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Influence of rotation and initial stress onPropagation of Rayleigh waves in fiber-reinforced solidanisotropic magneto-thermo-viscoelastic media.

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Abstract:

This paper is concerned with giving a mathematical model on the propagation of Rayleigh waves in a homogeneous magneto-thermo-viscoelastic, pre-stressed elastic half – space subjected to theinitial stress and rotation. The dispersion equation has been derived for a half-space, when both media are considered as pre-stressed and the effect of rotation and initial stressshown in earlier investigators.Numerical results have been obtained in the physical domain. Numerical simulated results are depicted graphically to show the effect of rotation and magnetic field and initial stress on Rayleigh wave velocity. Comparison was made with the results obtained in the presence and absence of the rotation , initial stressand magnetic field. The study shows that there is a variational effect of magneto-elasticityand rotation, initial stress on the Rayleigh wave velocity.

Keywords: Fibre-reinforced, viscoelastic, Rayleig wave velocity, rotation, magnetic field, the elastic medium.

1- Introduction:

In recent years, the theory of magneto- thermoelasticity which deals the interactions among the strain, temperature and magnetic field has drawn the attention of several researches due to its extensive uses in diverse fields , such as geophysics, for under standing the effects of the earthsmagnetic field on seismic waves, damping of acoustic waves in a magnetic field , emission of electromagnetic raditions from nuclear devices etc. Fiber-reinforcedcomposites are widely used in engineering structures, due to their superiority over the structural materials in appplications requiring high strength and stiffness in lightweight components.A continuum model is used to explain the mechanical properties of such materials. Several hypothesis of this type of context have been made and attempted by manypractitioners and researchers ([1]-[7]). Numerous authors have submitted their elaborated works on the study of wave propagation in different types of anisotropic media. Achenbach[9] rendered a complete discussion of seismic wave propagation in elastic layered media.

Stoneley[10] discussed the Rayleigh wave propagation in a heterogeneous medium. Dutta[11] investigated the possibility of Rayleigh wave propagation in an incompressible medium lying over a transversely isotropic semi-infinite base. Chattopadhyay[12] deliberated the propagation of shear wave in the crustal of visco-elastic material. Dey et al.[13] investigated the impact of initial stress on the reflection and refraction phenomenon at the boundary present between core-mantle of the Earth. Pal and Chattopadhyay[14] tried to demonstrate the reflection pattern of plane elastic waves in initially stressed homoge- neous orthotropic media. Chattopadhyay et al.[15] discussed the Rayleigh wave propagation under initial stress in cylindrical coordinates. Sharma and Gogna[16] obtained the solution to a differential equation representing the motion of elastic wave in a dissipative liquid filled viscoelastic porous solid and applied this solution to obtain the nature of waves. Fu and Rogerson[17] presented the nonlinear instability analysis of incompressible plate under the effect of initial stress. Again, Rogerson and Fu[18] used the concept of asymptotic analysis to build up the dispersion relation for wave propagating in an initially stressed incompressible elastic plate. In the last two decades, some of the results have been found by various authors related to seismic wave propagation in different types of anisotropic media under different types of physical situations. Abd-Alla et al.[19] performed impeccable effect of initial stress, orthotropy, and gravity field on Rayleigh wave propagation in a magneto-elastic half-space. Again, Abd-Allaet al.[20–21] made their efforts to confer the effect of various sorts of elastic parameters such as initial stress, gravity field, magnetic field, rotations, and relaxation time on the Rayleigh wave propagation. Sharma[22] perceived the influence of elasticity, pore-fluid viscosity, frequency, and pore characteristics numerically on the Rayleigh wave velocity in dissipative poro viscoelastic media. Afterwards, Sharma[23] made an attempt to discuss the propagation of Rayleigh waves in generalised thermo-elastic media with stress free boundaries, and solved the dispersion relation numerically for exact roots. Ahmed and Abo-Dahab[24] established the frequency equation for Rayleigh and Stoneley wave in a determinant form of orders twelve and eight, respectively, and obtained the phase velocity and attenuation coefficients for the waves. Ogden and Singh[25] derived an equation for plane wave motion with small amplitude in a rotating and initially have made their efforts to dissert the Rayleigh-type wave propagation in an initially stressed inhomogeneous incompressible visco-elastic medium situated over the same semi-infinite elastic medium. Wang et al.[26] remarkably studied the stop band properties of elastic waves in three-dimensional piezoelectric phononic crystals with the initial stress taking the mechanical and electrical coupling into account using the plane wave expansion (PWE) method. The elastic wave localization in disordered periodic piezoelectric rods with the initial stress was studied, and the effects of initial stress on the band gap characteristics were investigated using the transfer matrix and Lyapunovexponent method byWang et al.[27]. Some neoteric achievements in this domain have been done by numerous authors including Chatterjee et al.[28–29], Dhua and Chattopadhyay[30], Kumari et al.[31–32], and Khurana and Tomar[33]. Till now, no authors have made their efforts to dissert the Rayleigh-type wave propagation in an initially stressed inhomogeneous incompressible visco-elastic medium situated over the same semi-infinite elastic medium. Chatterjee and Chattopadhyay[34] studied the propagation, reflection, and transmission of SH-waves in slightly compressible, finitely deformed elastic media.

The purpose of the present research is to study the influences of rotation, magnetic field and specific heat, initial stresson the fiber-reinforced elastic anisotropicmagneto-thermo-viscoelastic media. Normal mode analysis is adopted to obtain the Frequency equation in closed form. These derived expressions are calculated numerically and depicted graphically to observe the effects of rotation, magnetic field and, initial stress specific heat **on Rayleigh wave velocity**.

