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An Improved group acceptance sampling plan for weighted binomial on time truncated testing strategy: Inverse Rayleigh Distribution

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Abstract

This paper elucidate an Improved group acceptance sampling plan (IGASP) using weighted binomial, when the lifetime of the test items follows inverse Rayleigh distribution. The optimum numbers of group are obtained for pre-defined parameters, acceptance number, quality levels, for different levels of consumer risk. The proposed plan compared with Naqvi and Bashir (2016). The results and comparison are discussed with the help of tables and figures. It was observed that sometimes proposed plan showed better results than existing plan, under the same parameter settings.

Keywords: Improved group acceptance sampling plan; Weighted Binomial; Inverse Rayleigh Distribution; Consumer risk.

1. Introduction

Acceptance sampling plans used as one of the tools, in statistical quality control (SQC), for lot sentencing which means either to accept or reject the lot. When dealing with a sampling plan(s) the two types of risk have automatically come into consideration known as consumer and producer risks. The probability of rejecting a good lot is known as producer's risk whereas the probability of accepting a bad lot is known as consumer's risk. The main purpose of sampling plans is to reduce such kind of risks as well as to get the cost effective or an efficient plan. The quality of an item usually evaluated through its average life. Let the average true life of an item is μ whereas it's specified average life is μ_0 then an appropriate null hypothesis can be $\mu \geq \mu_0$ against the alternative of lesser type.

When a single item is put on test then it is known as single sampling plan. When we put more than one items on a tester which form a group and the number of items in a group known as group size(r) so, dealing with group is known as group sampling plan. The key concern is to minimize the number of groups (g) which is as similar as to minimize the sample size (n) when a single item is put on a tester (n = rg).

Ample of work can be cited in this regard. The Rosaiah and Kantam (2005) develop usual acceptance sampling plan under the assumption that the lifetime of an item follows inverse

Rayleigh distribution. Further Rosaiah et al.(2008) develop the reliability plans for the inverse Rayleigh distribution. They concluded that for real data set the inverse Rayleigh has a better fit. Aslam and Jun (2009b) developed the group acceptance sampling plan on the truncated life test when the lifetime of a product follows Inverse Rayleigh or log-logistic distribution. Aslam et al.(2009) constructed group acceptance sampling plan using binomial distribution when the lifetime of an item follows pareto distribution of second kind. Aslam et al.(2011) constructed Improved group acceptance sampling plan for Dagum Distribution under percentiles lifetime. Aslam et al.(2011) discussed improved group sampling plans based on truncated life tests. Radhakrishna Rao (1977) proposed that weighted binomial distribution can be used in the construction of sampling plan. Radhakrishna Rao (2011a,b,c) constructed group acceptance sampling plan using weighted binomial distribution also Radhakrishna and Alagirisamy (2012) constructed a group acceptance sampling plan indexed through indifference quality level and invers Rayleigh distribution. Amburajan and Ramaswamy (2015) develop group acceptance sampling plan using weighted binomial distribution when the lifetime of an item follows Exponential and Weibull distribution. Naqvi and Bashir (2016) constructed an improve group acceptance sampling plan using weighted binomial distribution when the lifetime of an item follows exponential distribution. Naqvi and Bashir (2016) reported that their plan is efficient as compare to Amburajan and Ramaswamy (2015).

Although in this paper an improved group acceptance sampling plan (IGASP) proposed by Aslam et al.(2011) is reconsidered using weighted binomial distribution when the lifetime of an item follows the Inverse Rayleigh Distribution.

2. Design of the proposed plan/Methodology

Aslam et al.(2011) proposed IGASP as follows:

1) Select the number of groups g and allocate predefined r items to each group so that the sample size for a lot will be n = rg.
2) Select the acceptance numbers c for a group and the experiment time t_0.
3) Carry out the experiment for the g groups simultaneously and record the number of failures for each group.
4) Accept the lot if at most c failures occurs in each of, all groups.
5) Truncate the experiment, if more than c failures occur and reject the lot or, at time t_0.

