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Bigraph in GraphTheory

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Abstract

In this paper we study bigraph in graph theory and discussed properties bigraph of some type graph, we study odd complete graph and even complete graph has bigraph such that when partition graph into two part G_1 , G_2 , if even complete graph such G_1 is odd complete graph after partition and G_2 is not complete graph, either if odd complete graph such G_1 is even complete graph after partition and G_2 is not complete graph, we study regular graph for me bigraph too we get after partition G_1 either odd complete graph or even complete graph, will we discuss the status every bigraph is disconnected graph, also are looking at rest graphics achieve their properties B-bigraph for example we take Euler graph, square graph, Hamiltonian cycle graph , Hamiltonian path graph, This is the convention we use when trying to represent a bigroup by a graph. The vertices corresponds to the elements of the group, hence the order of the group G corresponds to the number of vertices in the graph.

Keyword: Bigraph; complete graph; Eulerian graph; square graph; regular graph; Hamiltonian graph.

1-introduction

in this paper, we introduce the concept bigraph in graph theory with result and counter examples, also we establish between graph and bigraph, then it has became rigorous area of researching of graph theory , the notion of bigroup on group was first introduced by P.L Maggu in 1994 .

Definition1.1 [Dr.R.Muthuraj and P.M.Sitharselvam, M.S.Muthuraman Department of Mathematics PSNA CET, (2010).**]:** A set $(G, +, *)$ with two binary operation $+$ and $*$is called an bigroup if there exist two proper subsets $G_1$and G_2 of G such that i. $G = G_1 \cup G_2$

ii. $(G_1, +)$ is a group. iii. $(G_2, *)$ is an group. A non-empty subset H of an bigroup $(G, +, *)$ is called an sub bigroup, if H itself is a bigroup under $+$ and $*$operations defined on G.

Definition 1.2 [Paul Van Dooren, 2009**]:** A graph $G = (V, E)$ is a pair of vertices (or nodes) V and a set of edges E, assumed finite i.e. $|V| = n$ and $|E| = m$.Here $V(G) = \{v_1, v_2, \ldots, v_5\}$ and $E(G) = \{e_1, e_2, \ldots, e_6\}$. An edge $e_k = (v_i, v_j)$ is incident with the vertices v_i and v_j . A simple graph has no self-loops or multiple edges.

Definition 1.3 [Clark. J. and Holton D. A., (1961).**]:** A complete graph is a simple graph in which each pair of distinct vertices is joined by an edge. If the complete graph has vertices $v_1, \ldots, v_n$ then the edges set can be given by $E = \{(v_i, v_j): i \neq j; i, j = 1, \ldots, n.\}$. It follows that the graph has $\frac{1}{2}n(n-1)$ edges .

Definition 1.4[Harary F., (1969)**]:** Let v be a vertex of the graph G. The degree $d(v)$ of v is the number of edges of G incident with v, counting the loop twice, i.e., it is the number of times v is an end vertex of an edge. If for some positive integer r, $d(v) = r$ for every vertex v of the graph G, then G is called $r-$regular. A regular graph is one that is $r-$regular for some r.

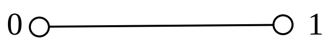
In this section we just recall some basic notions about some algebraic structures to make this paper a self contained .

Definition 1.5 [W.B.Vasantha Kandasamy Florentin Smarardache,(2009)].

A non empty set S on which is defined an associative binary operation * is called a semigroup; if for all $a, b \in S, a * b \in S$. A non empty set G is said to form a group if on G is defined an associative binary operation $*$ such that $a, b \in G$ then $a * b \in G$. There exists an element $e \in G$ such that $a * e = e * a = a$ $for\ all\ a \in G$. For every a $\in G$ there is an element a^{-1} $in\ G$ such that $a * a^{-1} = a^{-1} * a = e$ (existence of inverse in G). A group G is called an abelian or commutative if $a * b = b * a$ for all $a, b, \in G$.

Example 1.6 [W.B.Vasantha Kandasamy Florentin Smarardache,(2009)].