2-Formulation of the problem and basic equations

We investigate the dynamic interactions in a fiber-reinforced anisotropic half-spacewithrotation and magnetic field, intial stress under thermoelastic theory and x -axis is assumed to be pointing vertically into the medium so that the half-space occupies the region $x \geq 0$. We restrict our analysis to xy-plane. Since we are considering a two dimensional problem with $\vec{u} = (u, v, 0)$, so all the considered functions will depend on time t and the coordinates x and y. The field equations and constitutive relations for a fiber-reinforced linear thermoelastic anisotropic diffusive medium under rotation and magnetic field are given by [8] as

$$\begin{aligned}\sigma_{ij} &= D_\lambda e_{kk}\delta_{ij} + 2D\mu_T e_{ij} + D_\alpha\left(a_k a_m e_{km}\delta_{ij} + a_i a_j e_{kk}\right) + \\ &2\left(D_{\mu L} - D_{\mu T}\right)\left(a_i a_k e_{kj} + a_j a_k e_{ki}\right) + D_\beta a_k a_m e_{km} a_i a_j - \gamma\left(T - T_0\right)\delta_{ij} - P(\delta_{ij} + \omega_{ij}),\end{aligned} \quad (1)$$

where σij are the components of stress , e_{ij} are the components of strain,aninitialcompression P , ω_{ij} small rotation, D_λ, $D_{\mu T}$ are viscoelastic parameters , D_α, D_β, $\left(D_{\mu L} - D_{\mu T}\right)$are reinforcement viscoelastic parameters, $\gamma = \left(3D_\lambda + 2D_\mu\right)\alpha_t$, α_t is thermal expansion coefficient, δ_{ij} is the Kronecker delta, T is the temperature above reference temperature T_0, and $a \equiv (a_1, a_2, a_3)$, $a_1^2 + a_2^2 + a_3^2 = 1$. We choose the fiber-direction as $a \equiv (1, 0, 0)$.

The strains can be expressed in terms of the displacement u_i and small rotation as

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right), \omega_{ij} = \frac{1}{2}(u_{j,i} - u_{i,j}) \quad (2)$$

The elastic medium is rotating uniformly with an angular velocity $\underline{\Omega} = \Omega\,\underline{n}$ where $\underline{n}$ is a unit vector representing the direction of the axis of rotation . The displacement equation of motion in the rotating frame has two additional term centripetal acceleration , $\overline{\Omega} \times (\overline{\Omega} \times \overline{u})$ is the centripetal acceleration due to time varying motion only and $2\overline{\Omega} \times \overline{\dot{u}}$ is the Cariole's acceleration , and $\overline{\Omega} = (0, 0, \Omega)$.We assume that an induced magnetic field $\underline{h} \equiv (0,0,h)$ is developed due to the application of an initial magnetic field.

For a slowly moving homogenous electrically conducting elastic solid medium taking into account absence of the displacement current (SI)[29]

$$curl\ \vec{h} = \vec{J}$$

$$curl\ \vec{E} = -\mu_e \frac{\partial \vec{h}}{\partial t}$$

$$div\ \vec{h} = 0\ ,\ div\ \vec{E} = 0$$

$$where\quad \vec{h} = curl\ (\vec{u} \times \vec{H}_0)\ ,\ \vec{H} = \vec{H}_0 + \vec{h}(x, y, t)$$

where μ_e is the magnetic permeability and $\vec{u}$ is the dynamic displacement vector.

The components of the magnetic intensity vector are

$$H_X = 0, H_Y = 0, H_z = H_0 + h(x, y, t)$$

For plane strain deformation in the $x - y$ plane, displacement $\underline{u} = (u\ , v\ , 0)$, $\partial / \partial z = 0$. Eq. (1) then yields

$$\sigma_{xx} = A_{11} u_{,x} + A_{12} v_{,y} - \gamma (T - T_0) - P\ , \tag{3}$$

$$\sigma_{yy} = A_{12} u_{,x} + A_{22} v_{,y} - \gamma (T - T_0) - P\ , \tag{4}$$

$$\sigma_{zz} = A_{12} u_{,x} + D_{\lambda} v_{,y} - \gamma (T - T_0) - P\ , \tag{5}$$

$$\sigma_{xy} = D_{\mu L} (u_{,y} + v_{,x}) - P\omega_{12}\ ,\ \sigma_{yx} = D_{\mu L} (u_{,y} + v_{,x}) - P\omega_{21}\ \sigma_{xz} = \sigma_{yz} = 0\ , \tag{6}$$

where,

$$A_{11} = \lambda_0 + 2\alpha_0 + 4\mu_{L0} - 2\mu_{T0} + \beta_0 + (\lambda_1 + 2\alpha_1 + 4\mu_{L1} - 2\mu_{T1} + \beta_1) \frac{\partial}{\partial t}\ ,$$

$$A_{12} = \lambda_0 + \alpha_0 + (\lambda_1 + \alpha_1) \frac{\partial}{\partial t}\quad ,\quad A_{22} = \lambda_0 + 2\mu_{T0} + (\lambda_1 + 2\mu_{T1}) \frac{\partial}{\partial t}$$

$$D_{\lambda} = \lambda_0 + \lambda_1 \frac{\partial}{\partial t}, D_{\alpha} = \alpha_0 + \alpha_1 \frac{\partial}{\partial t}\ ,\quad D_{\beta} = \beta_0 + \beta_1 \frac{\partial}{\partial t}, D_{\mu T} = \mu_{T0} + \mu_{T1} \frac{\partial}{\partial t},$$

$$D_{\mu L} = \mu_{L0} + \mu_{L1} \frac{\partial}{\partial t}, D_{\mu T} = \mu_{T0} + \mu_{T1} \frac{\partial}{\partial t}, D_{\mu} = \mu_0 + \mu_1 \frac{\partial}{\partial t},\ \gamma = 3\lambda_0 + 2\mu_0 + (3\lambda_1 + 2\mu_1) \frac{\partial}{\partial t}$$

where λ_0, μ_0 are elastic constant and λ_1, μ_1 are the parameters associated with 1th order viscoelasticity .

The equation of motion in the context of the Green-Naghdi theory is

$$\rho\left[\ddot{u}_i + \left[\overline{\Omega}\times\left(\overline{\Omega}\times\overline{u}\right)\right]_i + \left(2\overline{\Omega}\times\dot{u}\right)_i\right] = \sigma_{ij,j} + \mu_e(\vec{J}\wedge\vec{H})_i\,, \quad i,j=1,2,3 \tag{7}$$

The heat conduction in the absence of heat sources under the G-N III theory is

$$K\,T_{,ii} + K^{*}\,\dot{T}_{,ii} = \rho\,C_E\,\ddot{T} + \gamma\,T_0\,\ddot{u}_{i,i}\,, \tag{8}$$

where ρ is the mass density, C_E is the specific heat at constant strain , K^{*} and K are respectively the material constant characteristic of the theory and thermal conductivity. When, $K^{*}\to 0$ "equation (8)," reduces heat conduction equation of the G-N II theory. Eq.s (7), (8) and (1) constitute the complete system of generalized thermoelasticity under the G-N III theory.

