0 \\ r_j \geq 0}}^{n} \left(\min\left(P_j z_j, r_j x_j\right), q_j y_j, \max\ (p_j x_j, r_j z_j)\right) \oplus$$

$$\sum_{\substack{j=1 \\ r_j<0}}^{n} \left(\min(p_j z_j, r_j z_j)\ ,\ q_j y_j,\ \max\ (p_j x_j, r_j x_j)\right) \oplus$$

$$\sum_{k=1}^{n} \sum_{\substack{j=1 \\ s_j \geq 0}}^{n} \left(\min(s_{kj} x_k x_j, v_{kj} x_k x_j)\ ,\ t_{kj}\ y_k y_j,\ \max\ (s_{kj}.z_k z_j, v_{kj}.z_k z_j)\right) \oplus$$

$$\sum_{k=1}^{n} \sum_{\substack{j=1 \\ s_j<0 \\ v_j \geq 0}}^{n} \left(\min\left(s_{kj} z_k z_j, v_{kj} x_k x_j\right), t_{kj} y_k y_j, \max\left(s_{kj} x_k x_j, v_j z_k z_j\right)\right) \oplus$$

$$\sum_{k=1}^{n} \sum_{\substack{j=1 \\ v_j<0}}^{n} \left(\min\left(s_{kj} z_k z_j, v_{kj} z_k z_j\right), t_{kj} y_k y_j, \max\left(s_{kj} x_k x_j, v_{kj} x_k x_j\right)\right)$$

subject to

$$\sum_{\substack{j=1\\ a_{ij}\geq 0}}^{n} \left(\min\,(a_{ij}x_j, c_{ij}x_j), b_{ij}y_j, \max(a_{ij}z_j, c_{ij}z_j)\right)\oplus$$

$$\sum_{\substack{j=1\\ a_{ij}<0\\ c_{ij}\geq 0}}^{n} \left(\min\,(a_{ij}z_j, c_{ij}x_j), b_{ij}y_j, \max(a_{ij}x_j, c_{ij}z_j)\right)\oplus$$

$$\sum_{\substack{j=1\\ c_{ij}<0}}^{n} \left(\min\,(a_{ij}z_j, c_{ij}z_j), b_{ij}y_j, \max(a_{ij}x_j, c_{ij}x_j)\right)\leq (b_i, g_i, h_i)$$

$$(x_j, y_j, z_i)\in F(R),\quad i=1,\ldots,m.$$

Step4. Using Definition 9, the (FFQP)3 problem can be written as follows:

(FFQP)4:

$$Max\,(or\,Min)\widetilde{Z}=\sum_{\substack{j=1\\ p_j\geq 0}}^{n}\Big(0.5(p_j+r_j)\,x_j-0.5(r_j-p_j)\left|x_j\right|, q_jy_j,$$

$$0.5\,(p_j+r_j)z_j+0.5(r_j-p_j)\left|z_j\right|\Big)\oplus$$

$$\sum_{\substack{j=1\\ p_j<0\\ r_j\geq 0}}^{n}\Big(0.5(p_jz_j+r_jx_j)-0.5\left|r_jx_j-p_jz_j\right|,\,q_jy_j,$$

$$0.5\,(p_jx_j+r_jz_j)+0.5\left|r_jz_j-p_jx_j\right|\Big)\oplus$$

$$\sum_{\substack{j=1\\ r_j<0}}^{n}\Big(0.5(p_j+r_j)z_j-0.5(r_j-p_j)\left|z_j\right|, q_jy_j,$$

$$0.5(p_j+r_j)x_j+0.5(r_j-p_j)\left|x_j\right|\Big)\oplus$$

$$\sum_{\substack{j=1\\ r_j<0}}^{n}\Big(0.5(p_j+r_j)z_j-0.5(r_j-p_j)\left|z_j\right|, q_jy_j,$$

$$0.5(p_j+r_j)x_j+0.5(r_j-p_j)\left|x_j\right|\Big)\oplus$$

$$\sum_{k=1}^{n}\sum_{\substack{j=1\\ s_j\geq 0}}^{n}\Big(0.5(s_{kj}+v_{kj})x_kx_j-0.5(v_{kj}-s_{kj})\left|x_kx_j\right|, t_{kj}.y_ky_j,$$

