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Economic Development Process: A Compartmental Analysis of a Macroeconomic Model.

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Abstract

Economic growth and development have always been an interesting topic for economists. Since development is a dynamical process, we developed a new dynamical mathematical model describing the process of economic development. In this paper the compartmental approach, traditionally used in mathematical biology, is applied to built a macroeconomic model characterized by countries. Countries are divided into three classes based on their economic status: D(t) developing countries at time t, E(t) emerging countries at time t and A(t) advanced countries at time t. This minimalist model represents the different stage of development and encompasses the conditions under which a country can change its economic status. Numerical simulations on real economic data are provided in order to illustrate the theoretical part and to support discussion. On the other hand, a global sensitivity analysis is done to identify the factors that most influence the level of development of countries.

Keywords: Compartmental systems; economic development; dynamical system; global sensitivity analysis; fiscal policy.

1. Introduction

Economic growth and development have always been an interesting topic for economists. Reasons for such interests are simply due to the fact that economic growth is variable among nations. Generally, economic growth is a result of greater quantity and better quality of natural, human, capital resources, and also technological advances that boosts productivity. Recall that economic development is the process by which a nation enhances its standard of living (Chen, 2006)[6]. However economic development is not a random phenomenon, it is greatly influenced by the policies and attitudes of the governments, as well as by the international environment.

Several models of economic growth and development have been developed to give some insights to economic development problems, but the question of development remains a major concern, especially for African countries that are involved on programs to achieve the emergence on a given horizon. These models are based on econometric concepts, mathematical optimization models, and so on.

Since development is a dynamical process, it can be modeled by dynamical systems, and compartment models are among the most popular tools used to analyze dynamical systems. Compartmental models have been traditionally used in physiology to describe the distribution of a substance among different tissues of an organism. It has been extensively used in chemistry, medicine, epidemiology, ecology, and pharmacokinetics. Many of the models and methods developed in these fields can be usefully applied, by analogy, in the description of economical and social systems. There are not, however, so many applications in economic literature. Among these works, Fabio T., 2010 [2], used the compartmental approach applied to a macroeconomic model characterized by companies that can go bankrupt each time they are unable to repay their debts. Marc A. et al., 2013[5] developed a model composed of two compartments, Government and Company, with cash flows between the compartments and the outside world.

The objective of this work is to build a dynamical compartmental model describing the process of growth and economic development. For that, we consider three compartments represented by countries based on their economic status: D (t) developing countries at time t, E (t) emerging countries at time t and A (t) advanced countries at time t (that we call as DEA model). Then we rely on the properties of dynamical systems for model analysis. We prove the global stability of the coexistence equilibrium, which shows the interdependence of the world economy. Numerical simulations on real economics data further illustrate these analytical results. Finally, a global sensitivity analysis is done to see which factors influence more on the level of development of a country. The paper is organized as follows: we start by introducing the economic model in section 2 and proceed to section 3 where we describe the mathematical analysis of the model. In section 4 global sensitivity analysis is done. We conclude in Section 5 with possible extensions.

2. The DEA Model

Our model is composed of three compartments characterized by countries according to the stage of development of their economies. We use the World Economic Forum (WEF) classification in the Global Competitiveness Report (Schwab et al., 2016)[3] to represent the compartments. Based on this classification, we consider a first compartment named Developing D represented by countries whose Gross Domestic Product (GDP) per capita is less than 3,000 Dollars and characterized by four pillars of competitiveness that are: Institutions, Infrastructure, Macroeconomic stability, and Health and primary education. These pillars are the basic requirements for developing countries.

In the second compartment, we have Emerging Countries E. These countries have a GDP per capita between 3,000 Dollars and 17,000 Dollars and characterized by four pillars of competitiveness that are: Higher education and training, Goods market efficiency, Labor market efficiency, Financial market sophistication, Technological readiness and Market size. These pillars represent the efficiency enhancers that characterize Emerging Countries.