Using the summation convention, from Equations (3) – (6) we note that the third equation of motion in Eq. (7) identically satisfied and the first two equations become

$$\rho\left[\frac{\partial^2 u}{\partial t^2} - \Omega^2 u - 2\Omega\frac{\partial v}{\partial t}\right] = A_{11}\frac{\partial^2 u}{\partial x^2} + B_2\frac{\partial^2 v}{\partial x\partial y} + D_{\mu L}\frac{\partial^2 u}{\partial y^2} - \gamma\frac{\partial T}{\partial x} + \frac{P}{2}\frac{\partial^2 u}{\partial y^2} - \frac{P}{2}\frac{\partial^2 v}{\partial x\partial y} + \mu_e H_0^{2}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x\partial y}\right) \tag{9}$$

$$\rho\left[\frac{\partial^2 v}{\partial t^2} - v\,\Omega^2 + 2\Omega\frac{\partial u}{\partial t}\right] = A_{22}\frac{\partial^2 v}{\partial y^2} + B_2\frac{\partial^2 u}{\partial x\partial y} + D_{\mu L}\frac{\partial^2 v}{\partial x^2} - \gamma\frac{\partial T}{\partial y} - \frac{P}{2}\frac{\partial^2 u}{\partial x\partial y} + \frac{P}{2}\frac{\partial^2 v}{\partial x^2} + \mu_e H_0^{2}\left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 u}{\partial x\partial y}\right) \tag{10}$$

Where $B_2 = A_{12} + D_{\mu L}$

Which can written as

$$\rho\left[\frac{\partial^2 u}{\partial t^2} - \Omega^2 u - 2\Omega\frac{\partial v}{\partial t}\right] = A'_{11}\frac{\partial^2 u}{\partial x^2} + B'_2\frac{\partial^2 v}{\partial x\partial y} + B'_1\frac{\partial^2 u}{\partial y^2} - \gamma\frac{\partial T}{\partial x}$$

$$\rho\left[\frac{\partial^2 v}{\partial t^2} - v\,\Omega^2 + 2\Omega\frac{\partial u}{\partial t}\right] = A'_{22}\frac{\partial^2 v}{\partial y^2} + B'_2\frac{\partial^2 u}{\partial x\partial y} + B'_1\frac{\partial^2 v}{\partial x^2} - \gamma\frac{\partial T}{\partial y}$$

And the equation of heat conduction(8) yield to the equation

$$\varepsilon_2\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial^2 y}\right) + \varepsilon_3\left(\frac{\partial^2 \dot{T}}{\partial x^2} + \frac{\partial^2 \dot{T}}{\partial y^2}\right) - \frac{\partial^2 T}{\partial t^2} = \gamma\varepsilon_1\frac{\partial^2 e}{\partial t^2} \tag{11}$$

where,

$$B'_1 = D_{\mu L} + \frac{P}{2} = \mu_{L0} + \mu_{L1}\frac{\partial}{\partial t} + \frac{P}{2}\,,$$

$$B'_2 = A_{12} + D_{\mu L} - \frac{P}{2} + \mu_e H_0^{2} = \lambda_0 + \alpha_0 + \mu_{L0} + (\lambda_1 + \alpha_1 + \mu_{L1})\frac{\partial}{\partial t} - \frac{P}{2} + \mu_e H_0^{2}\,,$$

$$A'_{22} = A_{22} + \mu_e H_0^{2}\,,\; A'_{11} = A_{11} + \mu_e H_0^{2}$$

Equations (9), (10) can be written as

$$\frac{\partial^2 u}{\partial t^2} - \left(\Omega^2 u + 2\Omega \frac{\partial v}{\partial t}\right) = h_{11}\frac{\partial^2 u}{\partial x^2} + h_2 \frac{\partial^2 v}{\partial x \partial y} + h_1 \frac{\partial^2 u}{\partial y^2} - \frac{\gamma}{\rho}\frac{\partial T}{\partial x}, \qquad (12)$$

$$\frac{\partial^2 v}{\partial t^2} - \left(v\,\Omega^2 - 2\Omega \frac{\partial u}{\partial t}\right) = h_{22}\frac{\partial^2 v}{\partial y^2} + h_2 \frac{\partial^2 u}{\partial x \partial y} + h_1 \frac{\partial^2 v}{\partial x^2} - \frac{\gamma}{\rho}\frac{\partial T}{\partial y}, \qquad (13)$$

where,

$$(h_{11}, h_{22}, h_1, h_2) = (A'_{11}, A'_{22}, B'_1, B'_2)/\rho ,$$

$$\varepsilon_1 = T_0 / \rho C_E, \quad \varepsilon_2 = K / \rho C_E, \quad \varepsilon_3 = K^* / \rho C_E .$$

where ε_1 is usually the thermo elastic coupling factor, ε_2 is the characteristic parameter of the G-N theory of type II and ε_3 is the characteristic parameter of the G-N theory of type III.

3 – Solution of the problem

The normal mode analysis gives exact solutions without any assumed restrictions on temperature, displacement and stress distributions. It is applied to a wide range of problems in different branches; see [26-27]. The normal mode analysis is, in fact, to look for the solution in the Fourier transformed domain. Assume that all the field quantities are sufficiently smooth on the real line such that normal mode analysis of these functions exists.

The solution of the considered physical variable can be decomposed in terms of normal modes as the following form:

$$[u, v, \sigma_{ij}](x, y, t) = [u^*(x), v^*(x), \sigma^*_{ij}(x)] \exp(\omega t + i b y), \qquad (14)$$

Let

$$\theta = T - T_0, \theta(x, y, t) = \theta^*(x) e^{\omega t + iby}$$

where ω is a complex time constant, $i = \sqrt{-1}$, b is the wave number in the y- direction, $u^*(x), v^*(x), T^*(x)$ and $\sigma^*_{ij}(x)$ are the amplitudes of the field quantities .

By using Eq.(14) , then Eqs.(11)-(13) take the from

$$[h^*_{11} D^2 - A_1]\, u^* + \left[i\, b\, h^*_2\, D + 2\frac{\rho\omega\,\Omega}{\gamma^*}\right] v^* = D\,\theta^*, \qquad (15)$$

$$\left[-2\frac{\omega\Omega\rho}{\gamma^*} + i\, b\, h^*_2 D\right] u^* + [h^*_1\, D^2 - A_2]\, v^* = i\, b\,\theta^*, \qquad (16)$$

$$A_3 D\, u^* + i\, b\, A_3 v^* = (\varepsilon\, D^2 - A_4)\,\theta^* \quad , \qquad (17)$$

$$\sigma^*_{xx} = A^*_{11} D\, u^* + i\, b\, A^*_{12} v^* - \gamma^* \theta^* - P \qquad (18)$$

$$\sigma^*_{yy} = A^*_{12} D\, u^* + i\, b\, A^*_{22} v^* - \gamma^* \theta^* - P \qquad (19)$$