$$0.5(s_{kj}+v_{kj})\,z_kz_j+0.5(v_{kj}-s_{kj})\left|z_kz_j\right|\Big)\oplus$$

$$\sum_{k=1}^{n}\sum_{\substack{j=1\\ s_j<0\\ v_j\geq 0}}^{n}\Big(0.5(s_{kj}z_kz_j+v_{kj}x_kx_j)-0.5\left|v_{kj}x_kx_j-s_{kj}z_kz_j\right|,t_{kj}y_ky_j,$$

$$0.5(s_{kj}x_kx_j+v_{kj}z_kz_j)+0.5\left|v_{kj}z_kz_j-s_{kj}x_kx_j\right|\Big)\oplus$$

$$\sum_{k=1}^{n}\sum_{\substack{j=1\\ v_j<0}}^{n}\Big(0.5\,(s_{kj}+v_{kj})z_kz_j-0.5(v_{kj}-s_{kj})\left|z_kz_j\right|,t_{kj}y_ky_j,$$

$$0.5(s_{kj}+v_{kj})x_kx_j+0.5(v_{kj}-s_{kj})\left|x_kx_j\right|\Big)$$

subject to

$$\sum_{\substack{j=1\\ a_{ij}\geq 0}}^{n}\Big(0.5(a_{ij}+c_{ij})x_j-0.5\,(c_{ij}-a_{ij})\left|x_j\right|,b_{ij}y_j,$$

$$0.5(a_{ij}+c_{ij})\,z_j+0.5(c_{ij}-a_{ij})\left|z_j\right|\Big)+$$

$$\sum_{\substack{j=1\\ a_{ij}<0\\ c_{ij}\geq 0}}^{n}\Big(0.5(a_{ij}z_j+c_{ij}x_j)-0.5\,\left|c_{ij}x_j-a_{ij}z_j\right|,b_{ij}y_j,$$

$$0.5\,(a_{ij}x_j+c_{ij}z_j)+0.5\left|c_{ij}z_j-a_{ij}x_j\right|\Big)+$$

$$\sum_{\substack{j=1\\ a_{ij}\geq 0}}^{n}\Big(0.5(a_{ij}+c_{ij})x_j-0.5\,(c_{ij}-a_{ij})\left|x_j\right|,b_{ij}y_j,$$

$$0.5(a_{ij}+c_{ij})\,z_j+0.5(c_{ij}-a_{ij})\left|z_j\right|\Big)+$$

$$\sum_{\substack{j=1\\ a_{ij}<0\\ c_{ij}\geq 0}}^{n}\Big(0.5(a_{ij}z_j+c_{ij}x_j)-0.5\,\left|c_{ij}x_j-a_{ij}z_j\right|,b_{ij}y_j,$$

$$0.5\,(a_{ij}x_j+c_{ij}z_j)+0.5\left|c_{ij}z_j-a_{ij}x_j\right|\Big)+$$

$$\sum_{\substack{j=1\\ c_{ij}<0}}^{n}\Big(0.5(a_{ij}+c_{ij})\,z_j-0.5(c_{ij}-a_{ij})\left|z_j\right|,b_{ij}\,y_j,$$

$$0.5(a_{ij}+c_{ij})x_j+0.5\,(c_{ij}-a_{ij})\left|x_j\right|\leq(b_i,g_i,h_i),$$

$$(x_i,y_i,z_i)\in F(R)\;\;,i=1,\ldots,m$$

Step5. Using definition 7, the $(FFQP)_4$ problem can be converted into fuzzy quadratic programming **(**FQP) problem which can be written as:

