Journal of Progressive Research in Mathematics(JPRM)
ISSN: 2395-0218

SCITECH
RESEARCH ORGANISATION

Volume 14, Issue 3
Published online: January 10, 2019|

Journal of Progressive Research in Mathematics
www.scitecresearch.com/journals

On the globally coupled lattice system (GCM) associated with the Belusov-Zhabotinskii reaction (p, q)-points on the coupling constant $\varepsilon \in (0, 1]$*

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Abstract

This paper focuses on the Global Coupling System (GCM) associated with the Belusov-Zhabotinskii reaction:

$$x_n^{m+1} = (1-\varepsilon)f(x_n^m) + \frac{\varepsilon}{L}[f(x_0^m) + \ldots + f(x_{n-1}^m) + f(x_{n+1}^m) + \ldots + f(x_{L-1}^m)]$$

where m is discrete time index, n is lattice side index with system size L, $\varepsilon \in (0,1]$ is coupling constant and f_n is a continuous selfmap on $[0,1]$ for every $n \in \{1, 2, \cdots, L\}$.We prove that the system is distributionally (p,q)-chaotic on the non-zero coupling constant $\varepsilon \in (0,1)$, and its main metric is not less than $\frac{2}{3} + \sum_{n=2}^{\infty} \frac{1}{n} \frac{2^{n-1}}{(2^n+1)(2^{n-1}+1)}$

Key words:Globally coupled system ;distributionally (p,q)-chaos;Measure *d*

1 Introduction

The dynamical system(d.s. for short) (X, f) refers to a continuous self-mapping f for a tight metric space X and X. Chaos is derived from nonlinear dynamic systems. The study of chaos begins with the discovery of chaotic phenomena, which are seemingly irregular movements in the dynamic system. In 1975, Li Tianyan and his mentor Yorke gave the definition of chaos (see[1]) (ie Li-Yorke chaos definition) from the first strict mathematical definition. Since then, various chaos definitions have been given, such as Devaney chaos, distributed chaos and so on. After the definition of Li-Yorke chaos is proposed, the dynamic system has been continuously explored and studied in different fields of physics and biology. In 1983 Kaneko (see[4]) proposed a coupled map lattice

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(CML). CML is sensitive to initial conditions and boundary conditions due to its chaos in space and time. It becomes a study of nonlinear spatiotemporal chaos. An effective model of the phenomenon. Throughout this paper, we further develop the following global coupling system (GCM) which is related to BZ reaction,and discuss its chaotic nature:

$$x_n^{m+1} = (1-\varepsilon)f(x_n^m) + \frac{\varepsilon}{L}[f(x_0^m) + ... + f(x_{n-1}^m) + f(x_{n+1}^m) + ... + f(x_{L-1}^m)],$$

where m denotes a discrete time index, n denotes a lattice index with a system size of L, $\varepsilon \in (0,1)$ denotes a coupling constant, and f denotes a continuous mapping. In addition, we also calculated that the system maps to the triangle tent map $\Lambda(x) = 1 - |1-2x|, x \in [0,1]$ when it's the main metric is not less than $\frac{2}{3} + \sum_{n=2}^{\infty} \frac{1}{n} \frac{2^{n-1}}{(2^n+1)(2^{n-1}+1)}$.

2 Preliminaries

First, give a general summary of the basic Li-Yorke chaos concept (see[6]). Throughout this paper, $I = [0,1]$, X is a tight metric space measured as d, mapping $f : X \to X$ consecutively, (X, f) is a topology dynamical system.

Definition 2.1 *Giving a pair of points $x, y \in X$,$x \neq y$,called Li-Yorke chaos if it meets the following conditions*

$$\limsup_{n\to\infty} d(f^n(x), f^n(y)) > 0.$$

$$\liminf_{n\to\infty} d(f^n(x), f^n(y)) = 0.$$

Then point to (x, y) for a Li-Yorke climbing couple of the system (X, f), if there is an uncountable set $S \subset X$, $\#S \geq 2$ # Represents the cardinality of a collection, in which each pair of points is a Li-Yorke climbing couple, it is a Li-Yorke climbing set mapped, called mapping f is Li-Yorke chaos, referred to as chaos.