$$\sigma^*_{zz} = A^*_{12} D\, u^* + i\, bD^*_{\lambda}\, v^* - \gamma^* \theta^* - P \tag{20}$$

$$\sigma^*_{xy} = D^*_{\mu L}\left(i\, b\, u^* + D\, v^*\right) - \frac{P}{2}(Dv^* - ibu^*) \quad , \quad \sigma^*_{xz} = \sigma^*_{yz} = 0\,,$$

$$\sigma^*_{yx} = D^*_{\mu L}\left(i\, b\, u^* + D\, v^*\right) + \frac{P}{2}(Dv^* - ibu^*) \tag{21}$$

where,

$$\left(h^*_{11}\,,\, h^*_{22}\,,\, h^*_1\,,\, h^*_2\right) = \left(A'^*_{11}\,,\, A'^*_{22}\,,\, B^*_1\,,\, B'^*_2\right)/\,\gamma^*\,,$$

$$A'^*_{11} = \lambda_0 + 2\alpha_0 + 4\mu_{L0} - 2\mu_{T0} + \beta_0 + (\lambda_1 + 2\alpha_1 + 4\mu_{L1} - 2\mu_{T1} + \beta_1)\omega + \mu_e H_o^2\,,$$

$$A^*_{12} = \lambda_0 + \alpha_0 + (\lambda_1 + \alpha_1)\omega\,,\, D^*_{\mu T} = \mu_{T0} + \mu_{T1}\omega \quad ,$$

$$A'^*_{22} = \left(\lambda_0 + 2\mu_{T0}\right) + (\lambda_1 + 2\mu_{T1})\omega + \mu_e H_o^2\,,\; B^*_1 = \mu_{L0} + \mu_{L1}\omega + \frac{P}{2}\,,\, D^*_{\mu L} = \mu_{L0} + \mu_{L1}\omega \quad ,$$

$$B'^*_2 = \alpha_0 + \lambda_0 + \mu_{L0} + (\alpha_1 + \lambda_1 + \mu_{L1})\omega - \frac{P}{2} + \mu_e H_o^2\,,$$

$$A_1 = h^*_1 b^2 + (\omega^2 - \Omega^2)\frac{\rho}{\gamma^*}\,, \quad A_2 = h^*_{22} b^2 + (\omega^2 - \Omega^2)\frac{\rho}{\gamma^*}\,, \quad A_3 = \omega^2 \varepsilon_1 \gamma^* \quad ,$$

$$A_4 = \varepsilon\, b^2 + \omega^2\,, \quad \varepsilon = \varepsilon_2 + \varepsilon_3\, \omega\,, \quad D = \frac{d}{dx}\,,\gamma^* = 3\lambda_0 + 2\mu_0 + (3\lambda_1 + 2\mu_1)\omega\,.$$

Eliminating $T^*(x)$ and $v^*(x)$ between equations (15)-(17) we obtain the ordinary differential equation satisfied with $u^*(x)$

$$\left[D^6 - AD^4 + BD^2 - C\right]u^*(x) = 0\,, \tag{22}$$

where,

$$A = \frac{1}{h^*_1 h^*_{11}\varepsilon}\left[h^*_1 A_1 \varepsilon + h^*_1 h^*_{11} A_4 + A_2 h^*_{11}\varepsilon + h^*_1 A_3 - b^2 h^{*2}_2 \varepsilon\right], \tag{23}$$

$$B = \frac{1}{h^*_1 h^*_{11}\varepsilon}\Big[\; h^*_1 A_4 A_1 + h^*_{11} A_2 A_4 + A_1 A_2 \varepsilon + A_2 A_3 + b^2 A_3 h^*_{11} - b^2 h^{*2}_2 A_4 - 2h^*_2 A_3 b^2 + \frac{4\Omega^2 \omega^2 \varepsilon \rho^2}{\gamma^{*2}} \Big], \tag{24}$$

$$C = \frac{1}{h^*_1 h^*_{11}\varepsilon}\left[A_1 A_2 A_4 + A_1 A_3 b^2 + \frac{4\Omega^2 \omega^2 \rho^2}{\gamma^{*2}} A_4\right]. \tag{25}$$

In the similar manner, we can show that $v^*(x)$ satisfy the equation

$$\left[D^6 - AD^4 + BD^2 - C\right]v^*(x) = 0\,. \tag{26}$$

Eq.(22) can be factored as

$$\left(D^2 - k_1^2\right)\left(D^2 - k_2^2\right)\left(D^2 - k_3^2\right)u^*(x) = 0\,. \tag{27}$$

Eq.(26) represent the initial integral equation of six orders, has six roots, i.e.,

$$\left[k^{6} - Ak^{4} + Bk^{2} - C\right] = 0 \tag{28}$$

We have six roots three positive and three negative the positive roots have given an unbounded solution for $u^{*}(x)$ since $x \geq 0$ hence we should suppress the positive exponentials hence the solution of equation (26) has the form:

$$u^{*}(x) = \sum_{n=1}^{3} M_{n} e^{-k_{n}x} \tag{29}$$

And we can get $v^{*}(x)$ from the relation between $u^{*}(x)$ and $v^{*}(x)$:

$$\left[h_{11}^{*}D^{2} - A_{1} + \frac{2\Omega\,\omega\rho}{ib\,\gamma^{*}}D - h_{2}^{*}D^{2}\right]u^{*}(x) = \left[\frac{h_{1}^{*}}{ib}D^{3} - \frac{A_{2}}{ib}D - ibh_{2}^{*}D + \frac{2\omega\Omega\rho}{\gamma^{*}}\right]v^{*}(x) \tag{30}$$

One get:

$$v^{*}(x) = \sum_{n=1}^{3} H_{1n} M_{n} e^{-k_{n}x}, \tag{31}$$

And similarly for equation (15)

$$\theta^{*}(x) = \sum_{n=1}^{3} H_{2n} M_{n} e^{-k_{n}x}, \tag{32}$$

where M_{n} are parameters, $k_{n}^{2}\ (n = 1,2,3)$ are the roots of the characteristic equation (28) and

$$H_{1n} = \frac{ibh_{11}^{*}\gamma^{*}k_{n}^{2} - ib\gamma^{*}A_{1} - ibh_{2}^{*}\gamma^{*}k_{n}^{2} + 2\Omega\,\omega\rho\,k_{n}}{-h_{1}^{*}\gamma^{*}k_{n}^{3} + A_{2}\gamma^{*}k_{n} - b^{2}h_{2}^{*}\gamma^{*}k_{n} + 2i\omega b\rho\Omega}, \tag{33}$$

$$H_{2n} = \frac{A_{1}\gamma^{*} - \gamma^{*}h_{11}^{*}k_{n}^{2} + i\gamma^{*}bh_{2}^{*}k_{n}H_{1n} + 2\omega\Omega\rho H_{1n}}{k_{n}\gamma^{*}}. \tag{34}$$