(FQP):

$$Max\,(or\,Min)\widetilde{Z} = R\left(\sum_{\substack{j=1\\ p_j\ge 0}}^{n} (0.5(p_j + r_j)x_j - 0.5(r_j - p_j)\left|x_j\right|,\, q_j y_j,\right.$$

$$\left. 0.5\,(p_j + r_j)z_j + 0.5(r_j - p_j)\left|z_j\right|\right) +$$

$$R\left(\sum_{\substack{j=1\\ p_j<0\\ r_j\ge 0}}^{n} 0.5(p_j z_j + r_j x_j) - 0.5(r_j x_j - p_j z_j),\, q_j y_j\ ,\right.$$

$$\left. 0.5(p_j x_j + r_j z_j) + 0.5\left|r_j z_j - p_j x_j\right|\right) +$$

$$R\left(\sum_{\substack{j=1\\ r_j<0}}^{n} 0.5(p_j + r_j)z_j - 0.5(r_j - p_j)\left|z_j\right|,\, q_j y_j\ ,\right.$$

$$\left. 0.5\,(p_j + r_j)x_j + 0.5(r_j - p_j)\left|x_j\right|\right) +$$

$$R\left(\sum_{k=1}^{n}\ \sum_{\substack{j=1\\ s_j\ge 0}}^{n} \left(0.5(s_{kj} + v_{kj})x_k x_j - 0.5(v_{kj} - s_{kj})\left|x_k x_j\right|,\, t_{kj} y_k y_j,\right.\right.$$

$$\left.\left. 0.5\,(s_{kj} + v_{kj})z_k z_j + 0.5(v_{kj} - s_{kj})\left|z_k z_j\right|\right)\right) +$$

$$R\left(\sum_{k=1}^{n}\ \sum_{\substack{j=1\\ s_j<0\\ v_j\ge 0}}^{n} (0.5(s_{kj} z_k z_j + v_{kj} x_k x_j) - o.5\left|v_{kj} x_k x_j - s_{kj} z_k z_j\right|,\, t_{kj} y_k y_j,\right.$$

$$\left. 0.5\,(s_{kj} x_k x_j + v_{kj} z_k z_j) + 0.5\left|v_{kj} z_k z_j - s_{kj} x_{k\,j}\right|\right) +$$

$$R\left(\sum_{k=1}^{n}\ \sum_{\substack{j=1\\ v_j<0}}^{n} (0.5(s_{kj} + v_{kj})z_k z_j - 0.5\,(v_{kj} - s_{kj})\left|z_k . z_j\right|,\, t_{kj} y_k y_j\right.$$

$$\left. 0.5(s_{kj} + v_{kj})x_k x_j + 0.5(v_{kj} - s_{kj})\left|x_k x_j\right|)\right)$$

subject to

$$\sum_{\substack{j=1\\ a_{ij}\ge 0}}^{n} \left(0.5(a_{ij} + c_{ij})x_j - 0.5\,(c_{ij} - a_{ij})\left|x_j\right|,\, b_{ij} y_j,\right.$$

$$\left. 0.5(a_{ij} + c_{ij})\ z_j + 0.5(c_{ij} - a_{ij})\left|z_j\right|\right) +$$

$$\sum_{\substack{j=1 \\ a_{ij}<0 \\ c_{ij}\geq 0}}^{n} \Big(0.5(a_{ij}z_j + c_{ij}x_j) - 0.5\ \left|c_{ij}x_j - a_{ij}z_j\right|, b_{ij}y_j,$$

$$0.5\ (a_{ij}x_j + c_{ij}z_j) + 0.5\left|c_{ij}z_j - a_{ij}x_j\right|\Big) +$$

$$\sum_{\substack{j=1 \\ c_{ij}<0}}^{n} \Big(0.5(a_{ij} + c_{ij})\ z_j - 0.5(c_{ij} - a_{ij})\ \left|z_j\right|, b_{ij}\ y_j,$$

$$0.5(a_{ij} + c_{ij})x_j + 0.5\ (c_{ij} - a_{ij})\ \left|x_j\right| \leq (b_i, g_i, h_i),$$

$$(x_i, y_i, z_i)\ \in F(R)\ \ , i = 1, \ldots, m.$$

Step6. Using Definition 6 –8, the fuzzy quadratic programming problem obtained in step5 is converted into the following crisp quadratic programming (CQP) problem:

(CQP):

$$Max\,(or\,Min)Z = \frac{1}{4}\left(\sum_{\substack{j=1 \\ p_j \geq o}}^{n} (0.5(p_j + r_j)x_j - 0.5(r_j - p_j)\left|x_j\right|,\right.$$

$$\left. 2(q_j y_j) + 0.5(p_j + r_j)z_j + 0.5(r_j - p_j)\left|z_j\right|\ \right) +$$

$$\frac{1}{4}\left(\sum_{\substack{j=1 \\ p_j<0 \\ r_j\geq 0}}^{n} (0.5(p_j z_j + r_j x_j) - 0.5\ (r_j x_j - p_j z_j) + \right.$$

$$\left. 2(q_j y_j) + 0.5(p_j x_j + r_j z_j) + 0.5\left|r_j z_j - p_j x_j\right|\ \right) +$$

$$\frac{1}{4}\left(\sum_{\substack{j=1 \\ r_j<0}}^{n} (0.5(p_j + r_j)z_j - 0.5\ (r_j - p_j)\ \left|z_j\right| + \right.$$

$$\left. 2(q_j y_j) + 0.5(p_j + r_j)x_j + 0.5(r_j - p_j)\left|x_j\right|\ \right) +$$

$$\frac{1}{4}\left(\sum_{k=1}^{n} \sum_{\substack{j=1 \\ s_j\geq 0}}^{n} (0.5(s_{kj} + v_{kj})x_k x_j - 0.5\ (v_{kj} - s_{kj})\ \left|x_k x_j\right| + \right.$$

$$\left. 2(t_{kj} y_k y_j) + 0.5(s_{kj} + v_{kj})z_k\, z_j + 0.5(v_{kj} - s_{kj})\left|z_k z_j\right|\right) +$$

$$\frac{1}{4}\left(\sum_{k=1}^{n}\ \sum_{\substack{j=1\\ s_j<0\\ v_j\geq 0}}^{n} (0.5(s_{kj}z_k z_j + v_{kj}x_k x_j) - 0.5\left|v_{kj}x_k x_j - s_{kj}z_k z_j\right| + \right.$$

$$\left. 2(t_{kj}y_k y_j) + 0.5(s_{kj}x_k x_j + v_{kj}z_k z_k) + 0.5\left|v_{kj}z_k z_j - s_{kj}x_k x_j\right| \right) +$$

$$\frac{1}{4}\left(\sum_{k=1}^{n}\ \sum_{\substack{j=1\\ v_j<0}}^{n} (0.5(s_{kj}+v_{kj})z_k z_j - 0.5\,(v_{kj}-s_{kj})\left|z_k z_j\right| + \right.$$

$$\left. 2(t_{kj}y_k y_j) + 0.5(s_{kj}+v_{kj})x_k x_j + 0.5(v_{kj}-s_{kj})\left|x_k x_j\right|) \right)$$

subject to

$$\sum_{\substack{j=1\\ a_{ij}\geq 0}}^{n} \left(0.5(a_{ij}+c_{ij})x_j - 0.5(c_{ij}-a_{ij})\left|x_j\right|\right) +$$

$$\sum_{\substack{j=1\\ a_{ij}<0\\ c_{ij}\geq 0}}^{n} \left(0.5(a_{ij}z_j + c_{ij}x_j) - 0.5\left|c_{ij}x_j - a_{ij}z_j\right|\right) +$$

$$\sum_{\substack{j=1\\ c_{ij}<0}}^{n} \left(0.5(a_{ij}+c_{ij})z_j - 0.5(c_{ij}-a_{ij})\left|z_j\right|\right) \leq b_i,$$

$$\sum_{j=1}^{n} b_{ij}y_j \leq g_i,$$

$$\sum_{\substack{j=1\\ a_{ij}\geq 0}}^{n} \left(0.5(a_{ij}+c_{ij})z_j + 0.5(c_{ij}-a_{ij})\left|z_j\right|\right) +$$

$$\sum_{\substack{j=1\\ a_{ij}<0\\ c_{ij}\geq 0}}^{n} \left(0.5(a_{ij}x_j + c_{ij}z_j) + 0.5\left|c_{ij}z_j - a_{ij}x_j\right|\right) +$$

$$\sum_{\substack{j=1\\ c_{ij}<0}}^{n} \left(0.5(a_{ij}+c_{ij})x_j + 0.5\,(c_{ij}-a_{ij})\left|x_j\right|\right) \leq h_i,$$

$$y_j - x_j \geq 0,\ {}_j - y_j \geq 0,$$

$$x_j, y_j, z_j \in R \quad , i = 1,\ldots,m, \quad j = 1,\ldots,n, \qquad k = 1,\ldots,n$$

Step7. Solving the (CQP) problem by using MATLAB package and find the optimal solution x_j, y_j, z_j.