The stated plan is based on two known plan parameters g and c. The said plan reduces to the ordinary acceptance sampling plan when r = 1.the lot acceptance probability for the IGASP is as follows:

$L(P) = \sum_{i=1}^{c} \quad (rg - 1\ i - 1\ ) p^{i-1} (1-p)^{rg-i}$ (1)

Here p is the probability of failure of an item before termination time t_0.As it's known that p is a function of cumulative distribution function which is, in this paper, inverse Rayleigh hence

$$p = exp\left(\frac{-\theta_I^2}{t^2}\right) \qquad (2)$$

$ere\ t > 0$; θ_I (>0) is the scale parameter. The mean of this distribution is

$\mu_I = \sqrt{\pi}\,\theta_I$ (3)

Mukerjee and Saran (1984) studied the failure rate of an inverse Rayleigh distribution and reported that for a single parameter inverse distribution is increasing for t < $1.069(\theta_I)^{1/2}$ and decreasing for t >$1.069(\theta_I)^{1/2}$.

also it is convenient to determine the termination time t_0 as a multiple of specified average life μ_0. So we can consider

$t_0 = a\mu_0$ (4)

for a constant "a" e.g a = 0.5 means that the experiment time is just half of the specified average life (Aslam and June (2009)). After simple calculation by using the above equations (2,3,4)

$$p = exp\left(\frac{-1}{a^2\pi}\left(\frac{\mu}{\mu_0}\right)^2\right) \qquad (5)$$

The optimal number of groups can be obtained by satisfying the following inequality in (6)

$$\sum_{i=1}^{c} \quad (rg - 1\ i - 1)\left(exp\left(\frac{-1}{a^2\pi}\left(\frac{\mu}{\mu_0}\right)^2\right)\right)^{i-1}\left(1 - exp\left(\frac{-1}{a^2\pi}\left(\frac{\mu}{\mu_0}\right)^2\right)\right)^{rg-i} \leq \beta \qquad (6)$$

Where β is the consumer risk

3. Notations

A	-	Test termination time multiplier
C	-	Acceptance number
G	-	Number of groups
N	-	Sample size
P	-	Probability of failure
L(p)	-	Probability of acceptance
R	-	Number of items in a group
t_0	-	Termination time
A	-	Producer's risk
B	-	Consumer's risk
M	-	True average life
μ_0	-	Specified average life

4. Results and Discussion

The optimal number of groups (g) has been obtained with a predefined consumer's risk (β), acceptance numbers(c), group size(r), quality ratio($\mu = \mu_0$) its worst case, because otherwise we need to reject the null hypothesis $\frac{\mu}{\mu_0} \geq 1$. The minimum number of groups are obtained by satisfying a nonlinear equation (6) through simulations study as follows in Table-1.

Table 1: Minimum number of groups (g) for proposed plan

B	r	c	A					
			0.7	0.8	1.0	1.2	1.5	2.0
0.25	**2**	**0**	**2**	**2**	**2**	**1**	**1**	**1**
	3	**1**	**2**	**2**	**2**	**2**	**2**	**1**
	4	**2**	**2**	**2**	**2**	**2**	**2**	**1**
	5	**3**	**2**	**2**	**2**	**2**	**2**	**2**
	6	**4**	**2**	**2**	**2**	**2**	**2**	**2**
	7	**5**	**2**	**2**	**2**	**2**	**2**	**2**
0.10	**4**	**0**	**2**	**1**	**1**	**1**	**1**	**1**
	5	**1**	**2**	**2**	**1**	**1**	**1**	**1**
	6	**2**	**2**	**2**	**2**	**1**	**1**	**1**
	7	**3**	**2**	**2**	**2**	**1**	**1**	**1**
	8	**4**	**2**	**2**	**2**	**2**	**1**	**1**
	9	**5**	**2**	**2**	**2**	**2**	**1**	**1**
0.05	**5**	**0**	**1**	**1**	**1**	**1**	**1**	**1**
	6	**1**	**2**	**2**	**1**	**1**	**1**	**1**
	7	**2**	**2**	**2**	**1**	**1**	**1**	**1**
	8	**3**	**2**	**2**	**2**	**1**	**1**	**1**
	9	**4**	**2**	**2**	**2**	**1**	**1**	**1**
	10	**5**	**2**	**2**	**2**	**2**	**1**	**1**
0.01	**7**	**0**	**1**	**1**	**1**	**1**	**1**	**1**
	8	**1**	**2**	**2**	**1**	**1**	**1**	**1**
	9	**2**	**2**	**2**	**1**	**1**	**1**	**1**
	10	**3**	**2**	**2**	**2**	**1**	**1**	**1**
	11	**4**	**2**	**2**	**2**	**1**	**1**	**1**
	12	**5**	**2**	**2**	**2**	**1**	**1**	**1**

Table 1 shows the optimal group with a number of predefined plan parameters setting, acceptance numbers, test items, constant "a" as well as consumer's risk. Although different setting for plan parameters can take place in this regards. The maximum number of group 2 can be observed which is not seems much more expensive.