Using equations (28),(30),(31) into equations (18)-(19) we get the following relations:

$$\sigma_{xx}^{*} = \sum_{n=1}^{3} H_{3n} M_{n} e^{-k_{n}x} - P, \tag{35}$$

$$\sigma_{yy}^{*} = \sum_{n=1}^{3} H_{4n} M_{n} e^{-k_{n}x} - P, \tag{36}$$

$$\sigma_{zz}^{*} = \sum_{n=1}^{3} H_{5n} M_{n} e^{-k_{n}x} - P, \tag{37}$$

$$\sigma_{xy}^{*} = \sum_{n=1}^{3} H_{6n} M_{n} e^{-k_{n}x},\ \sigma_{yx}^{*} = \sum_{n=1}^{3} H_{7n} M_{n} e^{-k_{n}x}, \tag{38}$$

where,

$$H_{3n} = \left(-k_{n}A_{11}^{*} + ibA_{12}^{*}H_{1n} - \gamma^{*}H_{2n}\right), \tag{39}$$

$$H_{4n} = \left(-k_{n}A_{12}^{*} + ibA_{22}^{*}H_{1n} - \gamma^{*}H_{2n}\right), \tag{40}$$

$$H_{5n} = \left(-k_n A_{12}^* + ibD_\lambda^* H_{1n} - \gamma^* H_{2n}\right), \tag{41}$$

$$H_{6n} = \left(ib\,(D^*_{\mu L} + \frac{P}{2}) - (D^*_{\mu L} - \frac{P}{2}) k_n H_{1n} \right) \tag{42}$$

4- The Boundary conditions of the problem

The parameter has to be chosen such that the boundary conditions on the surface at $x = 0$ take the form:

(i) A thermal boundary conditions that the surface of the half-space is

$$\theta(0, y, t) = 0. \tag{43}$$

(ii) A mechanical boundary condition that the surface of the half-space is traction free

$$\sigma_{xx}(0, y, t) = -P,\ \sigma_{xy}(0, y, t) = \sigma_{yx}(0, y, t) = 0. \tag{44}$$

Using the expressions of the variables considered into the above boundary conditions (43) and (44), we get

$$\sum_{n=1}^{3} H_{2n} M_n = 0, \tag{45}$$

$$\sum_{n=1}^{3} H_{3n} M_n = 0, \tag{46}$$

$$\sum_{n=1}^{3} H_{6n} M_n = 0. \tag{47}$$

To determine the constants $M_n, n = 1,2,3,$ it's necessary that the determinant of the constant coefficients must be vanish, i.e.,

$$\begin{vmatrix} H_{21} & H_{22} & H_{23} \\ H_{31} & H_{32} & H_{33} \\ H_{61} & H_{62} & H_{63} \end{vmatrix} = 0 \tag{49}$$

where,

$$\left.\begin{aligned} H_{1n} &= \frac{ibh_{11}^*\gamma^* k_n^2 - ib\gamma^* A_1 - ibh_2^*\gamma^* k_n^2 + 2\Omega\,\omega\rho\,k_n}{-h_1^*\gamma^* k_n^3 + A_2\gamma^* k_n - b^2 h_2^*\gamma^* k_n + 2i\omega b\rho\Omega}, \\ H_{2n} &= \frac{A_1\gamma^* - \gamma^* h_{11}^* k_n^2 + i\gamma^* bh_2^* k_n H_{1n} - 2\omega\Omega\rho H_{1n}}{k_n\gamma^*}, \\ H_{3n} &= \left(-k_n A_{11}^* + ibA_{12}^* H_{1n} - \gamma^* H_{2n}\right), \\ H_{6n} &= \left(ib\,(D^*_{\mu L} + \frac{P}{2}) - (D^*_{\mu L} - \frac{P}{2}) k_n H_{1n} \right) \end{aligned}\right\} \tag{50}$$

Equation (48) determines the Rayleigh surface waves under the influences of the viscosity and rotation in Fiber- reinforced isotropic solid thermo-viscoelastic media, from determining this equation has complex roots. The real part (Re) gives the velocity of Rayleigh waves and the imaginary part (Im) gives the attenuation coefficient. We discuss this case and special cases in Green-Naghdi theory II and III.