The advanced countries A are in the third compartment with a GDP per capita of more than 17000 Dollars and characterized by two pillars: Business sophistication and Innovation.

For a developing country to be able to emerge, it must significantly improve its growth rate by acting on the efficiency factors. Thus, it will be able to increase its GDP per capita and be in the class of emerging economies. From the WEF equation of competitiveness (Schwab et al., 2007)[4], we consider the function $Y(\sigma_i) = \alpha_1 \sum_i \sigma_i$ where σ_i, i=1,2, …6, are the performances made on the efficiency factors, and α_1 define the stage of development. To reach the advanced countries compartment, emerging countries must increase their GDP per capita, by acting on the innovation and sophistication factors. Let $Z(\delta_i) = \alpha_2 \sum_i \delta_i$ where $\delta_i, i = 1,2$ are the innovation and sophistication factors.

Another fact that unfortunately affects economies is the economic crisis. Countries are fragile to economic and financial crises. Although the origin of the crises begins in the major financial centers of the developed countries, we can see how it also affected the developing and emerging countries (Bruno, 2010)[1]. Thus, we consider τ_1 and τ_2 to be the probabilities of economic crisis that can weaken the emerging and advanced countries respectively and make them change their economic status.

The process of economic development is illustrated in the figure (1).

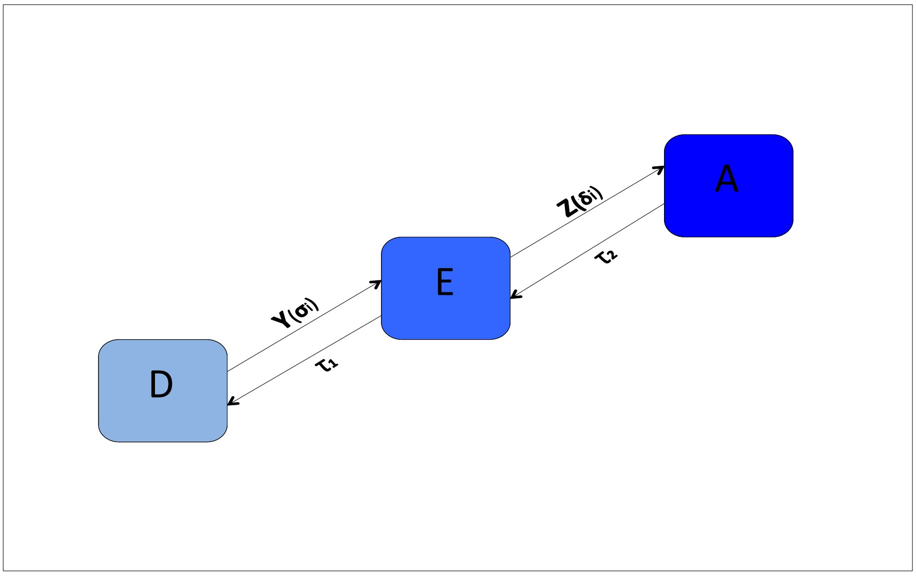


Figure 1: The Process of Economic Development.

The model is given analytically by the following system of ordinary differential equations:

$$\begin{cases} \dot{D}(t) = \tau_1 E - Y(\sigma_i)D, \\ E(\dot{t}) = Y(\sigma_i)D + \tau_2 A - (Z(\delta_i) + \tau_1)E, \\ \dot{A}(t) = Z(\delta_i)E - \tau_2 A. \end{cases} \quad (1)$$

Now we present the analysis of model (1) by using theories of ordinary differential equations.

3. Analysis of the model

3.1 Positivity and boundedness of solutions

The first step in understanding system is to show that solutions of the system remain nonnegative and are bounded, so that system is economically meaningful. We are interested only in the dynamics of system (1) in the first quadrant $\mathbb{R}^+$.

Lemma 3.1. Solutions of system (1) are nonnegative and bounded.