Definition 2.2 *Let the system (X, f) be a d.s.. For any x, $y \in X$ and any $n \in N$, defined the distributional function $F_{xy}^n : \mathrm{R}^+ \to [0,1]$ is as follows*

$$F_{xy}^n(t) = \frac{1}{n}\#\{i \in \mathrm{N} : d(f^i(x), f^i(y)) < t, 1 \leq i \leq n\},$$

where $\mathrm{R}^+ = [0, +\infty)$, represents the base. At this time, the upper and lower distribution functions are as follows

$$F_{xy}(t, f) = \liminf_{n\to\infty} F_{xy}^n(t)$$
$$F_{xy}^*(t, f) = \limsup_{n\to\infty} F_{xy}^n(t).$$

For any $0 \leq p \leq q \leq 1$, if the system (X, f) is in $\varphi > 0$ where is an uncountable set $S \subset X$ for any $x, y \in S$,$x \neq y$,$t \in (0, \varphi)$

$$F_{xy}(t, f) = p$$
$$F_{xy}^*(t, f) = q.$$

The mapping f is called distributionally (p, q)-chaos, and the system (X, f) distributes chaos when it satisfies the distributionally $(0, 1)$-chaos.

Definition 2.3 *The tent map* Λ *defined by* $\Lambda(x) = 1 - |1 - 2x|, x \in [0, 1]$*for any* $0 \leq p \leq q \leq 1$ *is* (p, q)*distribution chaos (see[8]). In recent years, some major metrics for the system* (X, f) *have been proposed.*

$$\mu_p(f) = \sup_{x,y \in X} \frac{1}{M} \int_0^{+\infty} \big(F_{xy}^*(t, f) - F_{xy}(t, f)\big) dt.$$

While f *is Triangle tent map ,* M *is the diameter of the space* X *(see[8]), they got* $\mu_p(\Lambda) = \frac{2}{3} + \sum_{n=2}^{\infty} \frac{1}{n} \frac{2^{n-1}}{(2^n+1)(2^{n-1}+1)}$.

3 Main results

Let the state space of the lattice dynamic system (p.s.) be a collection $\chi = \{x : x = \{x_i\}, x_i \in \mathrm{R}^d, i \in {}^D, ||x_i|| < \infty\}$ .Where d$\geq$ 1 is the spatial dimension map x_i, $D\geq$ 1 is the dimension of the lattice,

$$\|x\|_2 = \sqrt{\sum_{\mathrm{i} \in Z^D} |x_i|^2}$$

is often used to represent the l^2 paradigm (see[9]).

This paper considers the following GCM (related to the Belusov-Zhabotinskii reaction):

$$x_n^{m+1} = (1 - \varepsilon) f(x_n^m) + \frac{\varepsilon}{L} [f(x_0^m) + ... + f(x_{n-1}^m) + f(x_{n+1}^m) + ... + f(x_{L-1}^m)],$$

where m is a discrete time index, n is a system lattice index of size L, $\varepsilon \geq 0$ is a coupling constant, and $f : I \to I$ is a continuous map.

Usually,suppose that the system has one of the following periodic boundary conditions:

(1) $x_n^m = x_{n+L}^m$,
(2) $x_n^m = x_n^{m+L}$,
(3) $x_n^m = x_{n+L}^{m+L}$.

In this article, we will use the first condition as our system boundary condition. Let d represent the metric for space I^L, $(x_1, ..., x_L), (y_1, ..., y_L) \in I^L$ then

$$d((x_1, ..., x_L), (y_1, ..., y_L)) = \sqrt{\sum_{i=1}^{L} |x_i - y_i|^2}.$$

We define the map $F : (I^L, d) \to (I^L, d)$, where $F(x_1, ..., x_L) = (y_1, ..., y_L)$, $y_i = (1 - \varepsilon) f(x_i) + \frac{\varepsilon}{L} [f(x_0) + f(x_1) + ... + f(x_{i-1}) + f(x_{i+1}) + ... + f(x_{L-1})]$. It is easy to derive that the system (1) is equivalent to the system (I^L, F)(see[9],[10]).

Theorem 1 For any $p, q \in [0,1]$ with $p \leq q$, any $\varepsilon \in (0,1)$,system (1)

$$x_n^{m+1} = (1-\varepsilon)f(x_n^m) + \frac{\varepsilon}{L}[f(x_0^m) + ... + f(x_{n-1}^m) + f(x_{n+1}^m) + ... + f(x_{L-1}^m)] \quad (1)$$

is distributionally (p, q)-chaotic if $f = \Lambda$.