Let $Z_2 = \{0, 1\}$ be the group under addition modulo 2. The identity graph of Z_2 as $1 + 1 = 0(mod\ 2)$, figure as follows:



Definition 1.7[1]: let G be simple connected graph denoted by G^2 is defined to be the graph with same vertex set as G and in which two vertices u and v are joined by an edge if and only if in G we have $1 \leq d(u,v) \leq 2$.

2 –bigraph in graph theory.

In this section we introduce studying the type graph on properties bigraph and what the relation between graph and bigraph.

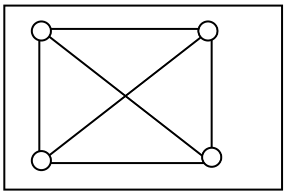
Definition 2.1: A set (G, v, e) with two element is vertices and $edges$ is called a bigraph if there exist two proper subsets G_1 and G_2 of G such that i. $G = G_1 \cup G_2$

ii. (G_1, v, e) is a graph. iii. (G_2, v, e) is an graph. A non- empty subset H of an bigraph (G, v, e) is called an sub bigraph, if H itself is a bigraph.

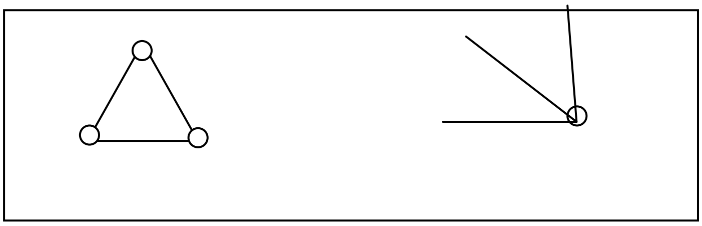
Example 2.2: we take k_4 complete graph, we apply properties bigraph.

Solution: Suppose that k_4 complete graph has 4 vertices , 6 edges , to show that k_4 is bigraph, we woke dividing the graph into two graphs G_1, G_2 such that $G = G_1 \cup G_2$

In the following complete



Now dividing the graph k_4 , we partition graph into two graph , so appear complete graph k_3 is G_1 and G_2 is one vertices has 3 edges as following:



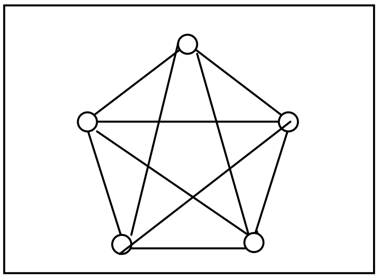
After dividing so union two graph such that definition union of graph [clark] Given two subgraphs G_1 and G_2 of G, the union $G_1 \cup G_2$ is the subgraph of G with vertices set consisting of all those vertices which are in either G_1 or G_2 (or both) and with edges set consisting of all those edges which are in either G_1 or G_2 (or both); symbolically $V(G_1 \cup G_2) = V(G_1) \cup V(G_2)$, $E(G_1 \cup G_2) = E(G_1) \cup E(G_2)$.

Thus after union G_1, G_2 so appear complete graph k_4 from new , thus application properties B-bigraph.

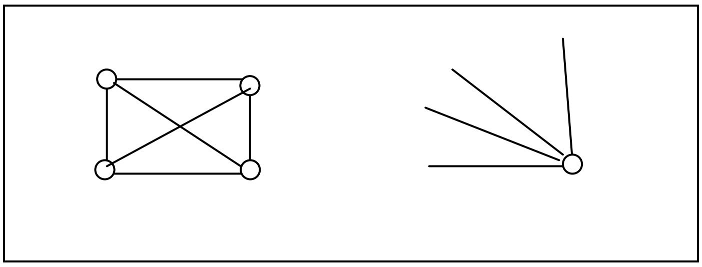
Example 2.3: we take k_5 complete graph, we apply properties bigraph.