Proof. Since $\dot{D}|_{D=0} > 0$, $\dot{E}|_{E=0} > 0$ and$\dot{A}|_{A=0} > 0$ solutions of system (1) remain nonnegative for $t \geq 0$. Furthermore, $\dot{D} + \dot{E} + \dot{A} = 0$, thus $D(t) + E(t) + A(t) = N$, where N is the total number of countries. Then all solutions of system are uniformly bounded.■

Since $\dot{D} + \dot{E} + \dot{A} = 0$ (the total number of countries is constant), system (1) is conservative. We can study the system (1) using only the first two equations, and A can be computed by means of the following equation: $A = N - D - E$. The system (1) becomes:

$$\begin{cases} \dot{D}(t) = \tau_1 E - Y(\sigma_i)D, \\ E(\dot{t}) = (Y(\sigma_i) - \tau_2)D + \tau_2 N - (Z(\delta_i) + \tau_1 + \tau_2)E. \end{cases} \quad (2)$$

In the following we will consider system (2).

3.2 Equilibium and stability

Equilibrium points of system (2) correspond to solutions of the algebraic system$\dot{D} = \dot{E} = 0$.

Proposition 3.2 The system (2) has a unique equilibrium (D^*, E^*) with,

$$D^* = \frac{\tau_1 \tau_2 N}{Y(\sigma_i)Z(\delta_i) + \tau_1 Y(\sigma_i) + \tau_1 \tau_2},$$

$$E^* = \frac{\tau_2 N Y(\sigma_i)}{Y(\sigma_i)Z(\delta_i) + \tau_1 Y(\sigma_i) + \tau_1 \tau_2}.$$

Remarks 1. This equilibrium characterizes the number of countries in the compartments D and E.

Theorem 3.3. The equilibrium (D^*, E^*) is locally asymptotically stable.

Proof. The Jacobian matrix of the system (2) is defined by

$$\boldsymbol{J} = \begin{pmatrix} -Y(\sigma_i) & \tau_1 \\ Y(\sigma_i) - \tau_2 & -(Z(\delta_i) + \tau_1 + \tau_2) \end{pmatrix}.$$

The characteristic polynomial is given by: $P(\lambda) = \lambda^2 - Trace\, J + Det\, J$, with

$$Trace\, J = -Y(\sigma_i) - (Z(\delta_i) + \tau_1 + \tau_2),$$
$$Det\, J = Y(\sigma_i)\, Z(\delta_i) + \tau_2 Y(\sigma_i) + \tau_1 \tau_2.$$

Since $Trace\, J < 0$ and $Det\, J > 0$, then the theorem is proved. ■

Theorem 3.4. The equilibrium (D^*, E^*) is globally asymptotically stable.
Proof. Let $\dot{D}(t) = f\ (D, E)$ and $\dot{E}(t) = g\ (D, E)$. We have:

$$\frac{\partial \mathrm{f}}{\partial \mathrm{D}} = -Y(\sigma_i),$$

$$\frac{\partial \mathrm{g}}{\partial \mathrm{E}} = -(Z(\delta_i) + \tau_1 + \tau_2).$$

Then $\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{D}} + \frac{\partial \mathrm{g}}{\partial \mathrm{E}} < 0$, according to the Dulac's criterion, the system (2) does not admit a limit cycle. And since the system is locally stable then it is globally stable. ■

Remarks 2. Stability allows us to move from an initial condition (D_0, E_0) to a new equilibrium state (D_1, E_1). Then it is possible to act on the parameters such that the number of developing countries decreases (D_0>D_1,), while that of emerging countries increases (E_0<E_1) as long as the system is stable.

Lemma 3.5. All the properties of system (2) is also valid for system (1) with the equilibrium point (D^*, E^*, A^*) where,

$$A^* = \frac{NY(\sigma_i)Z(\delta_i)}{Y(\sigma_i)Z(\delta_i) + \tau_1 Y(\sigma_i) + \tau_1\tau_2}.$$

In what follows, we present numerical simulation to illustrate the above results obtained.