i) G-N theory III

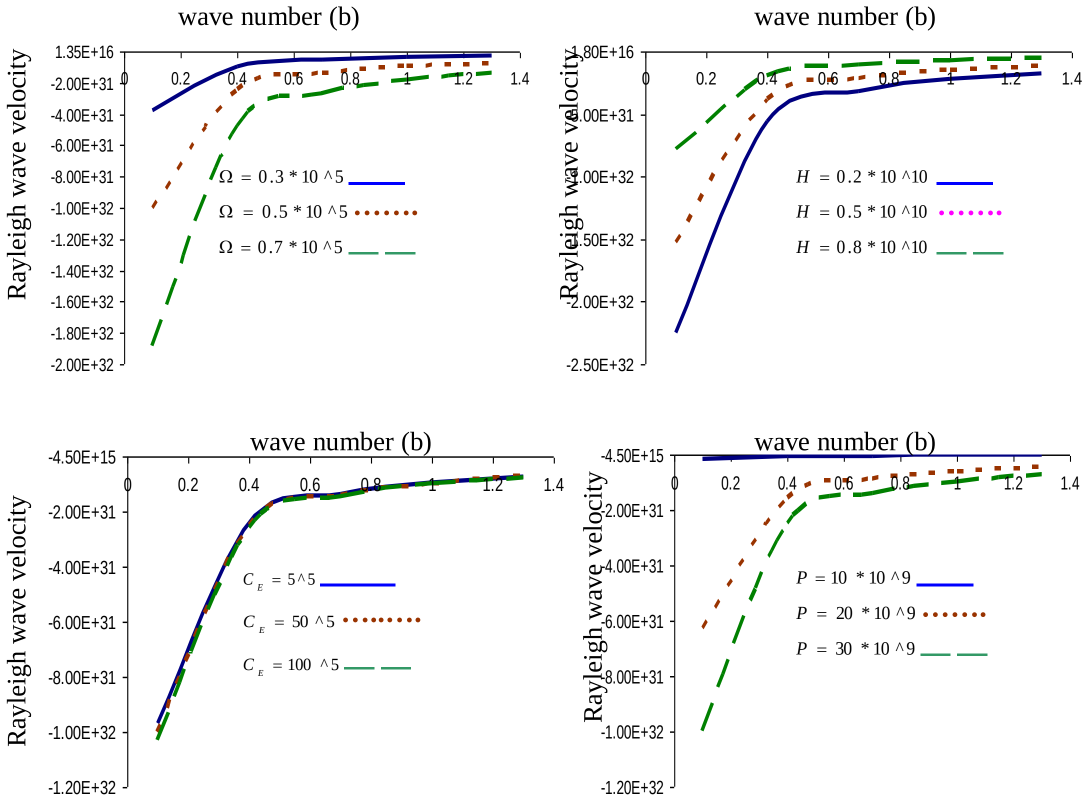
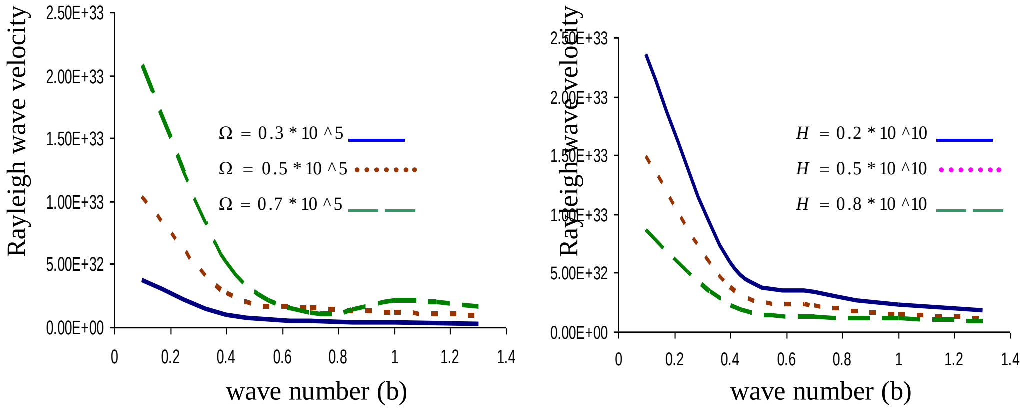


Fig. (1) Rayleigh wave velocity under effects of the rotation Ω, magnetic field H ,specific heat C_E and intial stress P with respect wave number

ii) G-N theory II , i.e $K^* \to 0$



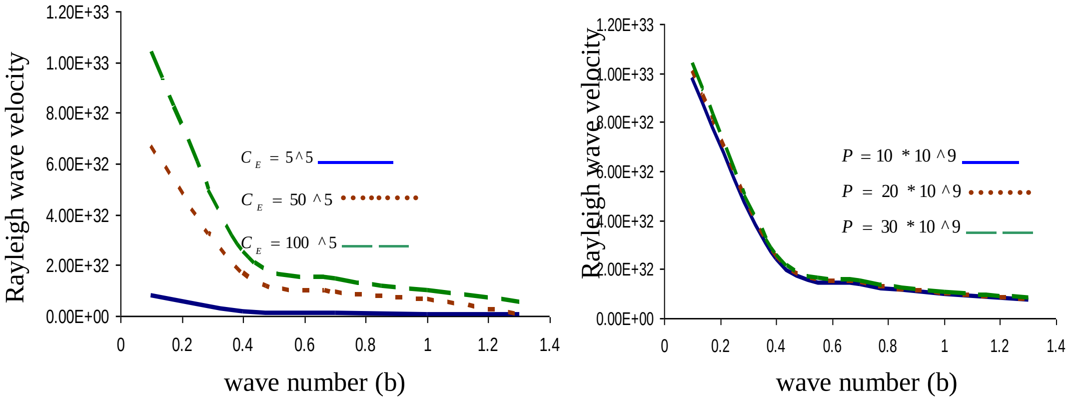


Fig. (2) Rayleigh wave velocity under effects of the rotation Ω, magnetic field H , specific heat C_E and intial stress P with respect wave number

5- Special case

i- If the rotation Ω are neglected:

i) G-N theory III

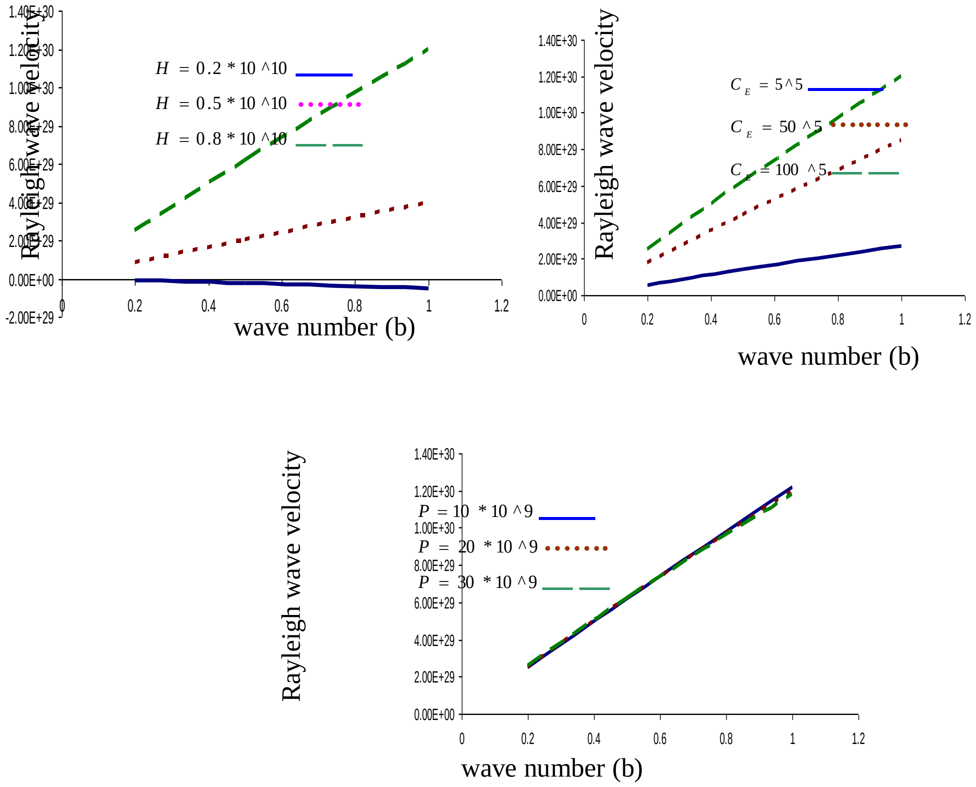


Fig. (3) Rayleigh wave velocity under effects of the magnetic field H , specific heat C_E and intial stress P with respect wave number