Solution: Suppose that $k_5$complete graph has 5 vertices , 10 edges , to show that k_5 is bigraph, we woke dividing the graph into two graphs G_1, G_2 such that $G = G_1 \cup G_2$

In the following complete



Now dividing the graph k_5 , we partition graph into two graph , so appear complete graph k_4 is $G_1$and G_2 is one vertices has 4 edges as following:



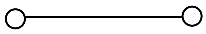
After dividing so union two graph such that definition union of graph [clark] Given two subgraphs G_1 and G_2 of G, the union $G_1 \cup G_2$ is the subgraph of G with vertices set consisting of all those vertices which are in either G_1 or G_2 (or both) and with edges set consisting of all those edges which are in either G_1 or G_2 (or both); symbolically $V(\,G_1 \cup G_2) = V(G_1) \cup V(G_2), E(\,G_1 \cup G_2) = E(G_1) \cup E(G_2)$.

Thus after union G_1, G_2 so appear complete graph k_4 from new, thus application properties B-bigraph.

In this section introduce we conclude when divided odd complete graph and even complete graph we get too even complete graph and odd complete graph literary.

Theorem 2.4: let k_{2n} be complete graph then $k_{2n}$is bigraph, such that $G_1 is\ k_n$ is odd complete graph and G_2 has one vertices - n edges, n expect zero.

Proof: suppose that k_2 be complete graph then has 2 vertices, 1 edges , to show that k_2 is bigraph, we woke dividing the graph into two graphs G_1, G_2 such that $G = G_1 \cup G_2$, In the following $k_2$complete graph,



Now dividing the graph k_2 , we partition graph into two graph , so appear complete graph k_1 is $G_1$and G_2 is one vertices - one edge as following bigraph:

Thus after union G_1, G_2 so appear complete graph k_2 from new , thus application properties B-bigraph. Either Suppose that $k_4$complete graph from example 2.2 we show that k_4 is B-bigraph. If k_6 we woke dividing the graph into two graphs G_1, G_2 such that $G = G_1 \cup G_2$, Now dividing the graph k_6 , we partition graph into two graph , so appear complete graph k_5 is $G_1$and G_2 is one vertices - 3 edge , Thus after union G_1, G_2 so appear complete graph k_6 from new and each G_1, G_2 is graph, thus application properties B-bigraph. Either reminder states the same method proof, we show that even complete graph is B- bigraph.

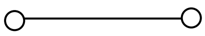
Theorem 2.5: let k_n be complete graph then $k_n$is B-bigraph, such that $G_1 is\ k_{2n}$ is even complete graph and G_2 has one vertices - 2n edges, n expect 1.

Proof: if suppose that k_3 be complete graph then has 3 vertices , 3 edges , to show that k_3 is bigraph, we woke dividing the graph into two graphs G_1, G_2 such that $G = G_1 \cup G_2$, Now dividing the graph k_3 , we partition graph into two graph , so appear complete graph k_2 is $G_1$and G_2 is one vertices - 2 edge , Thus after union G_1, G_2 so appear complete graph k_3 and each G_1, G_2 is graph, thus application properties B-bigraph. Either Suppose that $k_5$complete graph from example 2.3, we show that k_5 is B-bigraph. If k_7 we woke dividing the graph into two graphs G_1, G_2 such that $G = G_1 \cup G_2$, Now dividing the graph k_7 , we partition graph into two graph , so appear complete graph k_6 is $G_1$and G_2 is one vertices - 2 edge , Thus after union G_1, G_2 so appear complete graph k_7 from new and each G_1, G_2 is graph, thus application properties B-bigraph. Either reminder states the same method proof ,we show that even complete graph is B- bigraph.

In this theorem we study bigraph of regular such that G_1 appear either odd complete graph or even complete graph.

Theorem 2.6: let G be n-regular graph then G is bigraph.