Table 2: L(p) for weighted IGASP when lifetime of items follows inverse Rayleigh distribution

B	g	r	a	μ / μ_0					
				2	4	6	8	10	12
0.25	2	4	0.7	0.988499098	1	1	1	1	1
	2	4	0.8	0.941359823	0.999999998	1	1	1	1
	2	4	1.0	0.691778845	0.999991998	1	1	1	1
	2	4	1.2	0.391510083	0.999206397	0.999999998	1	1	1
	2	4	1.5	0.130604169	0.971410649	0.999991998	1	1	1
	1	4	2.0	0.615012901	0.978032294	0.999814213	0.999999767	1	1
0.10	2	6	0.7	0.956688258	1	1	1	1	1
	2	6	0.8	0.817165533	0.999999993	1	1	1	1
	2	6	1.0	0.366299734	0.999962969	1	1	1	1
	1	6	1.2	0.65952052	0.999763047	1	1	1	1
	1	6	1.5	0.374086732	0.990413131	0.999997693	1	1	1
	1	6	2.0	0.128688569	0.862270767	0.998297517	0.999997693	1	1
0.05	2	7	0.7	0.932799575	1	1	1	1	1
	2	7	0.8	0.740726709	0.999999988	1	1	1	1
	1	7	1.0	0.78029955	0.999995406	1	1	1	1
	1	7	1.2	0.516977638	0.999536424	0.999999999	1	1	1
	1	7	1.5	0.226344193	0.982305872	0.999995406	1	1	1
	1	7	2.0	0.050761796	0.78029955	0.996739879	0.999995406	0.999999999	1
0.01	2	9	0.7	0.871576894	1	1	1	1	1
	2	9	0.8	0.582148706	0.999999971	1	1	1	1
	1	9	1.0	0.602558248	0.999987256	1	1	1	1
	1	9	1.2	0.2884349	0.998757828	0.999999998	1	1	1
	1	9	1.5	0.072698874	0.957744853	0.999987256	1	1	1
	1	9	2.0	0.006751395	0.602558248	0.991628349	0.999987256	0.999999998	1

Tables 2 shows the probability of acceptance for different settings of plan parameters. The increasing trend of probability of acceptance can easily be observed with the increase in quality ratio μ / μ_0 which shows the proposed plan relevance and complying with the usual expectation.

5. Comparison

In this section the comparison of proposed and existing plan will be discussed with the help of tables and figures.

Table3: Comparison between proposed and *existing plan (at a=1.2)

Existing(IGASP)Exponential Distribution					**Proposed(IGASP)Inverse Rayleigh Distribution**				
c	0.25	0.10	0.05	0.01	c	0.25	0.10	0.05	0.01
0	1	1	1	1	0	1	1	1	1
1	2	1	1	1	1	2	1	1	1
2	2	1	1	1	2	2	1	1	1
3	2	2	2	1	3	2	1	1	1
4	2	2	2	2	4	2	2	1	1
5	2	2	2	2	5	2	2	2	1

*extracted the part of table 1 from Naqvi and Bashir (2016).

It can be observed in table 3 that with a varying acceptance numbers(c) the number of groups are decreasing when the consumer risk is increasing. Although the proposed plan is giving us better results at consumer‘s risk 0.05, 0.01 along acceptance numbers (3,4)and (4,5) respectively which is also clear from the following figure.

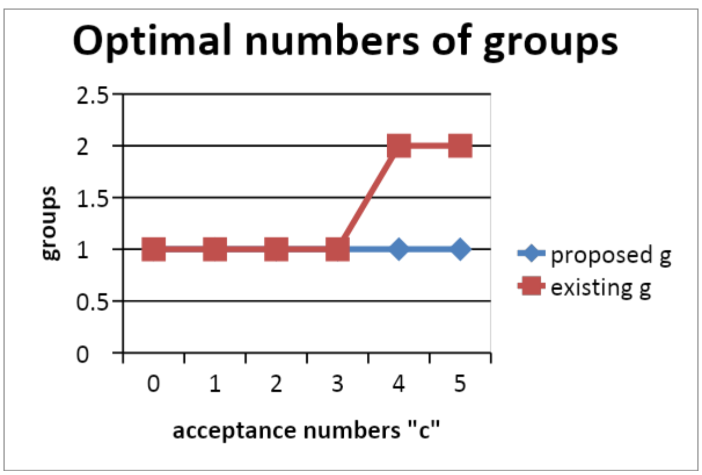

Figure1: Comparison of groups

Figure1 is the comparison of group “g” between the proposed and existing plan. When the acceptance numbers are increasing the proposed plan seems more efficient than the existing plan under the same parameter setting otherwise as efficient as the existing one.