ii) G-N theory II , i.e $K^{*} \to 0$

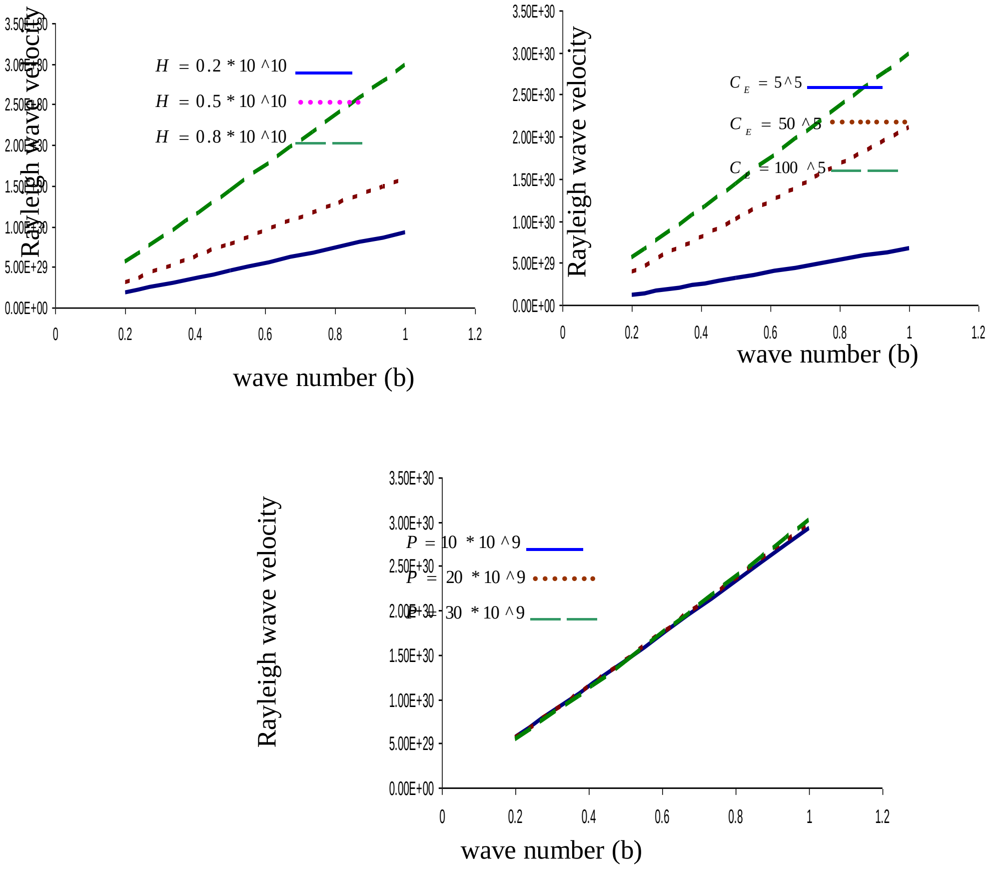
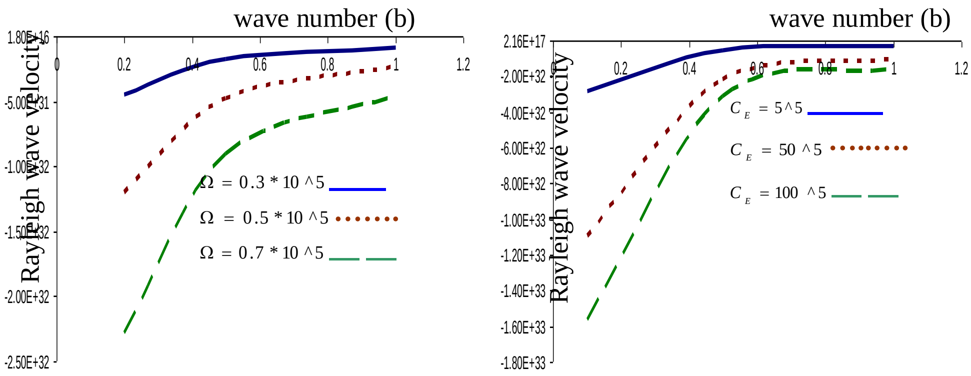


Fig. (4) Rayleigh wave velocity under effects of the magnetic field H , specific heat C_E **and intial stress P with respect wave number**

ii- If the magniticfield Hare neglected:

i) G-N theory III



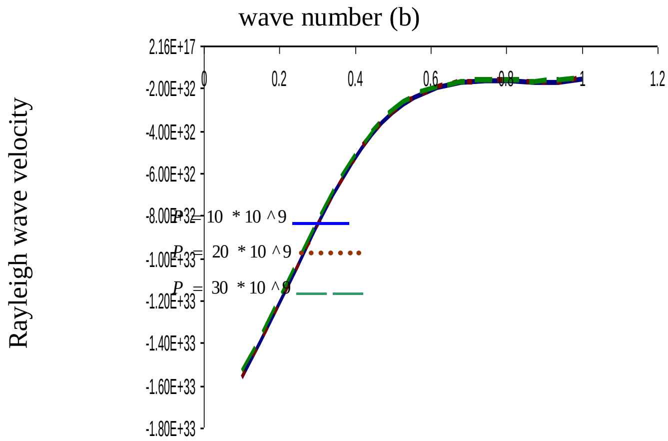


Fig. (5) Rayleigh wave velocity under effects of the rotation Ω , specific heat C_E and intial stress P with respect wave number