Proof. According to Proposition 3 in [7], for any $\leq q \leq 1$, Λ is distributionally (p, q)-chaotic, there is an uncountable set $\Gamma \subset I$ and $\varepsilon > 0$ makes for any $x, y \in \Gamma$ and $0 < t < \delta$,

$$F_{xy}(t, \Lambda) = p \quad (2)$$

$$F_{xy}^*(t, \Lambda) = q. \quad (3)$$

Let $\Delta = \{(x_1, x_2, ..., x_L) \in I^L : x_1 = x_2 = ...x_L \in\}$, where its $\vec{x} = (x, ..., x)$,$\vec{y} = (y, ..., y) \in \Delta$, and $\vec{x} \neq \vec{y}$. Then

$$F^n(\vec{x}) = (\Lambda^n(x), ..., \Lambda^n(x)) \quad (4)$$

$$F^n(\vec{y}) = (\Lambda^n(y), ..., \Lambda^n(y)). \quad (5)$$

By combing (2), (3), (4), (5), it is proved that for any $t \in \left(0, \sqrt{L}\delta\right)$,

$$\begin{aligned} F_{\vec{x}\vec{y}}(t, F) &= \lim_{n\to\infty} \inf \tfrac{1}{n}\# \left\{i \in N : d(F^i(\vec{x}), F^i(\vec{y})) < t, 1 \leq i \leq n\right\} \\ &= F_{xy}(\tfrac{t}{\sqrt{L}}, \Lambda) \\ &= p \end{aligned} \quad (6)$$

as well as

$$\begin{aligned} F_{\vec{x}\vec{y}}^*(t, F) &= \lim_{n\to\infty} \sup \tfrac{1}{n}\# \left\{i \in N : d(F^i(\vec{x}), F^i(\vec{y})) < t, 1 \leq i \leq n\right\} \\ &= F_{xy}^*(\tfrac{t}{\sqrt{L}}, \Lambda) \\ &= q. \end{aligned} \quad (7)$$

At this point, the system (1) is scrambled for (p, q) on $\varepsilon \in (0,1)$.
Corollary 1 For any $p, q \in [0,1]$, and $p \leq q$, when $\varepsilon = 1$, system (1)

$$x_n^{m+1} = (1-\varepsilon)f(x_n^m) + \frac{\varepsilon}{L}[f(x_0^m) + ... + f(x_{n-1}^m) + f(x_{n+1}^m) + ... + f(x_{L-1}^m)] \quad (8)$$

is distributionally (p, q)-chaotic if $f = \Lambda$.
Theorem 2 For any $\varepsilon \in (0,1)$, system (1),

$$x_n^{m+1} = (1-\varepsilon)f(x_n^m) + \frac{\varepsilon}{L}[f(x_0^m) + ... + f(x_{n-1}^m) + f(x_{n+1}^m) + ... + f(x_{L-1}^m)]$$

its main metric is no less than $\mu_p(f)$.

Proof. The proof is known that, if $\vec{x} = (x_1, ..., x_L) \in I^L, \vec{y} = (y_1, ..., y_L) \in I^L$ and $\vec{x} \neq \vec{y}$, as shown by the result (6) (7), for any $t > 0$, there are

$$F_{xy}(t, f) = F_{\vec{x}\vec{y}}(\sqrt{L}t, F) \tag{9}$$

$$F^*_{xy}(t, f) = F^*_{\vec{x}\vec{y}}(\sqrt{L}t, F), \tag{10}$$

then for any $\vec{x}, \vec{y} \in I^L$, I^L has a diameter of M, there is

$$\begin{aligned}
\mu_p(F) &\geq \sup_{x,y\in I} \frac{1}{M} \int_0^{+\infty} \Big(F^*_{\vec{x}\vec{y}}(t, F) - F_{\vec{x}\vec{y}}(t, F)\Big) dt \\
&= \sup_{x,y\in I} \frac{1}{\sqrt{L}} \int_0^{+\infty} \Big(F^*_{\vec{x}\ Vecy}(t, F) - F_{\vec{x}\vec{y}}(t, F)\Big) dt \\
&= \sup_{x,y\in I} \frac{1}{\sqrt{L}} \int_0^{+\infty} \Big(F^*_{xy}(\tfrac{t}{\sqrt{L}}, f) - F_{xy}(\tfrac{t}{\sqrt{L}}, f)\Big) dt \\
&= \sup_{x,y\in I} \int_0^{+\infty} \left(F^*_{xy}(t, f) - F_{xy}(t, f)\right) dt \\
&= \mu_p(f).
\end{aligned} \tag{11}$$

Therefore, theorem 2 is proved. On the basis of Definition 3 and Theorem 2, when $f = \Lambda$, the main metric of system (1) is no less than $\frac{2}{3} + \sum_{n=2}^{\infty} \frac{1}{n} \frac{2^{n-1}}{(2^n+1)(2^{n-1}+1)}$.

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