3.3 Sumulations and discussion

In this section, we present the numerical results that illustrate analytical solutions. We observe that the system admits a single point of equilibrium and for several initial conditions the system converges towards this stable equilibrium state, see fig (2)(a), (b). This equilibrium represents the number of countries in each compartment at each moment t. By varying our parameters, we have observed that the number of countries in the compartments is evolving simultaneously towards stable equilibrium. Depending on the compartment, some functions act the most:

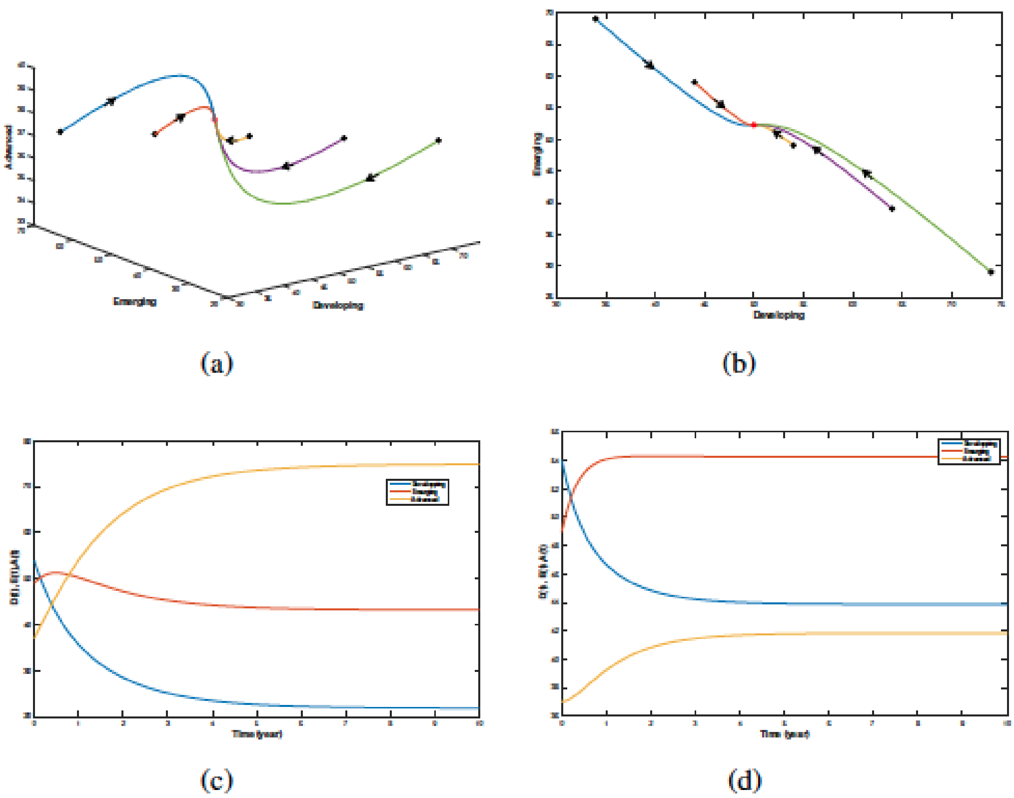


Figure 2. Plot of trajectories of system (1) using WEF data. Solutions converge to (50, 53, 37) for different initial conditions (Fig.a). Plot of trajectories of system (2) solutions converge to (50,53) (Fig.b). Fig. (c) gibes the trajectories in case when the

economic crises are minimal, developing countries towards zero and Fig. (d) for high values of economics crisis, developing countries increasing.