ii) G-N theory II , i.e $K^{*} \to 0$

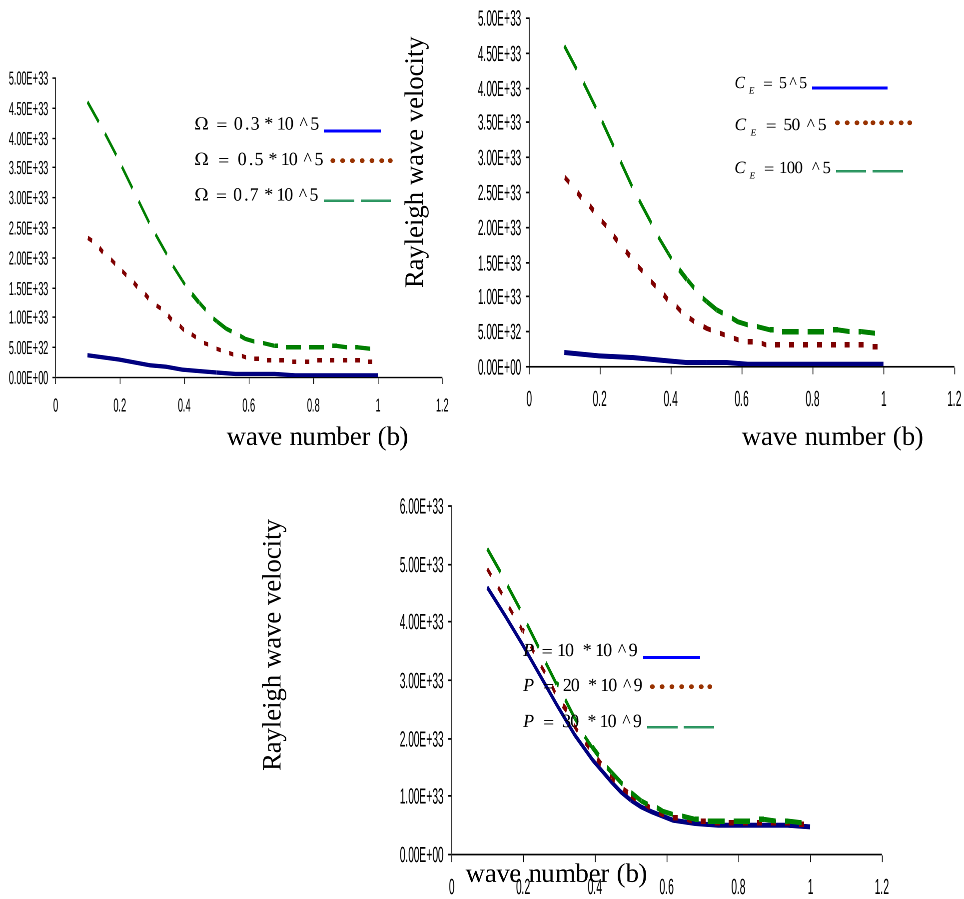


Fig. (6) Rayleigh wave velocity under effects of the rotation Ω , specific heat C_E and intial stress P with respect wave number

6 - Numerical results and discussion.

The purpose of the present study is to promote the wide applications of thermelastic process. The numerical work has been carried out with the help of computer programming using the softwareMobile and physical data for which is given [28].

$\rho = 7800 \ Kg / m^3$, $\lambda_0 = 5.65 \times 10^{10} \ Nm^{-2}$, $\lambda_1 = 2.25 \times 10^4$, $\mu_0 = 2.345 \times 10^{10}$,
$\mu_1 = 0.563 \times 10^{10}$, $\mu_{T0} = 2.46 \times 10^9 \ Nm^{-2}$, $\mu_{L0} = 5.66 \times 10^9 \ Nm^{-2}$, $\mu_{T1} = 2.46 \times 10^{10} \ Nm^{-2}$,
$\mu_{L1} = 5.66 \times 10^{10} \ Nm^{-2}$, $\beta_0 = 220.90 \times 10^9$, $\beta_1 = 220.90 \times 10^{10}$, $\alpha_0 = -1.28 \times 10^9 \ Nm^{-2}$, $\mu_e = 2 \times 10^{-7}$.
$\alpha_1 = -1.28 \times 10^{10}$, $C_E = 100 \times 10^5 \ J.kg^{-1}.K^{-1}$, $K = 10^7 \ w.m^{-1}.K^{-1}$, $K^* = 5 \times 10^4 \ w.m^{-1}.K^{-1}$,
$T_0 = 200 \ K$, $\omega_0 = -0.1$, $\xi = 0.45$, $\Omega = 0.5 \times 10^5$, $H = 0.8 \times 10^9$, $P = 20 \times 10^9$,

The numerical technique, outlined above, used study propagation of Rayleigh waves in Fiber reinforced anisotropic solid thermo-viscoelastic media under the effect of rotation,magenetic field and initial stress.

General case

i) G-N theory III

Fig. 1 show that the variation of the Rayleigh wave velocity with respect to wave number b for different values of rotation Ω, magnetic field H and initial stress P ,specific heat C_E. The Rayleigh wave velocitydecrease with increasing of rotation,initialstressandspecific heat, while it increasing with increasemagnetic field H .

ii) G-N theory II , i.e $K^* \to 0$

Fig. 2 show that the variation of the Rayleigh wave velocity with respect to wave number b for different values of rotation Ω, magnetic field H andinitialstress P ,specific heat C_E. The Rayleigh wave velocity increases with increasing of rotation, initial stressand specific heat, while it decreases with increasingmagnetic field.

<u>Special cases</u>

(i) If the rotation Ω is neglected:

Fig. 3 show that the variation of the Rayleigh wave velocity with respect to wave number b for different values of magnetic field H ,initial stress P and specific heat C_E in the absence of rotation Ω in (G-N theory III). The Rayleigh wave velocity increases with increasing of magnetic field, specific heat and decrease with increasing initial stress P .

Fig. 4 show that the variation of the Rayleig wave velocity with respect to wave number b for different values of magnetic field H ,initial stress P and specific heat C_E in the absence of rotation Ω in (G-N theory II). The Rayleigh wave velocity increase with increasing of magnetic field,initialstressand specific heat.

(ii) If the magnetic field is neglected:

Fig. 5 show that the variation of the Rayleig wave velocity with respect to wave number b , for different values of rotation Ω ,initial stress P and specific heat C_E in the absenceofmagnetic field H .in (G-N theory III) The Rayleigh wave velocity decrease with increasing ofrotation, specific heat while it increase with increasinginitialstress.

Fig. 6 show that the variation of the Rayleig wave velocity with respect to wave number b , for different values of rotation Ω ,initial stress P and specific heat C_E in the absenceofmagnetic field H .in (G-N theory II) The Rayleigh wave velocity increase with increasing ofrotation, specific heat ,initial stress.

7. Conclusion

The analysis of graphs permits us some concluding remarks

1. The Rayleigh wave velocity in a homogeneous, anisotropic, fibre-reinforced viscoelastic solid media under the effect of rotation,initialstress and magnetic field are investigated.
2. Rayleigh waves in a homogeneous, general thermo viscoelastic solid medium, we find that the wave velocity equation, proves that there is a dispersion of waves due to the presence of rotation, magnetic field and specific heat. The results are in complete agreement with the corresponding classical results in the absence of all fields.
3. ·The results presented in this paper will be very helpful for researchers in geophysics, designers of new materials and the study of the phenomenon of rotation and magnetic field is also used to improve the conditions of oil extractions.

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