Plan	β	R	C	A	G	$\frac{\mu}{\mu_0}$					
						2	4	6	8	10	12
Proposed	0.25	4	2	0.7	2	0.9884991	1	1	1	1	1
		4	2	0.8	2	0.9413598	1	1	1	1	1
		4	2	1.0	2	0.6917788	0.9999920	1	1	1	1
		4	2	1.2	2	0.3915101	0.9992064	1	1	1	1
		4	2	1.5	2	0.1306042	0.9714106	0.9999920	1	1	1
		4	2	2.0	1	0.6150129	0.9780323	0.9998142	0.9999998	1	1
*Existing	0.25	4	2	0.7	2	0.5338035	0.8631924	0.9451539	0.9730010	0.9848207	0.9906466
		4	2	0.8	2	0.4460934	0.8201303	0.9249710	0.9622869	0.9785232	0.9866507
		4	2	1.0	2	0.2995515	0.7261157	0.8766687	0.9354268	0.9622869	0.9761534
		4	2	1.2	2	0.1930821	0.6285373	0.8201303	0.9020302	0.9413528	0.9622869
		4	2	1.5	2	0.0944774	0.4888880	0.7261157	0.8421123	0.9020302	0.9354268
		4	2	2.0	1	0.6464673	0.9034294	0.9621059	0.9815310	0.9896755	0.9936614

Table 4: Comparison of probability of acceptance
* extracted the part of table 2 from Naqvi and Bashir (2016).

Both the plans shows increasing trend of probability of acceptance with the increasing quality ratio but at the same time it is also clear that the probability of acceptance is always higher for proposed plan as compared to existing plan. It can be reported that proposed plan is more efficient and cost effective under the said parameter setting than the existing plan. Although the different and may be better results can be observed for other distributions under the same parameter settings.

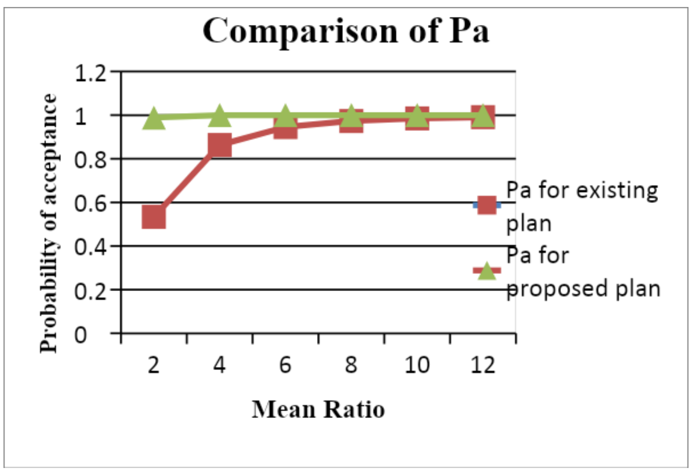

Figure 2: Comparison of probability of acceptance

Figure 2 shows when we double the quality of an item then there is a 98% chance that the lot will be accepted under proposed plan whereas there is only 53% chance for the existing plan, it means that the proposed plan is more efficient. Although when we increase the quality 10 times then both the plans give us the same results, which is not seems to be a vigilant decision.

6. Example

Let an engineer wants to test the quality of an electronic item for 1000h with the following sampling plan parameters:

Beta=0.5, r=5,c=0, a=0.8 then from table1 the required 5 electrical items for proposed plan whereas for the same parameter setting he requires 10 such electrical items, as double of existing plan(Naqvi and Bashir(2016),Table1) to conduct his experiment. Hence proposed plan seems economical in this case.

7. Conclusion.

In this paper an improved group acceptance sampling plan has been proposed for weighted binomial distribution when the lifetime of the test item follows inverse Rayleigh distribution. Under predefined parameter setting the optimal number of groups obtained. It has been observed that proposed plan, sometimes for the same parameter setting, more efficient than the existing plan.

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