Step8. Find the fuzzy optimal solution $\tilde{x}_j = (x_j, y_j, z_j)$.

Step9. Find the fuzzy optimal value by putting $\tilde{x}_j$ in

$$\sum_{j=1}^{n} c_j \otimes \widetilde{x}_j \oplus \sum_{k=1}^{n} \sum_{j=1}^{n} \left(\widetilde{x}_k d_{kj} \widetilde{x}_j\right).$$

5. Numerical example.

Consider the following (FFQP)$_1$ problem:

(FFQP)$_1$:

$$Max\ \hat{Z} = (-2,4,6) \otimes (x_1^2, y_1^2, z_1^2) \oplus (4,6,8) \otimes (x_2^2, y_2^2, z_2^2)$$

subject to

$$(-6,-4,0) \otimes (x_1, y_1, z_1) \oplus (12,14,16) \otimes (x_2, y_2, z_2) = (-60,-34-6),$$
$$(4,8,12) \otimes (x_1 y_1, z_1) \oplus (-4,-2,4) \otimes (x_2, y_2, z_2) = (-8,16,36),$$
$(x_1, y_1, z_1), (x_2, y_2, z_2)$ are unrestricted triangular fuzzy numbers.

Step1-2. By using the splitting technique writes (FFQP)$_1$ in the following form:

(FFQP)$_2$:

$$Max\ \hat{Z} = (-2,4,6) \otimes (x_1^2, y_1^2, z_1^2) \oplus (4,6,8) \otimes (x_2^2, y_2^2, z_2^2)$$

subject to

$$\left(\begin{bmatrix} (0,0,0) & (12,14,16) \\ (4,8,12) & (0,0,0) \end{bmatrix} \oplus \begin{bmatrix} (0,0,0) & (0,0,0) \\ (0,0,0) & (-4,-2,4) \end{bmatrix} \oplus \begin{bmatrix} (-6,-4,0) & (0,0,0) \\ (0,0,0) & (0,0,0) \end{bmatrix} \right) \otimes \begin{bmatrix} (x_1, y_1, z_1) \\ (x_2, y_2, z_2) \end{bmatrix} = \begin{bmatrix} (-60,-34,-6) \\ (-8,16,36) \end{bmatrix}$$

$$(x_j, y_j, z_j) \in F(R),\ j = 1, 2.$$

Step3. Using the arithmetic operations in Definition 9, the (FFQP)$_2$ can be written as:

(FFQP)$_3$:

$$Max\ \hat{Z} = \Big((\min(-2z_1^2, 6x_1^2), 4y_1^2, \max(-2x_1^2, 6z_1^2)\Big)$$
$$\oplus\ (\min(4x_2^2, 8x_2^2), 6y_2^2, \max(4z_2^2, 8z_2^2))$$

subject to

$$\big(\min(12x_2, 16x_2), 14y_2, \max(12z_2, 16z_2)\big) \oplus$$
$$\big(\min(-6z_1, 0), -4y_1, \max(-6x_1, 0)\big) = \big(-60, -34, -6\big),$$
$$\big(\min(4x_1, 12x_1), 8y_1, \max(4z_1, 12z_1)\big) \oplus$$
$$\big(\min(-4z_2, 4x_2), -2y_2, \max(-4x_2, 4z_2)\big) = (-8, +16, 36),$$

$$(x_j, y_j, z_j) \in F(R),\ j = 1, 2.$$

Step4. By using Definition 9, the $(FFQP)_3$ can be written as follows:

$(FFQP)_4$:

$$Max\,\hat{Z} = \left((-z_1^2 + 3x_1^2) - \left| z_1^2 + 3x_1^2 \right|, 4y_1^2, (-x_1^2 + 3z_1^2) + \left| x_1{}^2 + 3z_1^2 \right| \right) \oplus \left(6x_2^2 - 2\left| x_2^2 \right|, 6y_2^2, 6z_2^2 + 2\left| z_2^2 \right| \right)$$

subject to

$$\left(14x_2 - \left| 2x_2 \right|, 14y_2, 14z_2 + \left| 2z_2 \right| \right) \oplus (-3z_1 - 3\left| z_1 \right|, -4y_1, -3x_1 + 3\left| x_1 \right| = (-60, -34, -6),$$

$$\left(8x_1 - 4\left| x_1 \right|, 8y_1, 8z_1 + 4\left| z_1 \right| \right) \oplus (-2z_2 + 2x_2) - 2\left| z_2 + x_2 \right|, -2y, -2x_2 + 2z_2 + 2\left| z_2 + x_2 \right| \Big) = (-8, 16, 36)$$

$$(x_j, y_j, z_j) \in F(R),\ j = 1, 2.$$

Step5. By using Definition 7, the $(FFQP)_4$ problem is converted into the fuzzy quadratic programming(FQP) following form:

(FQP):

$$Max\,\hat{Z} = R\left((-z_1^2 + 3x_1^2) - \left| z_1^2 + 3x_1^2 \right|,\ 4y_1^2, (-x_1^2 + 3z_1^2) + \left| x_1{}^2 + 3z_1^2 \right| \right) \oplus R\left(6x_2^2 - 2\left|x_2^2\right|, 6y_2^2, 6z_2^2 + 2\left| z_2^2 \right| \right)$$

subject to

$$\left(14x_2 - \left| 2x_2 \right|, 14y_2, 14z_2 + \left| 2z_2 \right| \right) \oplus (-3z_1 - 3\left| z_1 \right|, -4y_1, -3x_1 + 3\left| x_1 \right| = (-60, -34, -6),$$

$$\left(8x_1 - 4\left| x_1 \right|, 8y_1, 8z_1 + 4\left| z_1 \right| \right) \oplus (-2z_2 + 2x_2) - 2\left| z_2 + x_2 \right|, -2y_2, -2x_2 + 2z_2 + 2\left| z_2 + x_2 \right| \Big) = (-8, 16, 36)$$

$$(x_j, y_j, z_j) \in F(R),\ j = 1, 2.$$

Step6. Using Definitions 6-8, the (FQP) problem is converted into the following form:

(CQP):

$$MaxZ = \frac{1}{4}\Big((-z_1^2 + 3x_1^2) - \left|z_1^2 + 3x_1^2\right| + 8y_1^2 + (-x_1^2 + 3z_1^2) + \left|x_1^2 + 3z_1^2\right| + 6x_2^2 - 2\left| x_2^2 \right| + 12y_2^2 + 6z_2^2 + 2\left| z_2^2 \right| \Big)$$

s.t

$$14x_2 - 2\left| x_2 \right| - 3z_1 - 3\left| z_1 \right| = -60,$$

$$14y_2 - 4y_1 = -34,$$

$$14z_2 + 2|z_2| - 3x_1 + 3|x_1| = -6,$$
$$8x_1 - 4|x_1| - 2z_2 + 2x_2 - 2|z_2 + x_2| = -8,$$
$$8y_1 - 2y_2 = 16,$$
$$8z_1 + 4|z_1| - 2x_2 + 2z_2 + 2|z_2 + x_2| = 36,$$
$$y_1 - x_1 \geq 0, z_1 - y_1 \geq 0, y_2 - x_2 \geq 0, z_2 - y_2 \geq 0,$$
$$x_j, y_j, z_j \in R, j = 1,2.$$

Step7. The optimal solution of CQP problem is

$$x_1 = 1, y_1 = 1.5, z_1 = 2, x_2 = -3, y_2 = -2, z_2 = -0.5$$

Step8. The fuzzy optimal solution is $\widetilde{x}_1 = (1, 1.5, 2)$, $\widetilde{x}_2 = (-3, -2, -0.5)$).