- The product $\tau_1\tau_2$ of the probability of economic crises, which can also mean underperformance,determines the number of countries in the developing countries compartment.Larger is this quantity, higher is the number of developing countries (fig(2)(c), (d)). This result confirms Bruno's work on the impact of economic crises for developing countries [1].
- The variables τ_2 and $Y(\sigma_i)$ characterize the number of emerging countries. So that to become emergent, developing countries will not only have to act on efficiency factors, but also control economic crises.
- Similarly with the WEF, a good improvement of the factors of efficiency and innovation $Y(\sigma_i)Z(\delta_i)$ can maximize the number of advanced countries.

This study shows that developing countries can change their economic status, but continue to be influenced by the international environment, especially the major financial centers, which are at the origin of economic crises. This interdependence, while advantageous, is also an obstacle for developing countries, which are much more affected by economic crises. Therefore, a good strategy would be to limit this dependence until it reaches a considerable level of growth in order to cope with economic crises.

Global analysis is performed in the next section in order to find relevant parameters to reach emerging and advanced countries.

4. Global sensitivity analysis

A possible definition of sensitivity analysis is the following: The study of how uncertainty in the output of a model (numerical or otherwise) can be apportioned to different sources of uncertainty in the model input (Saltelli et al., 2004)[7].

The objective of sensitivity analysis is to identify critical inputs (parameters and initial conditions) of a model and quantify how input uncertainty impacts models outcome(s). Sensitivity analysis can serve a number of useful purposes in the economy of modeling. It can surprise the analyst, uncover technical errors in the model, identify critical regions in the space of the inputs, establish priorities for research, simplify models and defend against falsifications of the analysis.

The local sensitivity analysis investigates the impact on model output, based on changes in factors only very close to the nominal values, while the global sensitivity analysis are implemented using Monte Carlo (MC) simulations.

We use the methodology for performing global uncertainty and sensitivity analysis developed by Marino et al. (2008) [8] and we apply it in our mathematical model as follows:

$$\left.\begin{array}{l}\dot{C} = g(C,\theta), C \equiv (D,E,A) \in \mathbb{R}^3 \\ \theta \equiv (\tau_1,\tau_2,\sigma_1,\sigma_2,\sigma_3,\sigma_4,\sigma_5,\sigma_6,\delta_1,\delta_2) \in \mathbb{R}^{10}\end{array}\right. \longrightarrow Y = f(C,\theta).$$

For each parameter, we do a sampling using a probability density function (pdf). In our case we choose an uniform distribution (assigning some hypothetical, but large range with minimum and maximum values for the parameters). To recreate input factor distributions through sampling, a large number of samples are likely required.

We use the Latin hypercube sampling (LHS), which belongs to the MC class of sampling method. The random parameter distributions are divided into N equal probability intervals, which are then sampled. N represents the sample size.

LHS is performed following the scheme:

$$\begin{cases}\tau_i \sim Unif\{1e^{-5},1\}, \\ (\sigma_i,\delta_i) \sim Unif\{1,7\}.\end{cases}$$

The LHS method assumes that the sampling is performed independently for each parameter. The sampling is done by randomly selecting values from each pdf. A matrix is generated (which we call the LHS matrix) that consists of

N rows for the number of simulations (sample size) and of k = 10 columns corresponding to the number of varied parameters. N model solutions are then simulated, using each combination of parameter values.