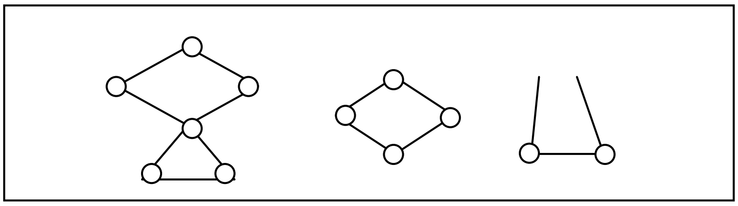
Proof: suppose that G is 1-reguar graph, then degree of graph is 1 , we woke dividing the graph into two graphs G_1, G_2 such that $G = G_1 \cup G_2$, In the following $1 - regular$ graph, as following:



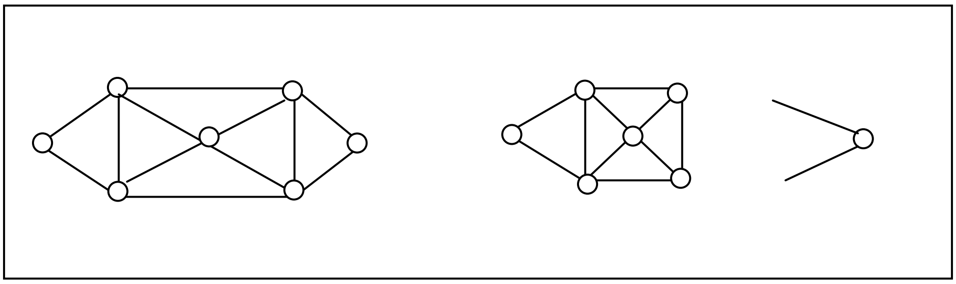
Now dividing the graph $1-regular$ graph , we partition graph into two graph , so appear one vertices is complete graph k_1 is $G_1$and G_2 is one vertices - one edge , Thus after union G_1, G_2 so appear complete graph $1-regular$ graph from new and each G_1, G_2 is graph, thus application properties B-bigraph. Either Suppose that $2-regular$ graph from example 2.2 we show that $2-rgular$ graph is B-bigraph since each complete graph is regular graph. If $3-regular$graph, we woke dividing the graph into two graphs G_1, G_2 such that $G = G_1 \cup G_2$, Now dividing the graph $3-regular$ graph, we partition graph into two graph , so appear complete graph $2-regular$ graph is $G_1$and G_2 is one vertices - 3 edge, Thus after union G_1, G_2 so appear complete graph $3-regular$ graph, and each G_1, G_2 is graph, thus application properties B-bigraph. Either reminder states the same method proof, we show that n-regular graph is B- bigraph.

Theorem 2.7: let G is eulerian graph then G is bigraph.

Proof: suppose that G is eulerian graph, then degree of graph is 2,4 , let the number vertices is 6, we woke dividing the graph into two graphs G_1, G_2 such that $G = G_1 \cup G_2$, In the following eulerian graph , bigraph, as following:



Now dividing eulerian graph , we partition graph into two graph , so appear G_1 is eulerian graph and G_2 is two vertices - 3 edge but disconnected and is not eulerian graph , Thus after union G_1, G_2 so appear eulerian graph and each G_1, G_2 is graph, thus application properties B-bigraph. Either Suppose that $euleian$ graph has degree is 4,2 but the number vertices 7, in the following G and G_1, G_2:



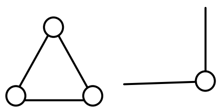
we woke dividing the graph into two graphs G_1, G_2 such that $G = G_1 \cup G_2$, Now euler trail graph is $G_1$and G_2 is one vertices - 2 edge , Thus after union G_1, G_2 so appear eulerian graph from new, and each G_1, G_2 is graph, thus application properties B-bigraph. Either reminder states the same method proof, we show that eulerian graph is B- bigraph.

Theorem 2.8: let G be Hamiltian cycle (Hamiltian) - Hamiltian path is bigraph.