Step9. The fuzzy optimal value is $\widetilde{z} = (-7, 33, 96)$.

6. Conclusions

In this paper an efficient method to find the fuzzy optimal solution of fully fuzzy quadratic programming FFQP problem with unrestricted variables and parameters has been proposed. This proposed method is based on converted FFQP problem into crisp quadratic programming CQP problem by using splitting technique for fuzzy coefficients matrix and ranking function which maps each Fuzzy number into the real line. Finally the method is illustrated by a numerical example.

References

[1] Allahviranloo,T., Lotfi H.F., Kiasary,M.Kh., Kiani,N.A., and Alizadeh,L.(2008) Solving Fully Fuzzy Linear Programming Problem by the Ranking Function. Applied Mathematical Sciences,2(1) 19 – 32.

[2] Bellman, R.E., and Zadeh, L.A. (1970) Decision Making in a Fuzzy Environment. Management Science, 17 141 – 164.

[3] Gabr,W.I.(2015) Quadratic and Non-Linear Programming Problems Solving and Analysis in Fully Environment. Alexandria Engineering Journal, 54 457 – 472.

[4] Jayalokshmi,M.,and Pandian,P. (2012) A New Method for Finding an Optimal Fuzzy solution for Fully Fuzzy Linear Programming Problems. International Journal of Engineering Research and Applications (IJERA), 2 (4) 247 – 254.

[5] Kour,J.,and Kumar,A.(2012) Exact Fuzzy Optimal solution of Fully Fuzzy Linear Programming Problems with Unrestricted Fuzzy Variables. Appl Intell, 37 145 – 154.

[6] Loganathan,C. (2017) Solving Fully Fuzzy Nonlinear Programming with Inequality Constraints. International Journal of Mechanical Engineering and Technology (IJMET), 8 (11) 354-362.

[7] Najafi,H., Edalatpanah, S.A., and Dutta,H. (2016) A Nonlinear Model for Fully Fuzzy Linear Programming with Fully Unrestricted Variables and Parameters. Alexandria

Engineering Journal, 55 2589 – 2595.

[8] Nasseri,S.H., Behmannesh, E., Taleshian, F., Abdolalipoor, M., and Taghinezhad, N.A.(2013) Fully Fuzzy Linear Programming with Inequality Constraints. Int. J. Industrial Mathematics, 5 (4) 309 – 316.

[9] Osman,M.,S., El-Banna, A.Z, and Elshafei,M. (1999) On Fuzzy Continuous Static Games (FCSG) (Stackelberg-Leader with Min – Max Followers. the Journal of Fuzzy Mathematics. Los Angeles, 7(2) 259 – 266.

[10] Pandian,P.(2013) Multi–objective Programming Approach for Fuzzy Linear Programming Problems. Applied Mathematical Science, 7 (37) 1811-1817.

[11] Rao,S.S.(1979) Optimization Theory and Applications. Wiley Eastern limited, USA.

[12] Sakawa,M., and Yano,H. (1989) Interactive Decision Making for Multi-Objective Non-Linear Programming Problems with Fuzzy Parameters.Fuzzy sets and Syst., 29 (3) 315 – 326.

[13] Safaei,N.(2014) A new method for solving Fully Fuzzy linear Fractional Programming with a triangular Fuzzy Numbers. APP. Math. and Comp. Intel., 3 (1) 273 – 281.

[14] Singh S.,and Haldar N.(2015) A New Method to Solve Bi-Level Quadratic linear Fractional Programming Problems.International Game Theory Review, 17 (2) 1540017 (18 Pages).

[15] Taha, H.A. (1976) Operations Research.Macmillan Publishing Co-Inc. New York.

[16] Tanaka,H., Okuda,T., and Asai, k. (1976) On Fuzzy Mathematical Programming. Journal of Cybernetics, 3 37 – 46.

[17] Zadeh, L.A. (1965) Fuzzy Sets.Information and computation, 8 (3) 338-353.

[18] Zimmermann, H.J. (1978) Fuzzy Programming and Linear Programming with Several Objective Functions. Fuzzy Sets and Systems, 1 45 – 55.