LHS MATRIX OUTPUT MATRIX

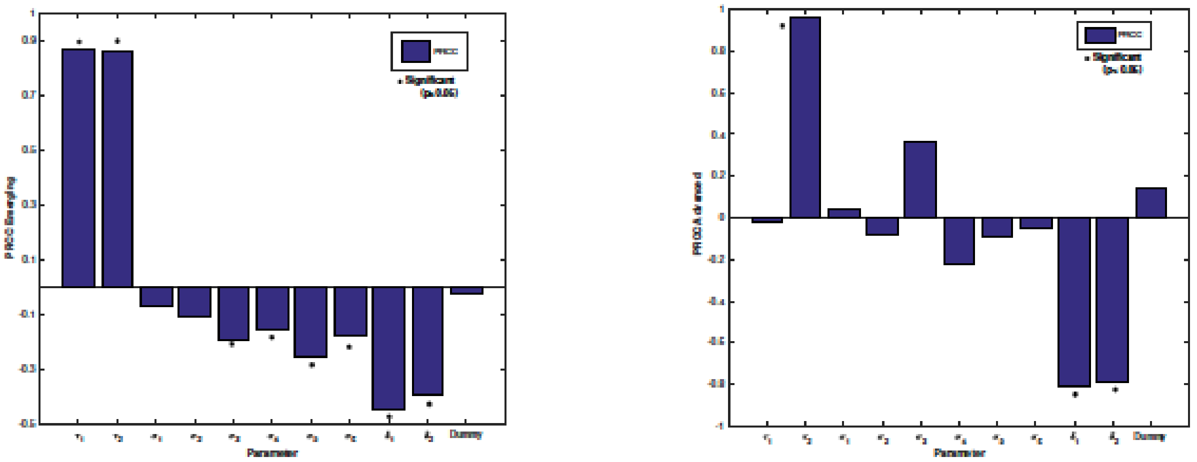
$$\begin{pmatrix} \tau_{11} & \tau_{21} & \sigma_{21} & \cdots & \delta_{21} \\ \vdots & & \ddots & & \vdots \\ \tau_{1N} & \tau_{2N} & \sigma_{2N} & \cdots & \delta_{2N} \end{pmatrix} \begin{pmatrix} y_1 = f(\tau_{11}, \tau_{21}, \cdots, \delta_{21}) \\ \vdots \\ y_N = f(\tau_{1N}, \tau_{2N}, \cdots, \delta_{2N}) \end{pmatrix}$$

The hypothetical model $C = g^{\cdot}(C, \theta)$ is then solved, the corresponding output generated, an stored in the matrix Y. The rank-transformed LHS MATRIX and output matrix (Y) are used to calculate the partial rank correlation coefficient (PRCC).

We use the following data from World Economic forum in the Report Global of competitiveness 2016 to analyze relevant parameters.

Parameters	Description	Range	Baseline
τ_1	Probability of economic crisis from emerging countries	$[1e^{-5}, 1]$	0.3
τ_2	Probability of economic crisis from advanced countries	$[1e^{-5}, 1]$	0.4
σ_1	Higher education and training	[1 ,7]	4.90
σ_2	Goods market efficiency	[1 ,7]	4.74
σ_3	Labor market efficiency	[1 ,7]	3.38
σ_4	Financial market development	[1 ,7]	3.81
σ_5	Technological readiness	[1 ,7]	4.15
σ_6	Market size	[1 ,7]	5.38
δ_1	Business sophistication	[1 ,7]	3.95
δ_2	Innovation	[1 ,7]	3.30

Figure (3) shows the results of PRCC with the most significant parameters:



(a) (b)

Figure 3. (a) PRCC for emerging countries, (b) PRCC for advanced countries. Sample size N=500. (*) denotes PRCCS that are significantly different from zero.

- For emerging countries all parameters are relevant but the most influential are economic crisis, labor market efficiency, technological readiness, business sophistication and innovation. Therefore, in order to become emergent, developing countries in particular need to focus on these parameters.
- Relevant parameters for Advanced Countries are economic crisis, business sophistication and innovation. Emerging countries must control economic crisis and at the same time act on the factors of innovation to reach the status of advanced countries.

5. Conclusion

In this paper, we described a process of economic development using a compartmental approach. The model, although minimalist, shows a global stability of the equilibrium that reflects the interdependence of the world economy. This model indicates the great influence of economic crisis on the stage of development of countries Based on the global sensitivity analysis, to reach emerging countries, developing countries should focus on the labor market efficiency and technological readiness and try to limit their dependence on advanced countries to reduce probabilities of economic crisis. There are several ways however in which the model can be extended. The model can be extended by using the openness rate, which plays an important role in development, and by highlighting the nature of relations between countries.

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