Proof: suppose that G is Hamiltian cycle – Hamiltian path graph, [clark] defined A *Hamiltonian path* in a graph G is a path which contains every vertices of G. Since, no vertex of a path is repeated, this means that a Hamilton path in G contains every vertex of G once and only once. A *Hamiltonian cycle* (or *Hamiltonian circuit*) in a graph G is a cycle which contains every vertex of G. Since, no vertex of a cycle is repeated apart from the final vertex being the same as the first vertex, this means that a Hamiltonian cycle in G with initial vertex v contains every other vertex of G precisely once and then ends back at v. we woke dividing the graph into two graphs G_1, G_2 such that $G = G_1 \cup G_2$, In the following Hamiltian path – cycle graph , bigraph, as following:

Hamiltian path- cycle

Now dividing Hamilitian path graph , we partition graph into two graph , so appear G_1 is eulerian graph and G_2 is one vertices , one edge but disconnected and is not eulerian graph , Thus after union G_1, G_2 so appear Hamiltian path graph from new , and each G_1, G_2 is graph, thus application properties B-bigraph. Either Suppose that Hamiltian cycle graph , in the following figure G_1, G_2:

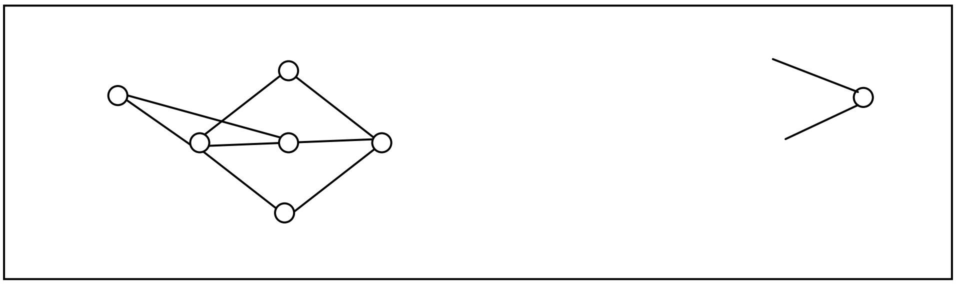


we woke dividing the graph into two graphs G_1, G_2 such that $G = G_1 \cup G_2$, Now eulerian graph is $G_1$and G_2 is one vertices - 2 edge , Thus after union G_1, G_2 so appear eulerian graph from new , and each G_1, G_2 is graph ,thus application properties B-bigraph. Either reminder states the same method proof, we show that eulerian graph is B- bigraph.

Theorem 2.9: let G^2 be square graph is bigraph**.**

Proof: suppose that G is simple connected graph and G^2 is connected graph, definition 1.7 we woke dividing the graph into two graphs G_1, G_2 such that $G = G_1 \cup G_2$, In the following G , G^2, the figure as following:

Now dividing G^2 graph , we partition graph into two graph , so appear G_1 has degree of vertices is 2,3 and G_2 is one vertices - 2 edge but is not $G^2$and is not graph G , Thus after union G_1, G_2 so appear G^2 graph from new , and each G_1, G_2 is graph, thus application properties B-bigraph. in the following $figure\ G_1^2, G_2^2$:



Either reminder states the same method proof, we show that graph is G^2 B-bigraph.

In this theorem we discussed relation between B-bigraph and graph.

Theorem 2.10: every B-bigraph is disconnected graph.

Proof: suppose that G is bigraph such that $G = G_1 \cup G_2$ from definition bigraph, we defined G into two graph is G_1, G_2 , thus is not found path connected link between the one graph $G_1$and the second graph G_2, A graph G is called connected if every two of its vertices are connected. A graph that is not connected is called disconnected, thus G is disconnected.

Conclusion

1- let k_n be complete graph then $k_n$is B-bigraph, such that $G_1 is\ k_{2n}$ is even complete graph and G_2 has one vertices - 2n edges, n expect 1.

2- let k_{2n} be complete graph then $k_{2n}$is bigraph, such that $G_1 is\ k_n$ is odd complete graph and G_2 has one vertices - n edges, n expect zero.

3- let G be n-regular graph then G is bigraph such that $G_1 is$ is odd – even complete graph.

4- let G is eulerian graph then G is bigraph.

5- let G be Hamiltian cycle (Hamiltian) - Hamiltian path is bigraph.

6- let G^2 be square graph is bigraph**.**

7- every B-bigraph is disconnected graph.

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